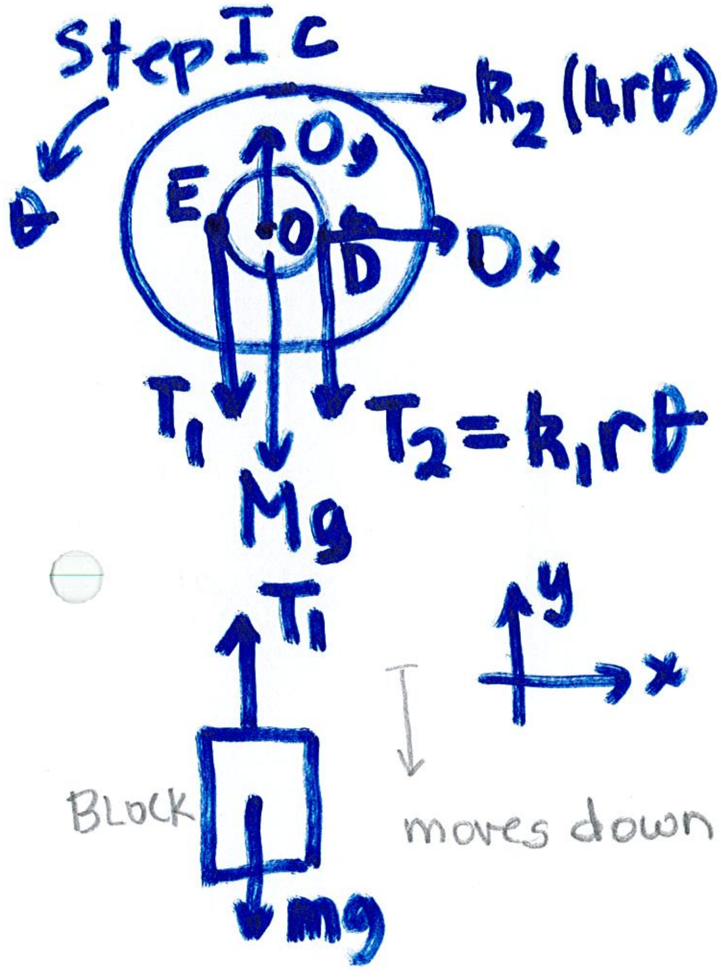
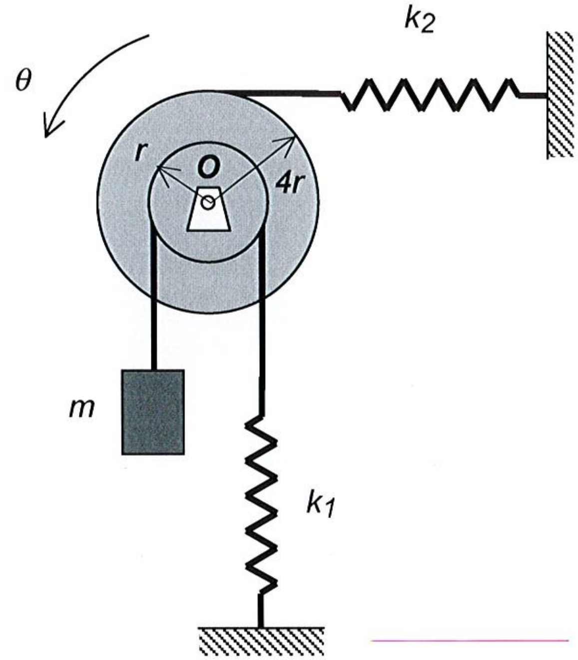
Example 6.B.5

Given: The pulley has a mass moment of inertia of I_O.

Find:
(a) Determine the EOM of this system in terms of the coordinate θ; and
(b) Determine the natural frequency of the system.





Step II $\quad \Sigma M_O = I_O \alpha \qquad \alpha = \ddot{\theta}$

$$-k_2(4r\theta)4r - \underbrace{(k_1 r\theta)}_{T_2} r + T_1 r = I_O\ddot{\theta} \qquad (1)$$

BLOCK $\quad T_1 - mg = -m a_E$ ← assuming downwards $\quad (2)$

Let's eliminate T_1: $\quad T_1 r = mgr - m a_E r$

Hence

$$\left[-16r^2k_2 - k_1 r^2\right]\theta + mgr - mra_E = I_0\ddot{\theta}$$

$$\therefore \quad I_0\ddot{\theta} + mra_E + (16r^2k_2 + k_1r^2)\theta = mgr \quad (**)$$

Step III Relate a_E (which is downwards) to $\alpha = \ddot{\theta}$

$$\bar{a}_E = \cancel{\bar{a}_0} + \ddot{\theta}\hat{k} \times \bar{r}_{E|O} - \omega^2 \bar{r}_{E|O}$$

$$a_{E_x}\hat{i} - a_{E_y}\hat{j} = \ddot{\theta}\hat{k} \times (-r\hat{i}) + \omega^2 r\hat{i}$$

$$\hat{j}: -a_{E_y} = -r\ddot{\theta}\hat{j} \longrightarrow a_{E_y} = r\ddot{\theta}$$

(*)

Step IV Solve sub (*) into (**) and collect $\theta, \dot{\theta}, \ddot{\theta}$ terms on LHS

$$(I_0 + mr^2)\ddot{\theta} + (16k_2 + k_1)r^2\theta = mgr$$

EOM (a)

$$\ddot{\theta} + \left(\frac{16k_2 + k_1}{I_0 + mr^2}\right)r^2\theta = \frac{mgr}{(I_0 + mr^2)}$$

(b)

$$\omega_n = \sqrt{\frac{r^2(16k_2 + k_1)}{I_0 + mr^2}} \text{ rad/s} \longrightarrow$$

Note: No damping in this system and so ~~because~~ there is no $\dot{\theta}$ term, and hence $\zeta = 0$.