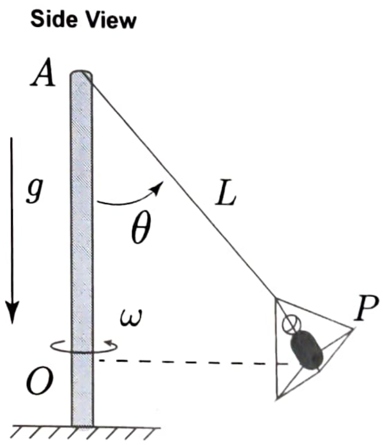
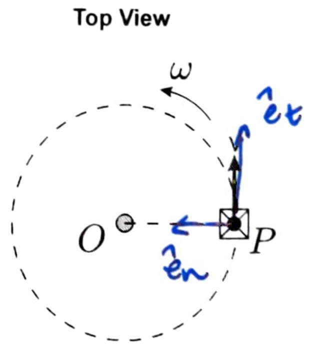
Problem 1 (20 points):

Given: A carnival ride features swinging chairs rotating about a vertical axis with a constant angular velocity $\vec{\omega}$ such that each swing is inclined at a constant angle θ as depicted.

Consider an individual sitting on a swing as a particle P. The cable length of the swing L (m), the combined mass of the swing and person M (kg), and the angle θ (°) are known.





Find: Note that your final answers should be in terms of, at most, the known variables L, θ, M, and the gravity g.

(a) Draw the free body diagram of the person and swing as a single particle P, clearly indicating your selection of coordinate system.

(b) Write Newton's 2nd Law expressions along each direction of your chosen coordinate system.

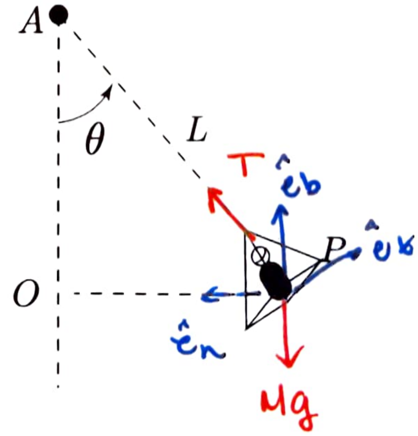
(c) Determine the magnitude of the tension force T in the cable.

(d) Determine the linear speed v of the swing and rider.

(e) Determine the constant angular speed ω of the ride.

Solution:

(a) Free body diagram including coordinate system.



(b) Write Newton's 2nd Law expressions.

$$+\leftarrow \Sigma F_n = T\sin\theta = Ma_n$$

$$+\uparrow \Sigma F_b = T\cos\theta - Mg = Ma_b$$

(c) Determine the magnitude of the tension force T in the cable.

$$a_b = 0 \Rightarrow T\cos\theta - Mg = 0$$

$$T = \frac{Mg}{\cos\theta} \text{ N}$$

(d) Determine the linear speed v of the swing and rider.

$$a_n = \frac{v^2}{\rho}, \quad \rho = L\sin\theta$$

$$T\sin\theta = \frac{Mv^2}{L\sin\theta} \rightarrow v^2 = \frac{TL\sin^2\theta}{M} = \frac{\cancel{M}g}{\cos\theta}\frac{L\sin^2\theta}{\cancel{M}}$$

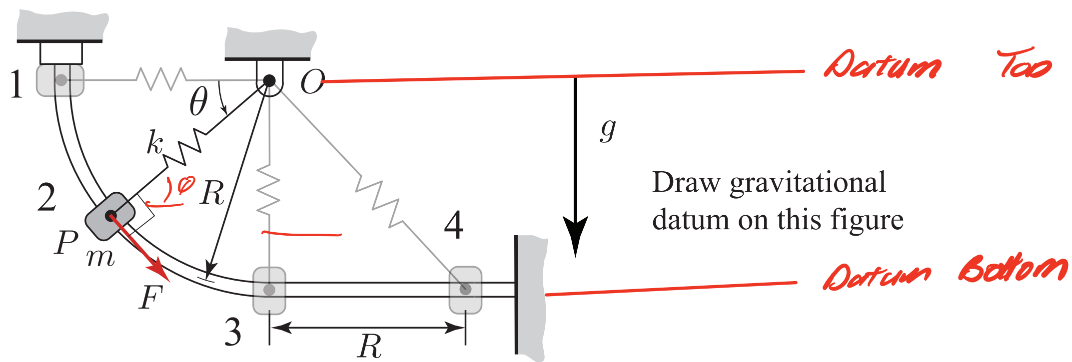
$$v = \sin\theta\sqrt{\frac{gL}{\cos\theta}} \ \frac{\text{m}}{\text{s}}$$

(e) Determine the constant angular speed ω of the ride.

$$v = \rho\omega = L\sin\theta\,\omega \rightarrow \omega = \frac{v}{L\sin\theta} = \sqrt{\frac{g}{L\cos\theta}} \ \frac{\text{rad}}{\text{s}}$$

Problem 2 (20 points):

Given: A collar P of mass m (kg) is attached to a spring of stiffness k (N/m). The spring is attached to a pivot joint at O and has an an unstretched length of $0.5R$ (m). The collar is released from rest at position 1 and slides along a smooth rod under the action of a constant force F (N). The force always acts tangent to the path of the collar as the collar moves from posotion 1 to position 4. The rod consists of a quarter circle of radius R (m) centered at point O, and a constant horizontal section. At position 2, collar P makes an angle θ and at position 3 collar P is directly below point O. Finally, at position 4 collar P is at a horizontal distance R (m) from position 3.



Find: Note that your final answers should be in terms of at most, the known variables R, k, m, F, θ and the gravity g.

(a) Draw a free body diagram of the collar at position 2.

(b) Indicate your gravitational datum on the figure and write the potential energy due to gravity (V_g) at positions 1, 2, 3, and 4.

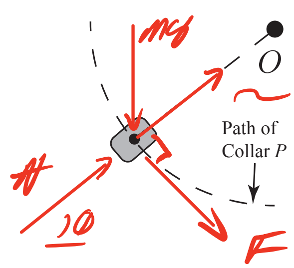
(c) Write the potential energy due to the spring (V_{sp}) at positions 1, 2, 3, and 4.

(d) Write the work done by the force F on the particle as it moves from positions 1 to 2, positions 1 to 3, and positions 1 to 4.

(e) Use the principle of work-energy to find the velocity of the collar at position 2.

Solution:

(a) Draw a free body diagram of the collar at position 2 on the figure below.



$F_{sp} = k(R - \frac{1}{2}R) = \frac{kR}{2}$

(b) Indicate your gravitational datum on the figure and write the potential energy due to gravity (V_g) at positions 1 ,2, 3, and 4. Place your final answers in the table below.

either is acceptable

V_g at 1	V_g at 2	V_g at 3	V_g at 4
Top 0	$-mgR\sin\theta$	$-mgR$	$-mgR$
Bottom mgR	$mgR - mgR\sin\theta$	0	0

(c) Write the potential energy due to the spring (V_{sp}) at positions 1, 2 , 3, and 4. Place your final answers in the table below.

V_{sp} at 1	V_{sp} at 2	V_{sp} at 3	V_{sp} at 4
$\frac{1}{2}k\Delta^2 = \frac{1}{2}k(R-\frac{1}{2}R)^2 = \frac{1}{8}kR^2$	$\frac{1}{8}kR^2$	$\frac{1}{8}kR^2$	$\frac{1}{2}k\Delta^2 = \frac{1}{2}k(\sqrt{2}R-\frac{1}{2}R)^2 = 0{,}42kR^2$

(d) Write the work done by the force F on the particle at moves from positions 1 to 2, positions 1 to 3, and positions 1 to 4. Place your final answers in the table below.

$\sum U_{1\to2}^{NC}$	$\sum U_{1\to3}^{NC}$	$\sum U_{1\to4}^{NC}$
$F\Delta_{12} = FR\theta$	$F\Delta_{13} = FR\pi/2$	$FR\pi/2 + FR = FR(1+\pi/2) = 2{,}57FR$

(e) Use the principle of work-energy to find the speed of the collar at position 2.

$T_1 + V_1 + \sum U_{1-2}^{nc} = T_2 + V_2$ use datum at top

$0 + 0 + \cancel{\frac{1}{8}kR^2} + FR\theta = \frac{1}{2}mv^2 - mgR\sin\theta + \cancel{\frac{1}{8}kR^2}$

$\frac{1}{2}mv^2 = R(F\theta + mg\sin\theta)$

$v = \sqrt{\dfrac{2R(F\theta + mg\sin\theta)}{m}}$ Ans

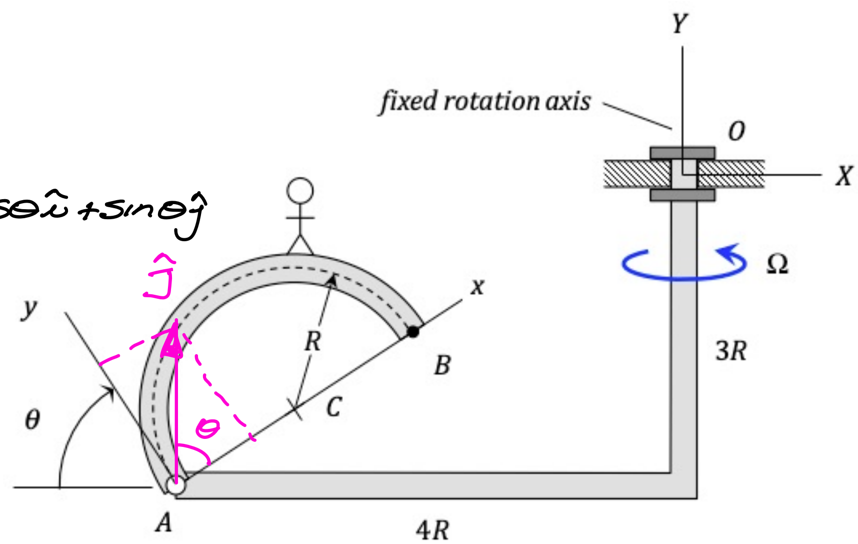
NOTE: *You are asked to provide justification (written and/or through equations) for your answers here in Problem 3. A correct response, alone, will receive only partial credit. Your work will be graded.*

Problem 3A(8 points): ______

Given: Arm OA rotates about a fixed axis with a constant rate of Ω. Semi-circular member AB rotates with respect to OA with a constant rate of $\dot{\theta}$. It is desired to determine the acceleration of point B on AB using the following moving reference frame kinematics equation:

$$\vec{a}_B = \vec{a}_A + (\vec{a}_{B/A})_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times (\vec{v}_{B/A})_{rel} + \omega \times (\omega \times \vec{r}_{B/A})$$

$\hat{J} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$



For this equation, an observer is attached on AB, and the xyz-axes are also attached to AB. Provide the $\hat{i}\hat{j}\hat{k}$ components for the following four terms in the above kinematics equation for an arbitrary angle θ:

$\vec{\omega} = \Omega\hat{J} - \dot{\theta}\hat{k} = \Omega(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) - \dot{\theta}\hat{k}$

$\vec{\alpha} = \cancel{\dot{\Omega}}\hat{J} + \Omega\cancel{\dot{\hat{J}}} - \cancel{\ddot{\theta}}\hat{k} - \dot{\theta}\dot{\hat{k}} = -\dot{\theta}(\vec{\omega} \times \hat{k})$

$= -\dot{\theta}\left[\Omega\cos\theta\,\hat{i} + \Omega\sin\theta\,\hat{j}\right] \times \hat{k}$

$= -\dot{\theta}\Omega\sin\theta\,\hat{i} + \dot{\theta}\Omega\cos\theta\,\hat{j}$

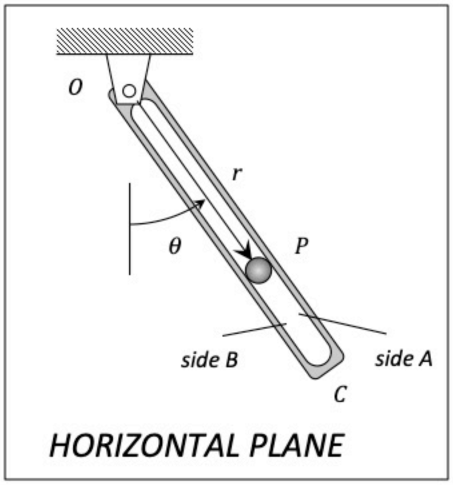
$(\vec{v}_{B/A})_{rel} = \vec{0}$

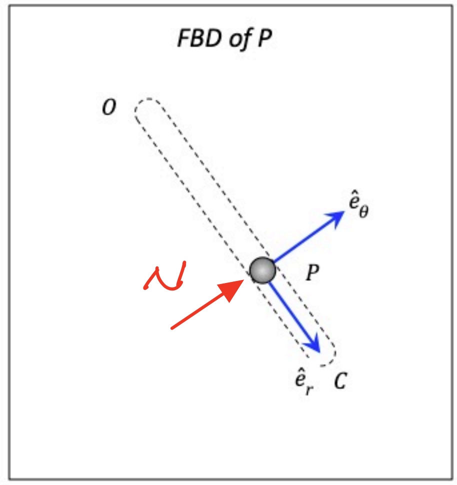
$(\vec{a}_{B/A})_{rel} = \vec{0}$

B is on <u>same</u> rigid body as the observer

Problem 3B(4 points):

Given: A block slides to the right with a constant speed v_O. A slotted arm OC is pinned to the block at O, with OC rotating in the counterclockwise direction with a constant rate of $\dot{\theta}$. Particle P is able to slide within the smooth slot in OC. At the instant of interest, P is sliding toward O with $\dot{r} < 0$. All motion for this system lies in a horizontal plane.





Circle the correct answer below regarding the contact of P with the slot:

$\boxed{\text{(a) P is in contact with side A of the slot.}}$

(b) P is in contact with side B of the slot.

(c) P is in contact with neither side of the slot.

(d) More information is needed for answering this question.

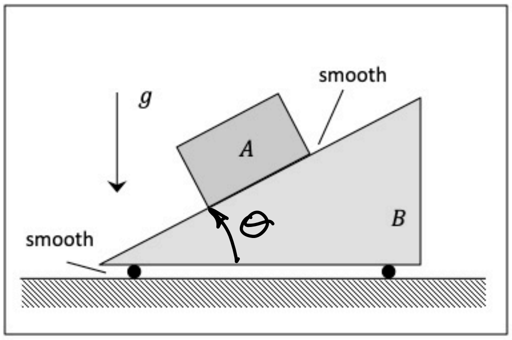
Justification: It is suggested that you draw and use an FBD above of P alone.

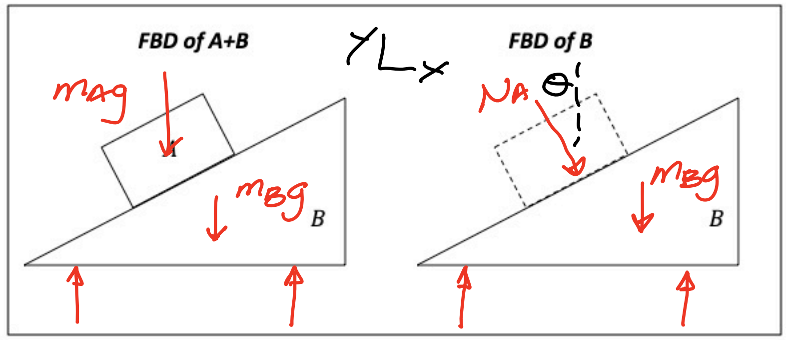
$$\sum F_\theta = N = m a_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$\hookrightarrow N = 2m\underbrace{\dot{r}}_{<0}\underbrace{\dot{\theta}}_{>0} \Rightarrow N < 0 \Rightarrow \text{contacting side A}$$

Problem 3C (4 points):

Given: Wedge-shaped block B is able to move along a smooth, horizontal surface. Block A is able to slide on the smooth, inclined surface of B. The system is released from rest.





Circle the correct TRUE/FALSE response for each of the two questions that follow regarding the subsequent motion of the system:

(1) Mechanical energy is conserved for block B alone: TRUE or $\boxed{\text{FALSE}}$

Justification: It is suggested that you draw and use an FBD above of block B alone.

B: $\sum F_x = N_A \sin\theta \neq 0 \Rightarrow$ N_A does work on B

$\Rightarrow$ energy NOT conserved for B

(2) Mechanical energy is conserved for blocks A and B together: $\boxed{\text{TRUE}}$ or FALSE

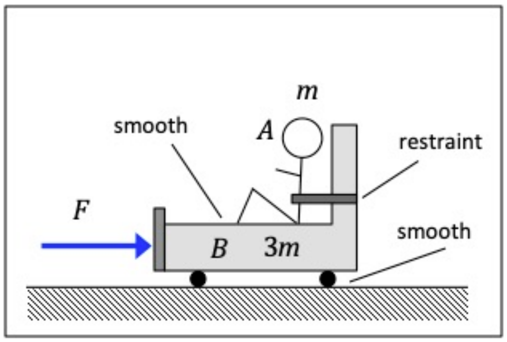
Justification: It is suggested that you draw and use an FBD above of blocks A and B together.

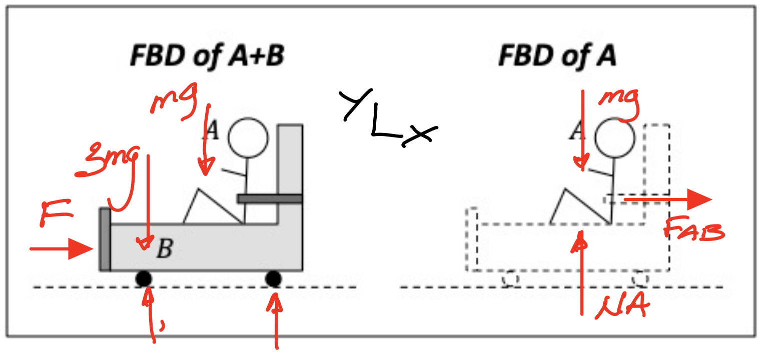
A+B: $\sum F_x = 0 \Rightarrow$ no work done on A+B

$\Rightarrow$ energy for A+B conserved

Problem 3D (4 points): ____

Given: Cart B (having a mass of $3m$) is able to move along a smooth, horizontal surface. This cart carries a passenger A (having a mass of m). A force F acts to the right on the cart, as shown in the figure. The passenger is held securely on the cart through a safety restraint, as shown in the figure. Let F_{AB} be the magnitude of the force on the passenger by the safety restraint.





Circle the correct response below regarding the F_{AB}:

$\boxed{\text{(a) } F > F_{AB}}$

(b) $F = F_{AB}$

(c) $F < F_{AB}$

(d) More information is needed to answer this.

Justification: It is suggested that you draw and use the FBDs above of A and B together and of A alone.

A+B: $\sum F_x = F = (3m+m)a = 4ma$

A: $\sum F_x = F_{AB} = ma$

$\therefore \quad F = 4F_{AB}$