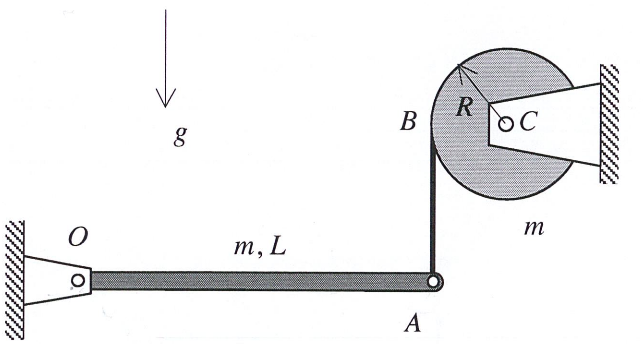
ME 274 – Summer 2010
Examination No. 2
PROBLEM NO. 3

Name ________________________________

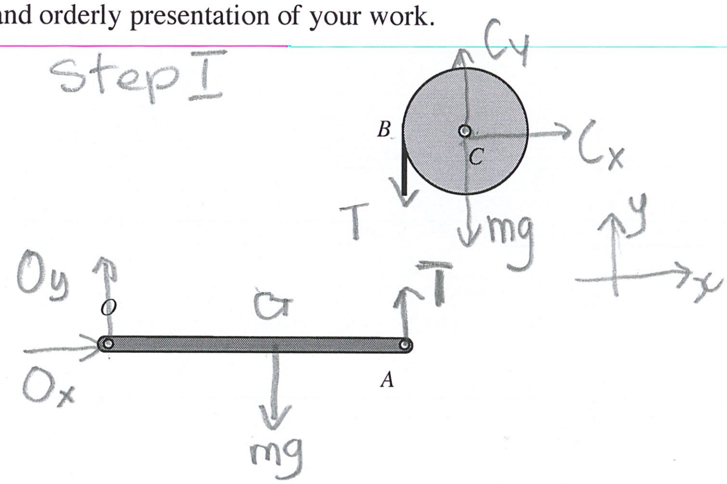
Given: Homogeneous bar OA (of length L and mass m) is pinned to ground at end O. End A of the bar is connected to a cable that is wrapped around a homogeneous disk (of mass m and outer radius R), with the disk being pinned to ground at its center C. Assume that the cable does not slip on the disk. The system is released from rest with OA being horizontal and the cable being vertical.

Find: Determine the *angular acceleration of the disk* on release. Use the following: $m = 10kg$, $L = 4\,meters$ and $R = 2\,meters$.

Please clearly indicate the four steps in a neat and orderly presentation of your work.



Step I



Step II — Kinetics

$$\Sigma M_{O_{BAR}} = I_O \alpha_{BAR} \qquad I_O = I_G + m\left(\frac{L}{2}\right)^2 = \frac{1}{12}mL^2 + m\frac{L^2}{4} = \frac{1}{3}mL^2$$

CCW +ve

$$T \cdot \cancel{L} - mg\frac{\cancel{L}}{2} = \frac{1}{3}mL^{\cancel{2}}\alpha_{BAR} \qquad (1)$$

DISK

$$\Sigma M_C = I_B \alpha_{DISK} = \frac{1}{2}mR^2\alpha_{DISK}$$

$$TR = \frac{1}{2}mR^2\alpha_{DISK} \qquad (2)$$

(1) × R − (2) (eliminate T for the equations)

$$-\frac{mgR}{2} = \frac{1}{3}mLR\alpha_{BAR} - \frac{1}{2}mR^2\alpha_{DISK} \quad — (3)$$

step [II] Kinematics — want to relate α_{DISK} to α_{BAR}

Point A and Point A are the connecting points. $a_{By} = a_{Ay}$.

DISK

$$\bar{a}_B = \cancel{\bar{a}_C} + \bar{\alpha}_{DISK} \times \bar{r}_{B|C} - \cancel{\omega^2_{DISK}} \bar{r}_{B|C}$$

$\cancel{\bar{a}_C} \to 0$ fixed point

$\cancel{\omega^2_{DISK}} \to 0$ released from rest.

$$a_{Bx}\hat{i} + a_{By}\hat{j} = \alpha_{DISK}\hat{k} \times (-R\hat{i}) =$$

$$= -\alpha_{DISK} R\hat{j}$$

At this instant. $a_{Bx} = 0$

$$a_{By} = -\alpha_{DISK} R. \qquad (4)$$

Do same with the Bar. $\bar{a}_A = \bar{\alpha}_{BAR} \times \bar{r}_{A|O} = +\alpha_{BAR}\hat{k} \times L\hat{i}$

$$a_{Ay} = +\alpha_{BAR} \cdot L \rightarrow \qquad (5)$$

$\therefore$ (4) & (5) $\rightarrow$ $\alpha_{BAR} L = -\alpha_{DISK} \cdot R$

$$\alpha_{BAR} = -\frac{\alpha_{DISK} R}{L} \qquad (6)$$

Note: a_{By} and a_{Ay} must be equal (rope does not stretch)

If you assumed α_{DISK} was clockwise then this would be a plus sign.

Here the −sign tells us that the bar and disk are rotating in opposite directions: BAR CW, DISK CCW

Step IV Solve

Substitute 6 → 3

$$-mg\frac{R}{2} = \frac{1}{3}mR\cancel{L}\left(-\frac{\alpha_{DISK}R}{\cancel{L}}\right) - \frac{1}{2}mR^2\alpha_{DISK}$$

$$\cancel{m}g\frac{\cancel{R}}{2} = \left[\frac{1}{3}\cancel{m}R^{\cancel{2}} + \frac{1}{2}\cancel{m}R^{\cancel{2}}\right]\alpha_{DISK}$$

$$\alpha_{DISK} = \frac{3g}{5R}\ \text{rad/s}^2$$

$$\bar{\alpha}_{DISK} = \left(\frac{3g}{5R}\right)\hat{k}\ \text{rad/s}^2$$