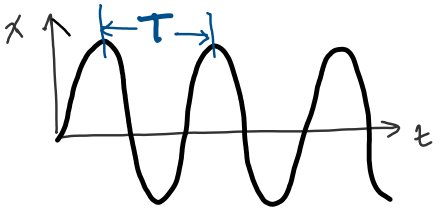
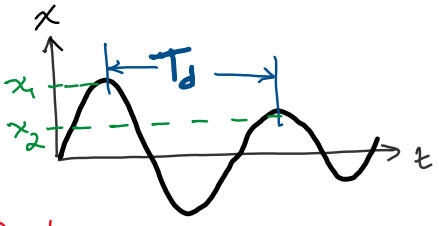
6.B FREE VIBRATIONS - SUMMARY

Wednesday, April 19, 2023 6:11 PM

EOM (standard form): $\ddot{x} + \frac{c}{M}\dot{x} + \frac{K}{M}x = \ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$

$2\zeta\omega_n = \frac{c}{M}$, $\omega_n = \sqrt{\frac{K}{M}}$

Case	ζ	λ_1, λ_2	Plot
Undamped	$\zeta = 0$	$\pm i\omega_n$	 $T = \frac{2\pi}{\omega_n}$ Solution: $x(t) = A\cos\omega_n t + B\sin\omega_n t$
Underdamped	$0 < \zeta < 1$	$-\zeta\omega_n \pm i\omega_d$ $\omega_d = \omega_n\sqrt{1-\zeta^2}$	 $T_d = \frac{2\pi}{\omega_d}$ $\zeta = \frac{1}{2\pi}\ln\left(\frac{x_1}{x_2}\right)$ Solution: $x(t) = e^{-\zeta\omega_n t}[A\cos\omega_d t + B\sin\omega_d t]$
critically damped	$\zeta = 1$	$\lambda_1 = \lambda_2 = -\omega_n$	
overdamped	$\zeta > 1$	$\left(\zeta \pm \sqrt{\zeta^2 - 1}\right)\omega_n$ $\in \mathbb{R}$