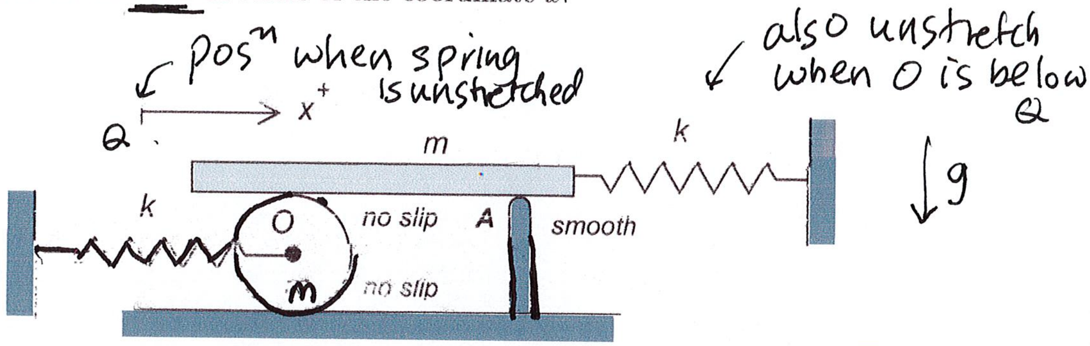
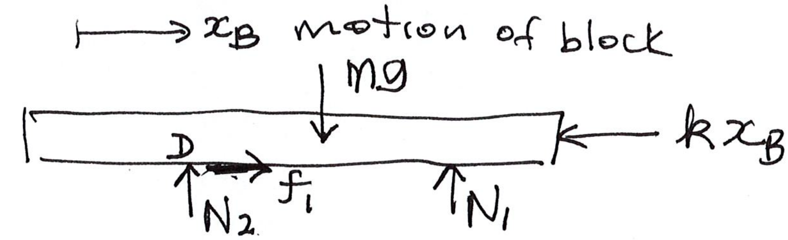
Example 6.A.3

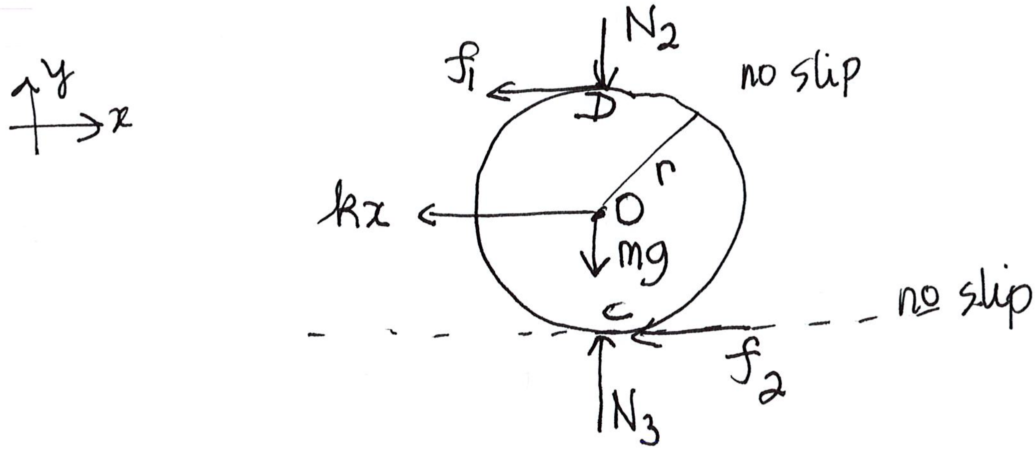
Given: A homogeneous disk of mass of m and radius r rolls without slipping on a rough horizontal surface. A spring, having a stiffness of k, is attached between the disk center O and ground, as shown below. A block, also of mass m, is in *no-slip* contact with the top surface of the disk and with a smooth vertical support at A. A second spring of stiffness k is connected between the block and ground. Let x describe the position of O, where the springs are unstretched when $x = 0$.

Find: Determine the EOM for the disk in terms of the coordinate x.



Step I
BLOCK





Step II

DISK $\quad \Sigma M_c = I_c \alpha = \left(\frac{1}{2}mr^2 + mr^2\right)\alpha = \frac{3mr^2}{2}\alpha$

$$kxr + f_1 \cdot 2r = \frac{3mr^2}{2}\alpha \qquad (1)$$

BLOCK $\quad \Sigma F_x = m\ddot{x}_B \Rightarrow \quad f_1 - kx_B = m\ddot{x}_B \qquad (2)$

Step III Use kinematics to express $\ddot{x}_B$ and α in terms of $\ddot{x}$ and x_B in terms of x.

$$\bar{a}_D = \bar{a}_C + \bar{\alpha} \times \bar{r}_{D/C} - \omega^2 \bar{r}_{D/C}$$

$$a_{Dx}\hat{i} + a_{Dy}\hat{j} = a_C\hat{j} + \alpha\hat{k} \times (2r\hat{j}) - \omega^2 2r\hat{j}$$

$\hat{i}$: $a_{Dx} = -2r\alpha \longrightarrow \alpha = \dfrac{-\ddot{x}_B}{2r} = -\dfrac{\ddot{x}}{r}$ (3)

and note $a_{Dx} = \ddot{x}_B$ for no slip.

Similarly $\bar{a}_O = a_O\hat{i} = \ddot{x}\hat{i}$ moves in a straight line.

$$\ddot{x} = -r\alpha \quad \left(= \tfrac{1}{2}\ddot{x}_B\right) \qquad (4)$$

Also $x_B = 2x$ (5)
moves twice as far as the center of disk.

Step IV Solve. Use (3), (4) and (5) in (1) & (2)

(1a): $\dfrac{kx}{2} + f_1 = \dfrac{3}{4} m \cancel{r} \left(-\dfrac{\ddot{x}}{\cancel{r}}\right)$

(2a): $-k(2x) + f_1 = m 2\ddot{x}$

Eliminate f_1 (1a) − (2a): $2.5kx = -\left(\dfrac{3}{4}m + 2m\right)\ddot{x}$

Rearrange with all the x terms on L.H.S. $\dfrac{11m\ddot{x}}{4} + \dfrac{5kx}{2} = 0$

$$11m\ddot{x} + 10kx = 0 \quad \text{ANS}$$