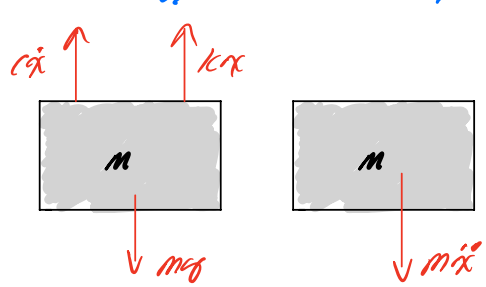
Example 6.B.12

Given: The system shown below is released from rest under the action of gravity.

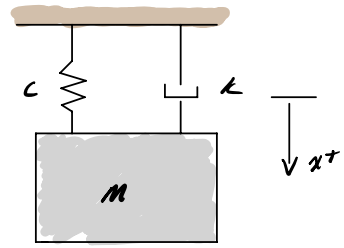
Find: Determine the initial overshoot past the static equilibrium of the system.

Used the following parameters in your analysis:
$m = 5\,kg$, $c = 6\,N\cdot s/m$, $k = 180\,N/m$



$+\downarrow \Sigma F_x: \; -c\dot{x} - kx = m\ddot{x}$

$m\ddot{x} + c\dot{x} + kx = 0$



Static: $\dot{x} = \ddot{x} = 0$ $\quad kx_{ST} = mg \rightarrow x_{ST} = mg/k = \frac{5 \cdot 9.81}{10}$

Dynamics around equilibrium pt.

$x = x_{ST} + z, \quad \dot{x} = \dot{z}, \quad \ddot{x} = \ddot{z}$

$m\ddot{z} + c\dot{z} + kz + \cancel{kx_{ST}} = \cancel{mg}$

$m\ddot{z} + c\dot{z} + kz = 0$

<u>Parameter Identification</u>

$\omega_n = \sqrt{k/m} = \sqrt{180/5} = 6\,rad/s$

$2\zeta\omega_n = c/m \rightarrow \zeta = c/2m\omega_n = 6/2(5)6 = 1/10 = 0.10$

Solution to $z(t)$

$z(t) = e^{-\zeta\omega_n t}(C\cos\omega_d t + S\sin\omega_d t)$

$\dot{z}(t) = -\zeta\omega_n e^{-\zeta\omega_n t}(C\cos\omega_d t + S\sin\omega_d t)$
$\quad + e^{-\zeta\omega_n t}(-\omega_d C\sin\omega_d t + \omega_d S\cos\omega_d t)$

Ok we need C and S here are our initial conditions

$$\left.\begin{aligned} x(0) &= 0 \\ \dot{x}(0) &= 0 \end{aligned}\right\}$$ there are in "x" and not "z"

$x(0) = x_{ST} + z(0) = 0 \qquad z(0) = -x_{ST}$

$\dot{x}(0) = \dot{z}(0) = 0$

The new initial conditions

$z(0) = -x_{ST}$

$\dot{z}(0) = 0$

$$-x_{ST} = \cancelto{1}{e^{-\xi\omega_n 0}}\left(C\cancelto{1}{\cos\omega_d 0} + S\cancelto{0}{\sin\omega_d 0}\right)$$

$$-x_{ST} = C$$

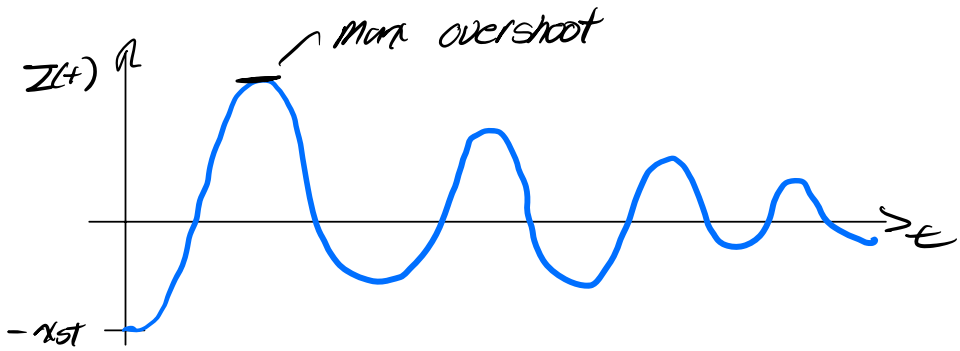
$$0 = -\xi\omega_n \cancelto{1}{e^{-\xi\omega_n 0}}\left(C\cancelto{1}{\cos\omega_d 0} + S\cancelto{0}{\sin\omega_d 0}\right)$$
$$+ \cancelto{1}{e^{-\xi\omega_n 0}}\left(-\omega_d C\cancelto{0}{\sin\omega_d 0} + \omega_d S\cancelto{1}{\cos\omega_d 0}\right)$$

$$0 = -\xi\omega_n C + \omega_d S$$

$$S = \frac{\xi\omega_n C}{\omega_d} = -\frac{\xi\omega_n}{\omega_d} x_0 = +\frac{\xi}{\sqrt{1-\xi^2}} x_{ST}$$

$$x(t) = -x_{ST} e^{-\xi\omega_n t}\left(\cos\omega_d t - \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right)$$

Need to plot use Matlab excel



or would solve for t_* when $\dot{z}(t_*) = 0$

and plug t_* in $z(t)$ to get max overshoot