FORCED VIBRATIONS SUMMARY

SYSTEM	EOM	F(t)	STEADY STATE RESPONSE
Undamped $\zeta = 0$	$M\ddot{x} + Kx = F(t)$	$F_0 \sin \omega t$	$x_p(t) = A\sin\omega t + B\cos\omega t$ $B = 0$ $A = \dfrac{F_0/K}{1-(\omega/\omega_n)^2}$
Undamped $\zeta = 0$	$M\ddot{x} + Kx = F(t)$	$F_0 \cos \omega t$	$x_p(t) = A\sin\omega t + B\cos\omega t$ $A = 0$ $B = \dfrac{F_0/K}{1-(\omega/\omega_n)^2}$
Underdamped $0 < \zeta < 1$	$M\ddot{x} + C\dot{x} + Kx = F(t)$	$F_0 \sin \omega t$ or $F_0 \cos \omega t$	$x_p(t) = X\sin(\omega t - \varphi)$ $X = \dfrac{F_0/K}{\left[\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2\right]^{\frac{1}{2}}}$ $= \dfrac{F_0/K}{\left[(k-M\omega^2)^2 + (C\omega)^2\right]^{\frac{1}{2}}}$ $\varphi = \tan^{-1}\left(\dfrac{2\zeta\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right)$ $= \tan^{-1}\left(\dfrac{C\omega}{k-M\omega^2}\right)$

FORCED VIBRATIONS - SPECIAL CASES (UNDAMPED)

CASE	EOM	F_0
Base excitation $y = Y\sin\omega t$ or $y = Y\cos\omega t$	$M\ddot{x} + Kx = KY\sin\omega t$ or $M\ddot{x} + Kx = KY\cos\omega t$	KY KY
Rotating Imbalance	$(M+m)\ddot{x} + Kx = me\omega^2\cos\omega t$ or $(M+m)\ddot{x} + Kx = me\omega^2\sin\omega t$	$me\omega^2$