Name (print) SOLUTION **ME 274: Basic Mechanics II**

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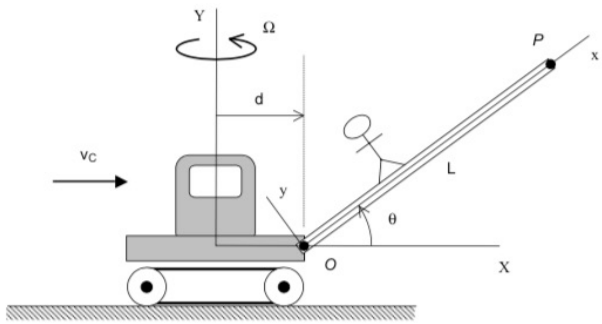
NOTE: *You are NOT asked to provide justification for your answers here in Problem 1. A correct response will receive full credit. Any work provided will not be graded, only the final answer.*



$$\hat{I} = \cos\theta\,\hat{i} - \sin\theta\,\hat{j}$$
$$\hat{J} = \sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

PROBLEM NO. 1 - PART A

A crane moves along a flat, horizontal surface with a constant speed of v_C. The cab of the crane rotates about a fixed, vertical axis at a *constant* rate of Ω while the boom OP is being raised at a *constant* rate of $\dot{\theta}$. The acceleration of point P at the end of the boom is to be written using the following moving reference frame equation, where *the observer for this equation is attached to the boom*:

$$\vec{a}_P = \vec{a}_O + \left(\vec{a}_{P/O}\right)_{rel} + \vec{\alpha}\times\vec{r}_{P/O} + 2\vec{\omega}\times\left(\vec{v}_{P/O}\right)_{rel} + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}_{P/O}\right)$$

Provide expressions for the following terms in the above equation. Write your answers *in terms of their xyz-components* and leave the answers in terms of, *at most*: Ω, θ, $\dot{\theta}$, d and L.

1.A.1: 2 points – $\vec{\alpha} =$ $\vec{\omega} = \Omega\hat{J} + \dot{\theta}\hat{k}$

$$\vec{\alpha} = \cancel{\dot{\Omega}}\hat{J} + \Omega\cancel{\dot{\hat{J}}} + \cancel{\ddot{\theta}}\hat{k} + \dot{\theta}\dot{\hat{k}} = \dot{\theta}(\vec{\omega}\times\hat{k})$$
$$= \dot{\theta}(\Omega\hat{J} + \dot{\theta}\hat{k})\times\hat{k} = \dot{\theta}\Omega(\sin\theta\,\hat{i} + \cos\theta\,\hat{j})\times\hat{k}$$
$$= \dot{\theta}\Omega(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

1.A.2: 2 points – $\left(\vec{a}_{P/O}\right)_{rel} =$ $\vec{0}$

1.A.3: 2 points – $\vec{a}_O =$ $-d\Omega^2\hat{I} = -d\Omega^2(\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

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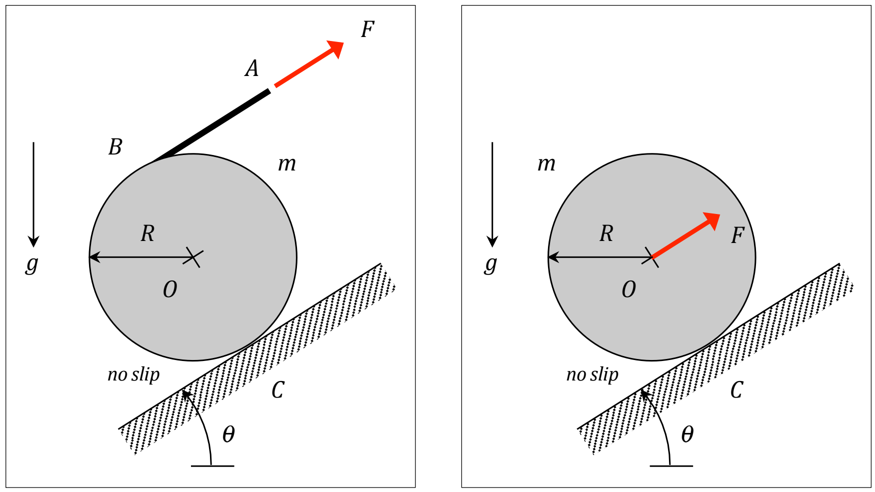
PROBLEM NO. 1 - PART B



System I **_System II_**

In System I shown above, a homogeneous disk of mass m and outer radius R is able to roll without slipping on an inclined surface. A cable is wrapped around the disk, with a constant force F being applied to the free end of the cable at A. In System II, an identical disk is moved up the same incline with the same constant force F now being applied at the center of the disk. Let $U_{1\to2}^{(I)}$ and $U_{1\to2}^{(II)}$ be the work done by F is moving the disk center O up the incline the same distance d for Systems I and II, respectively.

1.B.1: 2 points **-** Choose the correct response below regarding the size of these work terms:

a) $\boxed{U_{1\to2}^{(I)} > U_{1\to2}^{(II)}}$

b) $U_{1\to2}^{(I)} = U_{1\to2}^{(II)}$

c) $U_{1\to2}^{(I)} < U_{1\to2}^{(II)}$

d) More information is needed to answer this question.

Since $C = IC \Rightarrow$

$$\left.\begin{array}{l} V_O = R\omega \Rightarrow d_O = R\theta \\ V_A = 2R\omega \Rightarrow d_A = 2R\theta \end{array}\right\} d_A = 2d_O$$

$$\therefore\ U_{1\to2}^{(I)} = F d_A = 2F d_O$$

$$U_{1\to2}^{(II)} = F d_O$$

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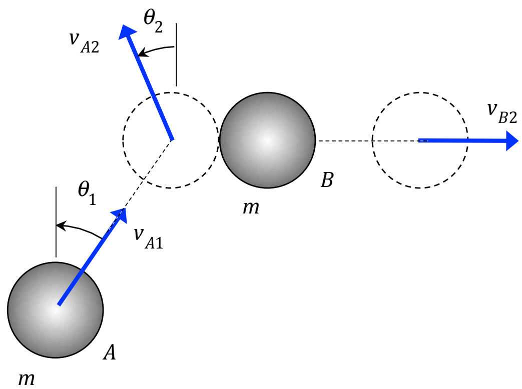
PROBLEM NO. 1 - PART C



Particle A is traveling with a speed of v_{A1} in the direction of $\theta_1 = 30°$. Particle A impacts a stationary Particle B, with the coefficient of restitution given by $e = 1$. Let θ_2 be the rebound angle for Particle A, as shown in the figure.

1.C.1: 2 points **-** Choose the correct response below:

a) $\theta_2 = -90°$

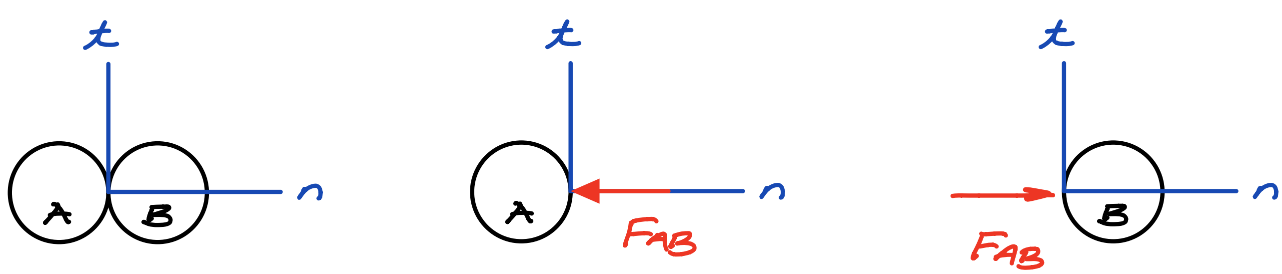
b) $-90° < \theta_2 < 0$

c) $\boxed{\theta_2 = 0}$

d) $0 < \theta_2 < 90°$

e) $\theta_2 = 90°$

f) Not relevant since A stops after impact.



A: $\Sigma F_t = 0 \Rightarrow v_{At2} = v_{At1} = v_{A1}\cos\theta_1$ (1)

A+B: $\Sigma F_n = 0 \Rightarrow \cancel{m} v_{An1} + \cancel{m} \cancel{v_{Bn1}}^{\,0} = \cancel{m} v_{An2} + \cancel{m} v_{Bn2}$ (2)

COR: $e = \dfrac{v_{Bn2} - v_{An2}}{v_{An1} - \cancel{v_{Bn1}}^{\,0}} \Rightarrow -e\, v_{An1} = v_{An2} - v_{Bn2}$ (3)

Add (2) & (3): $v_{An1}(1-e) = 2 v_{An2} \Rightarrow v_{An2} = 0$

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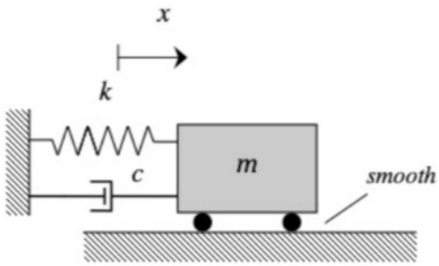
PROBLEM NO. 1 - PART D



The equation of motion for the system shown above is known to be: $m\ddot{x}+c\dot{x}+kx=0$. For a particular set of parameters, the *damping ratio* for the system is known to be $\zeta=0.2$ (underdamped).

1.D.1: 1 point – If the value of *m* is now *doubled*, with *k* and *c* being held constant, then:

a) the value of ζ will *increase*.

b) the value of ζ will *not change*.

c) the value of ζ will *decrease*.

$$\zeta = \frac{c}{2\sqrt{k(2m)}} = \frac{1}{\sqrt{2}}\left(\frac{c}{2\sqrt{km}}\right)$$

1.D.2: 1 point – If, instead, the value of *k* is now *doubled*, with *m* and *c* being held constant, then:

a) the value of ζ will *increase*.

b) the value of ζ will *not change*.

c) the value of ζ will *decrease*.

$$\zeta = \frac{c}{2\sqrt{(2k)m}} = \frac{1}{\sqrt{2}}\left(\frac{c}{2\sqrt{km}}\right)$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$2\zeta\omega_n = \frac{c}{m} \Rightarrow \zeta = \frac{c}{2m\omega_n} = \frac{c}{2\sqrt{km}}$$

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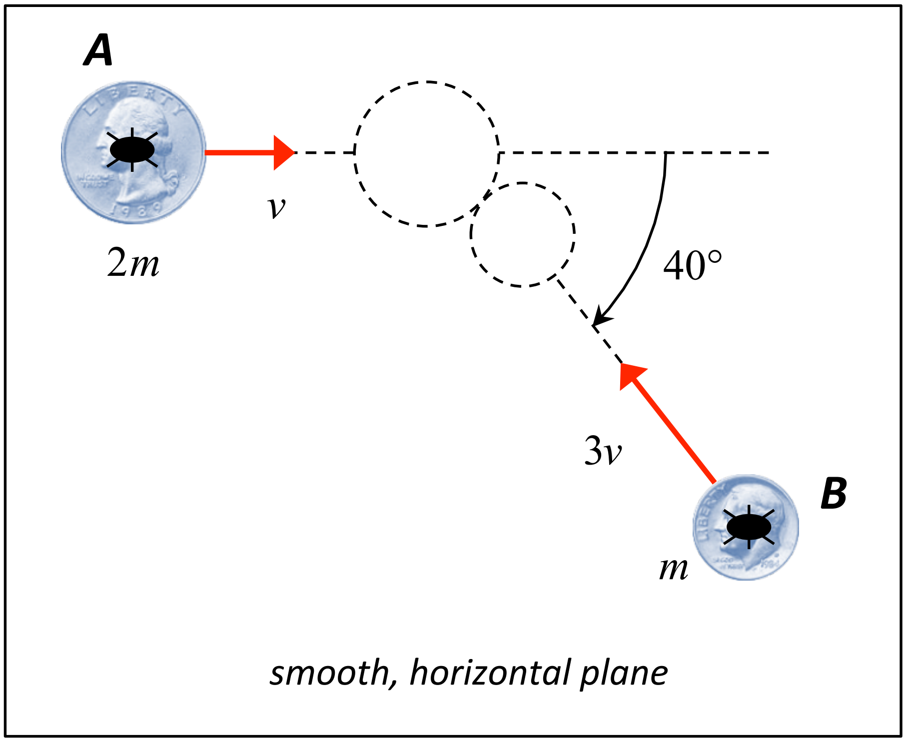
PROBLEM NO. 1 - PART E



Coins A and B approach each other with speeds of v and $3v$, respectively, as shown above. An insect rides along on each coin (with no motion relative to the coin) as the coins move. Let the combined insect+coin mass for A and B be $2m$ and m, respectively. Let $|\vec{a}_A|$ and $|\vec{a}_B|$ be the magnitudes of the acceleration of insects A and B, respectively, during the time of impact of the two coins.

1.E.1: 2 points - Choose the correct answer below regarding the sizes of $|\vec{a}_A|$ and $|\vec{a}_B|$:

a) $|\vec{a}_A| > |\vec{a}_B|$

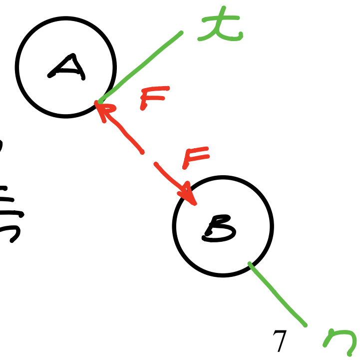
b) $|\vec{a}_A| = |\vec{a}_B|$

c) $\boxed{|\vec{a}_A| < |\vec{a}_B|}$

d) More information is needed in order to answer this question.

$$A: \sum F_n = -F = 2ma_A \Rightarrow |a_A| = \frac{F}{2m}$$

$$B: \sum F_n = F = ma_B \Rightarrow |a_B| = \frac{F}{m}$$



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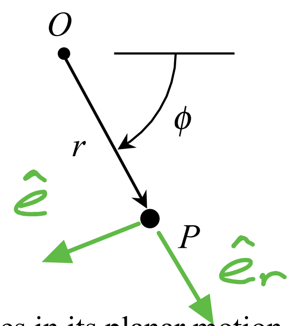
PROBLEM NO. 1 - PART F



The position of point P as it moves in its planar motion is described by the coordinates of r and ϕ, where O is a fixed point. When P is at a position given by $r = 3m$ and $\phi = 35°$, it is known that: $\dot{r} = 8m/s$, $\dot{\phi} = 2rad/s$, and $\ddot{r} = \ddot{\phi} = 0$.

1.F.1: 2 points - At this position (circle the correct response):

a) $\boxed{\text{The speed of P is increasing.}}$
b) The speed of P is constant.
c) The speed of P is decreasing.
d) More information is needed to answer this question.

$$\vec{V} = \dot{r}\hat{e}_r + r\dot{\phi}\hat{e}_\phi = (8\hat{e}_r + 6\hat{e}_\phi)\frac{m}{s}$$

$$\vec{a} = (\ddot{r} - r\dot{\phi}^2)\hat{e}_r + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\hat{e}_\phi = (-12\hat{e}_r + 32\hat{e}_\phi)\frac{m}{s^2}$$

$$\dot{v} = \frac{\vec{V}}{|\vec{V}|}\cdot\vec{a} = \left(\frac{8\hat{e}_r + 6\hat{e}_\phi}{10}\right)\cdot(-12\hat{e}_r + 32\hat{e}_\phi)$$

$$= 9.6\frac{m}{s^2} > 0$$

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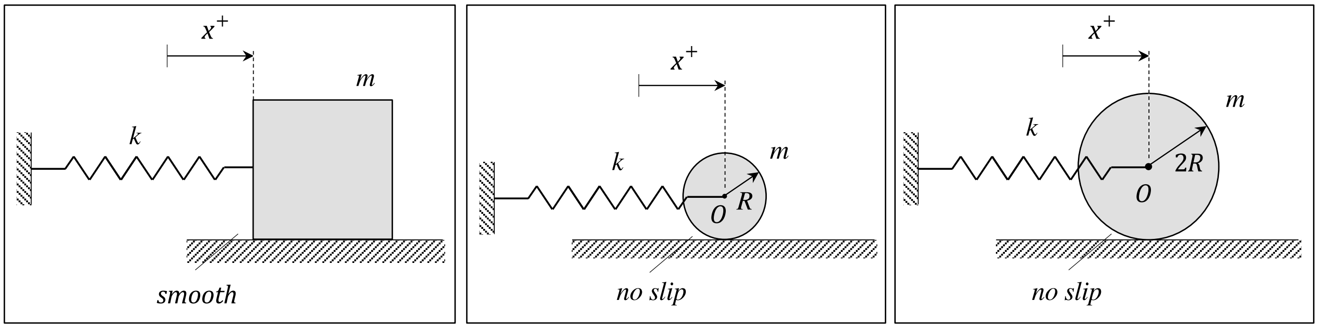
PROBLEM NO. 1 - PART G



System 1 **System 2** **System 3**

Consider systems 1, 2 and 3 shown above, where the disks in Systems 2 and 3 are homogeneous. Let ω_{n1}, ω_{n2} and ω_{n3} represent the undamped natural frequencies for Systems 1, 2 and 3, respectively. Choose the correct responses below regarding the sizes of these natural frequencies:

1.G.1: 2 points

a) $\boxed{\omega_{n1} > \omega_{n2}}$

b) $\omega_{n1} = \omega_{n2}$

c) $\omega_{n1} < \omega_{n2}$

d) More information is needed to answer this question.

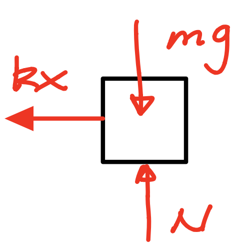
1.G.2: 2 points

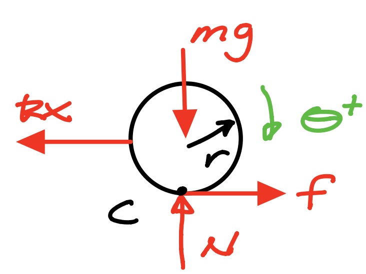
a) $\omega_{n2} > \omega_{n3}$

b) $\boxed{\omega_{n2} = \omega_{n3}}$

c) $\omega_{n2} < \omega_{n3}$

d) More information is needed to answer this question.





$$\Sigma F_x = -kx = m\ddot{x}$$

$$\hookrightarrow m\ddot{x} + kx = 0$$

$$\hookrightarrow (\omega_n)_1 = \sqrt{\frac{k}{m}}$$

$$\Sigma M_C = -(kx)r = I_C\ddot{\theta}$$

$$\hookrightarrow \left(\frac{3}{2}mr^2\right)\frac{\ddot{x}}{r} + krx = 0$$

$$\hookrightarrow \omega_n = \sqrt{\frac{2}{3}}\sqrt{\frac{k}{m}}$$

& ind. of "r"

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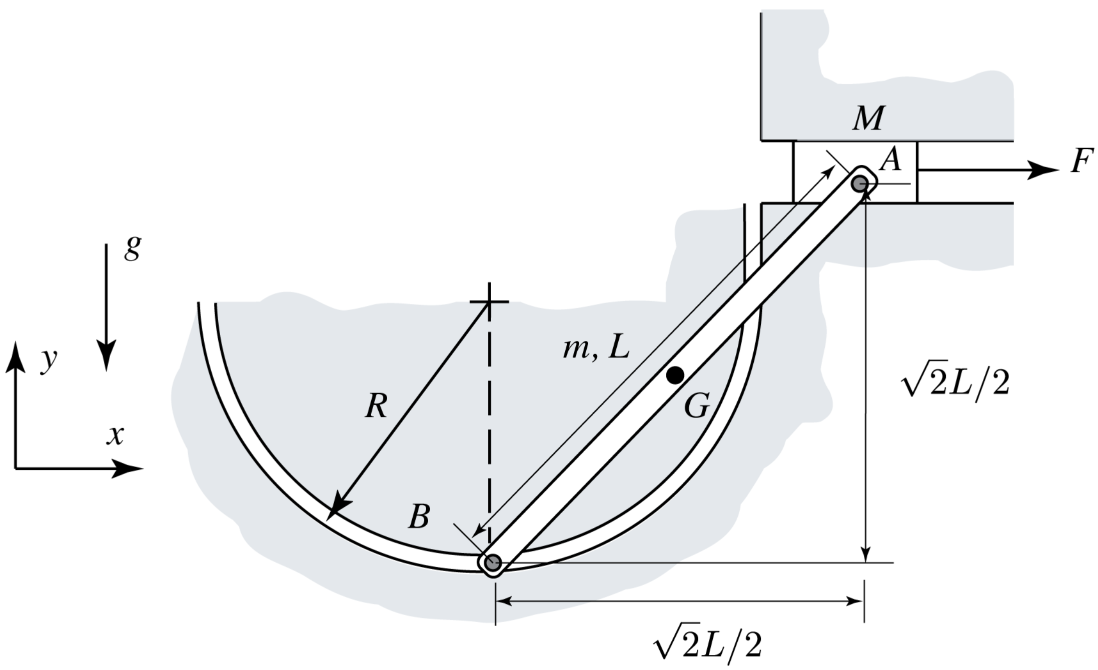
PROBLEM NO. 2 **(***20 points)*

Given: At the instant shown a rod BA with mass m is constrained at B to slide along a semicircular surface of radius R. BA is connected to slider of M at A that is acted on by a horizontal force F. The slider and pin A are constrained to move in the x-direction. Both slider and rod BA are initially at rest.



Find: Show that the **angular acceleration of rod *BA* at this instant equal zero** and **determine the acceleration of the center of mass of the bar (point *G*)** using the following steps.

(a) Draw separate free body diagrams (FBDs) of the bar and slider. Indicate the coordinate system you will use for the FBDs.

(b) Write the appropriate kinetic equations needed to find the acceleration of the bar's center of mass and the angular acceleration of the bar.

(c) If you have more unknowns than kinetic equations, write down the additional kinematic equations needed to find the unknowns in (b).

(d) Show that the angular acceleration of rod BA at this instant equal zero (Hint: this is not because the system is at rest initially), and determine the acceleration of point G in Cartesian vector form in terms of m, M, g, R, F, and L. Some terms may not be used.

PROBLEM NO. 2 **Extra Pages for Work**

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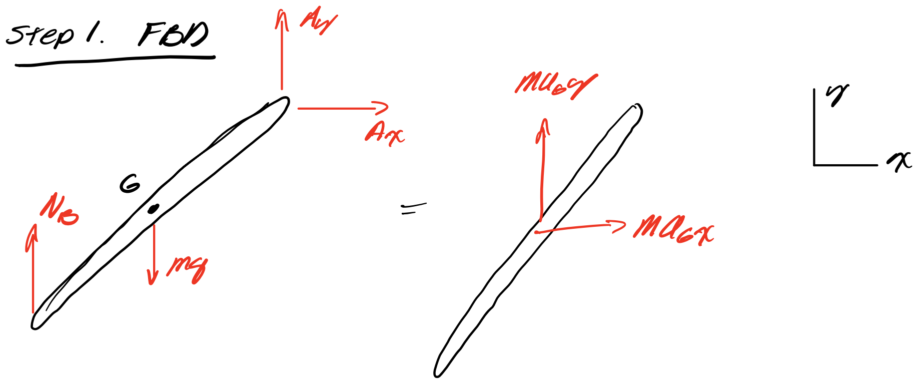
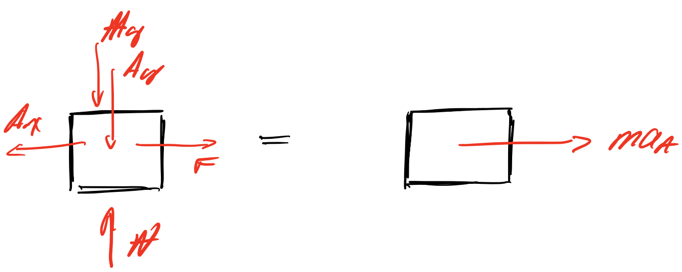
Step 1. FBD





Step 2 Kinetics

Slider $\xrightarrow{+} \Sigma F_x$: $F - A_x = \cancel{M} a_A$ 1)

$+\uparrow \Sigma F_y$: $N - A_x - \cancel{A_y} = 0$ 2)

Rod $\xrightarrow{+} \Sigma F_x$: $A_x = m a_{Gx}$ 3)

$+\uparrow \Sigma F_y$: $A_y + N_B - mg = m a_{Gy}$

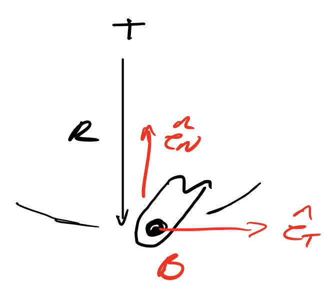
$\circlearrowleft \Sigma M_G$: $-A_x \sqrt{2}L/4 + A_y \sqrt{2}L/4 - N_B \sqrt{2}L/4 = I_G \alpha$ $\quad I_G = 1/12\, mL^2$

Step 3 Kinematics

Point B moves in a circular path

$$a_B = \dot{v}_B \hat{e}_T + \frac{v_B^2}{R}\hat{e}_n \quad \text{at rest } v_B = 0$$

$$a_B = \dot{v}_B \hat{e}_T = \dot{v}_B \hat{i}$$



Point B is on a rigid body

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} + \omega^2 \vec{r}_{B/A} \quad \text{at rest } \omega = 0$$

k $\quad \hat{j}$

$$\vec{a}_B = a_A \hat{i} + \alpha \hat{k} \times (-\sqrt{2}L/4\, \hat{i} - \sqrt{2}L/4\, \hat{j}) + 0$$

$$\vec{a}_B = a_A \hat{i} - \alpha\sqrt{2}L/4\, \hat{j} + \alpha\sqrt{2}L/4\, \hat{i} = \dot{v}_B \hat{i}$$

PROBLEM NO. 2 **Extra Pages for Work**

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$\hat{i} \Rightarrow \quad \dot{v}_B = a_A \quad + \alpha\sqrt{2}L/4 \quad \longrightarrow \quad \dot{v}_B = a_A = a_B$

$\hat{j} \Rightarrow \quad 0 = \alpha\sqrt{2}L/4 \quad \longrightarrow \quad \alpha = 0$

Now, $\quad \vec{a}_G = \vec{a}_A + \cancel{\vec{\alpha} \times \vec{r}_{G/A}}^{\,0} - \cancel{\omega^2 \vec{r}_{G/A}}^{\,0}$

$\vec{a}_G = \vec{a}_A = \vec{a}_B$

Step 4: Solve

$F - A_x = M a_A \quad 1) \quad \longrightarrow \quad A_x = F - M \cancel{a_A}^{\,a_{Gx}}$

$A_x = m a_{Gx} \quad 3) \quad \longrightarrow \quad F - M a_{Gx} = m a_{Gx}$

$a_{Gx} = \dfrac{F}{m+M}$

$\vec{a}_{Gx} = \left(\dfrac{F}{m+M}\right)\hat{i}$

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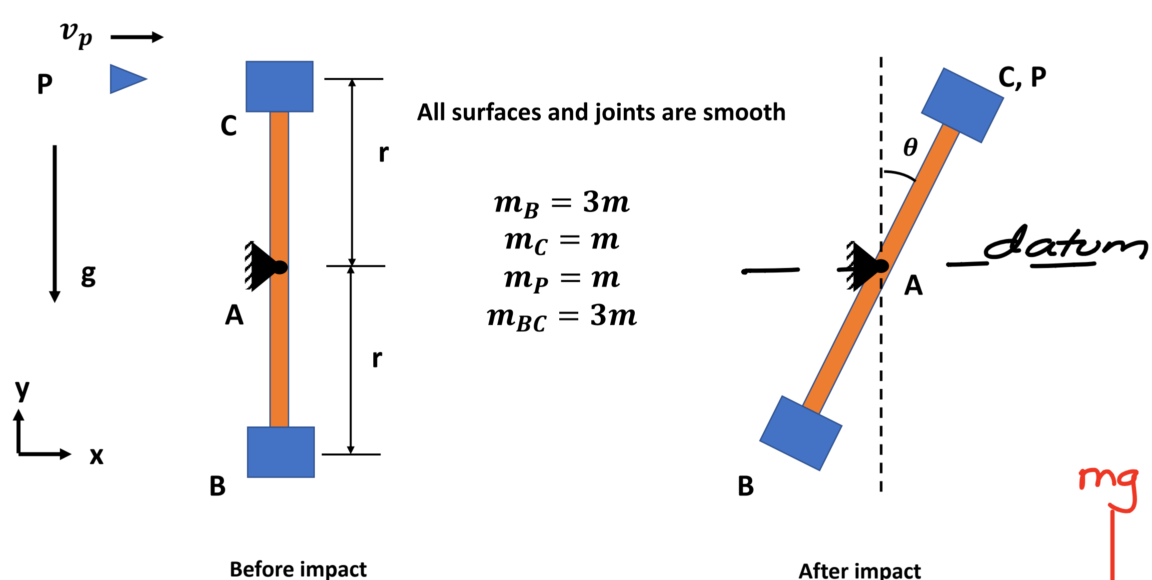
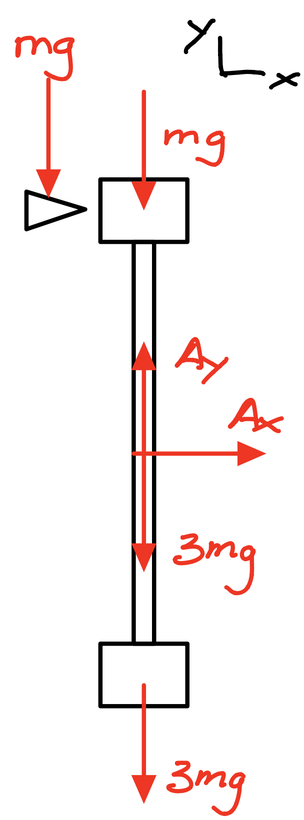
PROBLEM NO. 3 (*20 points*)

Given: As shown in the figure below, in a vertical plane, two blocks B and C are connected to a rigid bar BC with a length of $2r$. The bar is connected to a fixed-point A with a smooth pin at the center of the bar. A bullet P is striking block C at a velocity of v_p and it sticks to the block after impact.

The mass of block B is $m_B = 3m$, the mass of block C is $m_C = m$, the mass of bullet P is $m_p = m$ and the mass the bar is $m_{BC} = 3m$. Initially, the bar and blocks B and C are at rest.





(1) If all blocks (B and C), the bullet P and the bar BC are treated as a system:

a) Is the **linear momentum** in the x direction conserved **during impact**?
Please circle your answer: YES $\boxed{\text{NO}}$

b) Is the **angular momentum about A** conserved **during impact**?
Please circle your answer: $\boxed{\text{YES}}$ NO

c) Is the **linear momentum** in the x direction conserved **after impact**?
Please circle your answer: YES $\boxed{\text{NO}}$

d) Is the **angular momentum about A** conserved **after impact**?
Please circle your answer: YES $\boxed{\text{NO}}$

(a) $\Sigma F_x \neq 0$
(b) $\Sigma M_A = 0$
(c) $\Sigma F_x \neq 0$
(d) $\Sigma M_A \neq 0$

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PROBLEM NO. 3 **(continued)**

Part (2) Find the velocity of block B and block C as well as the angular velocity of bar BC immediately after impact, express your answer in terms of m, r, v_p and g.

1. FBD: See on preceeding page.

2. Kinetics

$$\sum \vec{M}_A = \vec{0} \Rightarrow \vec{H}_{A1} = \vec{H}_{A2}$$

w/ $$\vec{H}_{A1} = m\vec{r}_{C/A} \times \vec{v}_{P1} = m(r\hat{j}) \times (v_p\hat{i}) = -mrv_p\hat{k}$$

$$\vec{H}_{A2} = \underbrace{m\vec{r}_{C/A} \times \vec{v}_{P2}}_{P} + \underbrace{m\vec{r}_{C/A} \times \vec{v}_{C2}}_{C} + \underbrace{3m\vec{r}_{B/A} \times \vec{v}_{B2}}_{B} + \underbrace{I_A\vec{\omega}}_{bar}$$

$$I_A = \frac{1}{12}(3m)(2r)^2 = mr^2$$

3. Kinematics

$$\vec{v}_{P2} = -r\omega\hat{i} = \vec{v}_{C2}$$

$$\vec{v}_{B2} = r\omega\hat{i}$$

4. Solve

$$\vec{H}_{A2} = 2(m)(r\hat{j}) \times (-r\omega\hat{i}) + (3m)(-r\hat{j}) \times (r\omega\hat{i}) + I_A\omega\hat{k}$$

$$= (5mr^2 + mr^2)\omega\hat{k} = 6mr^2\omega\hat{k}$$

$$\vec{H}_{A1} = \vec{H}_{A2} \Rightarrow -\cancel{m}\cancel{r}v_p = 6\cancel{m}r^{\cancel{2}}\omega \Rightarrow \vec{\omega} = -\frac{v_p}{6r}\hat{k}$$

$$\vec{v}_B = r\omega\hat{i} = -\frac{v_p}{6}\hat{i}$$

Part (3)

$$T_2 + V_2 + \cancelto{0}{U_{2\to3}^{(nc)}} = \cancelto{0}{T_3} + V_3$$

$$T_2 = \frac{1}{2}mv_{P2}^2 + \frac{1}{2}mv_{C2}^2 + \frac{1}{2}3mv_{B2}^2 + \frac{1}{2}I_A\omega^2$$

$$= \frac{1}{2}m(r\omega)^2 + \frac{1}{2}m(r\omega)^2 + \frac{1}{2}(3m)(r\omega)^2 + \frac{1}{2}(mr^2)\omega^2$$

$$= \frac{m}{2}[1 + 1 + 3 + 1]r^2\omega^2 = 3mr^2\omega^2$$

$$V_2 = mgr + mgr + 3mg(-r) = -mgr$$

***PROBLEM NO. 3* Extra Pages for Work**

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$$V_3 = mgr\cos\theta + mgr\cos\theta + 3mg(-r\cos\theta)$$
$$= -mgr\cos\theta$$

$$\therefore \quad 3\cancel{m}r^{\cancel{2}}\omega^2 - \cancel{m}g\cancel{r} = -\cancel{m}g\cancel{r}\cos\theta$$

$$\hookrightarrow \cos\theta = 1 - 3\frac{r}{g}\left(\frac{v_P}{6r}\right)^2 = 1 - \frac{v_P^2}{12gr}$$

$$\hookrightarrow \theta_{max} = \cos^{-1}\left(1 - \frac{v_P^2}{12gr}\right)$$

θ_{max}

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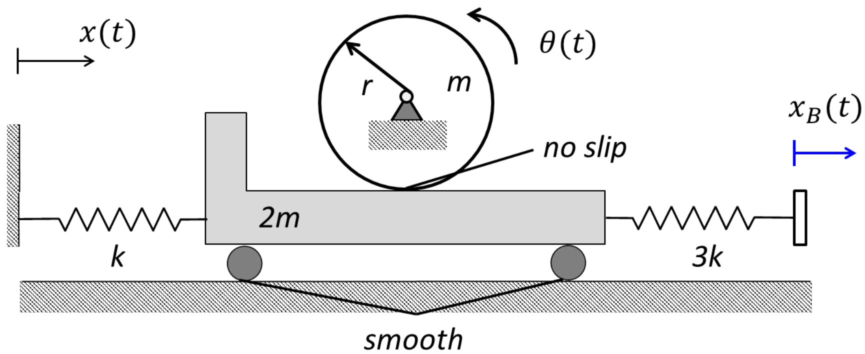
PROBLEM NO. 4 **(***20 points)*

Given:
A homogeneous disk of radius r and mass m is connected to the ground at point O. The disk is in no-slip contact with a block of mass $2m$ that is connected to a grounded wall through a spring of stiffness k. The block is further connected to a second spring of stiffness $3k$ and a massless base subjected to a prescribed base displacement $x_B(t) = b \sin \Omega t$. Let $x(t)$ represent the horizontal motion of the block, where the left spring is unstretched when $x = 0$. Let $\theta(t)$ represent the angular motion of the disk.



Find:

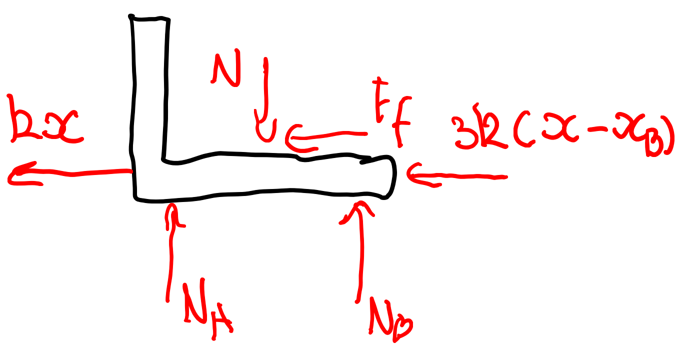
a) Derive the dynamical equation of motion (EOM) of the system in terms of $\theta(t)$.

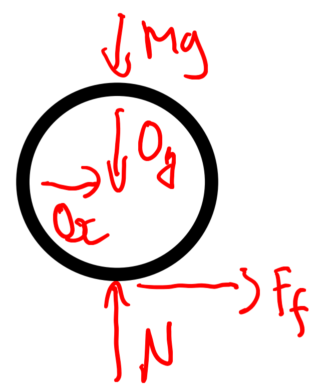
b) Determine the undamped natural frequency of the system.

c) Derive the *forced response* of the system $\theta_p(t)$. Show the complete derivation process.

d) If the forcing frequency is $\Omega = \sqrt{\frac{3k}{2m}}$. Under these conditions, would the system be in-phase, or out-of-phase? Justify your answer.

1) FBD



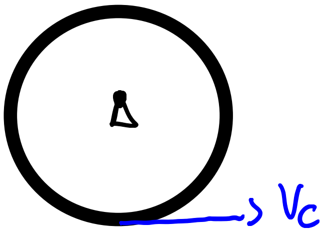


2) EOM

$\Sigma M_O: R \cdot F_f = I_O \ddot{\theta}$ (1)

$\Sigma F_x: -kx - 3k(x - x_B) - F_f = 2m\ddot{x} \longrightarrow 2m\ddot{x} + 4kx + F_f = 3kx_B$ (2)

3) Kinematics



$V_c = \dot{\theta} \cdot R$

$\dot{x} = \dot{\theta} \cdot R$ (3)

$F_f = I_0 \ddot{\theta} / R$

Sub (1) and (3) into (2)

$$2m(\ddot{\theta} R) + 4k(\theta R) + \frac{I_0}{R}\ddot{\theta} = 3k x_B(t)$$

$$2m\ddot{\theta} R + \frac{I_0}{R}\ddot{\theta} + 4k(\theta R) = 3k x_B(t) \quad ; \; I_0 = \frac{1}{2} mR^2$$

$$2m\ddot{\theta} R + \frac{1}{2} mR\ddot{\theta} + 4k\theta R = 3k x_B(t)$$

$$\frac{5}{2} m\ddot{\theta} + 4k\theta = 3k x_B \quad ; \; x_B(t) = b\sin\Omega t$$

a)

$$\frac{5}{2} m\ddot{\theta} + 4k\theta = \frac{3kb}{R}\sin\Omega t$$

b) $M = \frac{5m}{2}$ $\quad K = 4k$ $\quad F = \frac{3kb}{R}$

$$\omega_n = \sqrt{\frac{8k}{5m}}$$

c) $\theta_p = A\sin\Omega t + B\cos\Omega t$

standard form: $\ddot{\theta} + \omega_n^2\theta = \frac{6kb}{5mR}\sin\Omega t$

$$-\Omega^2(A\sin\Omega t + B\cos\Omega t) + \omega_n^2(A\sin\Omega t + B\cos\Omega t) = \frac{6}{5}\frac{kb}{mR}\sin\Omega t$$

balance terms: $B = 0$

characteristic equation: $(-\Omega^2 + \omega_n^2) A = \frac{6}{5}\frac{kb}{mR}$

$$\omega_n^2 = \frac{8k}{5M}$$

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PROBLEM NO. 4 **Extra Pages for Work**

PLEASE SCAN ALL PAGES

$$A = \frac{\frac{6}{5}kb/mR}{-\Omega^2 + \omega_n^2} \longrightarrow A = \frac{\frac{6}{5}kb/mR\left(\frac{1}{\omega_n^2}\right)}{1 - \frac{\Omega^2}{\omega_n^2}} \; ; \; A = \frac{\frac{6kb}{5MR}\,\frac{5M}{8k}}{1 - \frac{\Omega^2}{\omega_n^2}}$$

$$A = \frac{\frac{3b}{4R}}{1 - \frac{\Omega^2}{\omega_n^2}} \longrightarrow \theta_p = A \cdot \sin \Omega t$$

d) if $\Omega = \sqrt{\frac{2k}{3M}}$ $\longrightarrow \omega_n = \sqrt{\frac{8k}{5M}}$

$$\Omega < \omega_n$$

$$A = \frac{\frac{3b}{4R}}{1 - \frac{2k/3M}{8k/5M}} \longrightarrow A = \frac{\frac{3}{4}b/R}{1 - \frac{10}{24}} \longrightarrow A > 0$$

$$\boxed{\text{in-phase}}$$