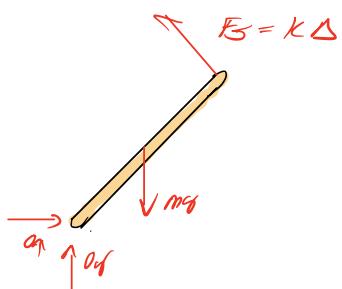


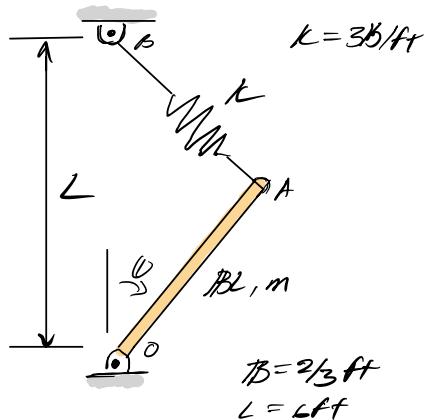
Given: The spring acting on a thin, homogeneous bar of weight $W = 15 \text{ lb}$, is unstretched when $\theta = 0$.

Find: Determine the angular speed of the bar at $\theta = 0^\circ$, if the bar just reaches $\theta = 90^\circ$ before it comes to rest.

FBD

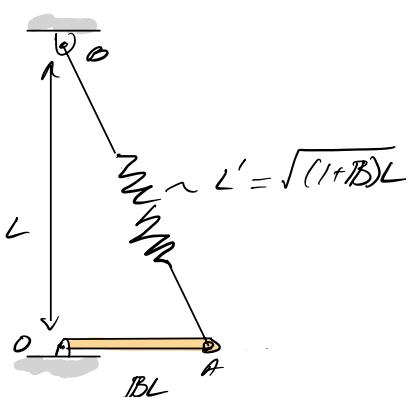
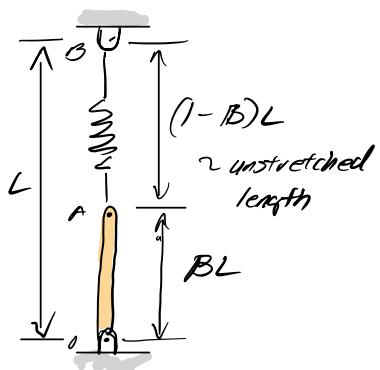


Position 1



$$B = 2/3 \text{ ft}$$

$$L = 6 \text{ ft}$$



Kinetics

$$T_1 + V_1 + \sum U_{1-2}^{nc} = T_2 + V_2$$

$$\frac{1}{2} I_0 \omega_1^2 + mg \frac{BL}{2} + 0 = \frac{1}{2} I_0 \cancel{\omega_2^2} + \frac{1}{2} k L^2 (\sqrt{1+B} + (1-B))^2$$

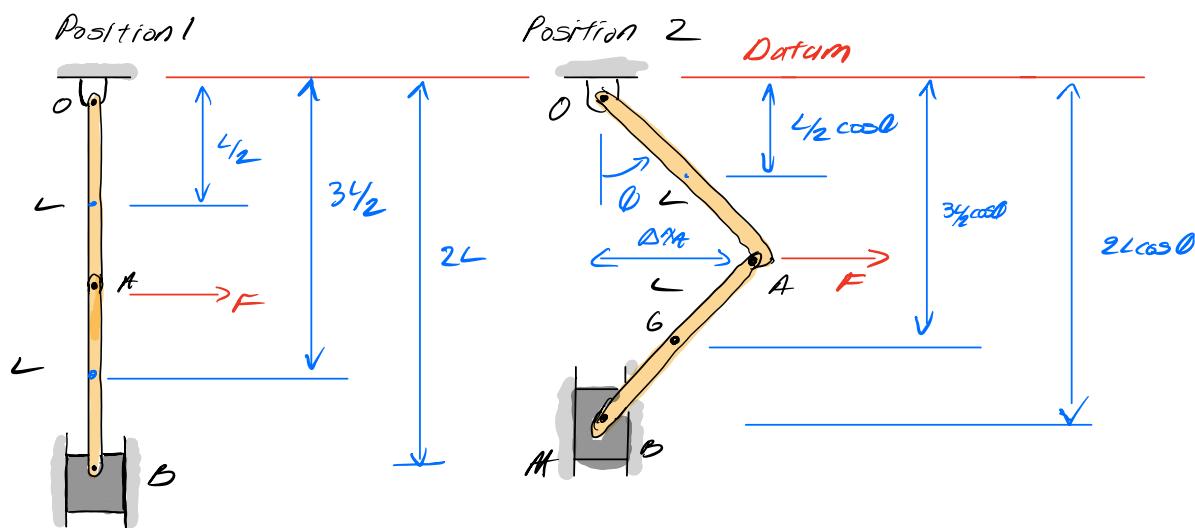
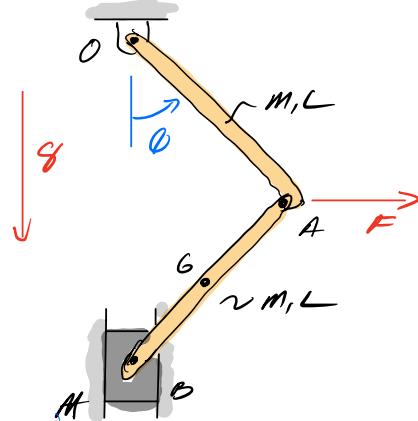
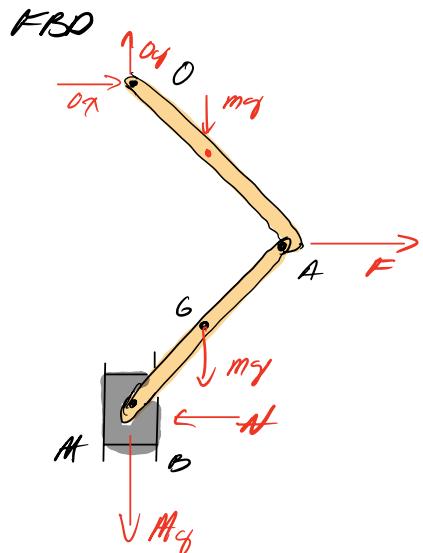
$$I_0 = \frac{1}{3} m L^2$$

Solving for ω_1 ,

$$\omega_1 = \sqrt{\frac{3k}{Bm} (\sqrt{1+B^2} + 1-B) - \frac{g}{BL}} \quad \underline{\text{Ans}}$$

Given: This system shown is released from rest when $\theta=0$. A constant force F acts horizontally at pin B for all time. Both links are homogeneous and have a mass of m . The slider block at B has a mass of M . The system moves in a vertical plane.

Find: Determine the speed of B at angle θ .



$$\text{Kinetics : } T_1 + V_1 + \sum U_{1-2}^{\text{nc}} = T_2 + V_2$$

$$T_1 = 0, \quad V_1 = -mgL/2 - \frac{3}{2}mgL - 2Agl$$

$$\sum U_{1-2}^{\text{nc}} = \int_{s_1}^{s_2} \vec{F} \cdot \hat{ds} = \int_{x_1}^{x_2} F \cdot \hat{dx} = F(x_2 - x_1) = Fl \sin\theta$$

$$= Fl \sin\theta$$

$$T_2 = \underbrace{\frac{1}{2} I_0 \omega_A^2}_{\text{Rod OA}} + \underbrace{\frac{1}{2} I_0 \omega_B^2}_{\text{Rod BA}} + \underbrace{\frac{1}{2} m v_G^2}_{\text{Circumferential Mass}} + \underbrace{\frac{1}{2} M V_B^2}_{\text{Circumferential Mass}}$$

Putting it together

$$-2mgL - 2Agl + fl \sin\theta = \frac{1}{2} I_0 \omega_A^2 + \frac{1}{2} I_0 \omega_B^2 + \frac{1}{2} m v_G^2 + \frac{1}{2} M V_B^2 - 2gL \cos\theta - 2Agl \cos\theta$$

$$I_0 = \frac{1}{3} m L^2, \quad I_0 = \frac{1}{12} M L^2$$

Simplifying

$$-2gL(m+M) + fl \sin\theta = -2gL \cos\theta (m+M) + \frac{1}{2} (\frac{1}{3} m L^2) \omega_A^2 + \frac{1}{2} (\frac{1}{12} M L^2) \omega_B^2 + \frac{1}{2} m v_G^2 + \frac{1}{2} M V_B^2$$

Kinematics using vector equation

$$1) \vec{v}_A = \vec{v}_0 + \vec{\omega}_{OA} \times \vec{r}_{OA}$$

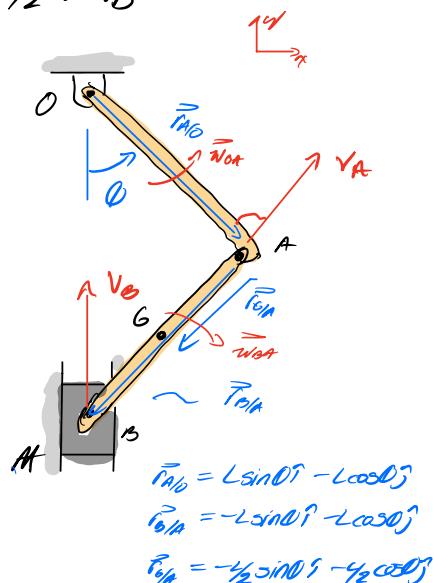
$$v_{Ax} i + v_{Ay} j = \vec{v}_0 + \vec{\omega}_{OA} \hat{k} \times L(\sin\theta i - \cos\theta j)$$

$$v_{Ax} i + v_{Ay} j = \vec{\omega}_{OA} L (\cos\theta i + \sin\theta j)$$

$$|\vec{v}_A| = v_A = \vec{\omega}_{OA} L$$

$$2) \vec{v}_B = \vec{v}_A + \vec{\omega}_{BA} \times \vec{r}_{BA}$$

$$v_{Bx} i = v_{Ax} i + v_{Ay} j + \vec{\omega}_{BA} \hat{k} \times L(-\sin\theta i - \cos\theta j)$$



$$\vec{V}_{Bj} = \vec{V}_{Ax} + \vec{V}_{Ay} + \omega_{BAL} (\cos\theta - \sin\theta)$$

$\omega_{Ax} \cos\theta$ $\omega_{Ay} \sin\theta$

$$\begin{aligned} \vec{V}_O &= \omega_{BAL} \cos\theta + \omega_{BAL} \cos\theta \\ \vec{V}_B &= \omega_{BAL} \sin\theta - \omega_{BAL} \sin\theta \end{aligned}$$

Solving... $\underline{\omega_{BA} = -\omega_{BA}}$

$$\underline{V_B = -2\omega_{BAL} L \sin\theta}$$

$$3) \vec{V}_B = \vec{V}_A + \vec{\omega}_{BAL} \times \vec{r}_{BA} \quad \text{Note } \omega_{BA} = \omega_{BAL}$$

$$V_{Axj} + V_{Ayj} = V_{Ax} + V_{Ay} + \omega_{BAL} \frac{L}{2} (-\sin\theta - \cos\theta)$$

$$V_{Axj} + V_{Ayj} = V_{Ax} + V_{Ay} + \omega_{BAL} \frac{L}{2} (\cos\theta - \sin\theta)$$

$$V_{Axj} + V_{Ayj} = (\omega_{BAL} \cos\theta + \omega_{BAL} \frac{L}{2} \cos\theta)j + (\omega_{BAL} \sin\theta - \omega_{BAL} \frac{L}{2} \sin\theta)j$$

$$V_{Axj} + V_{Ayj} = -\omega_{BAL} \frac{L}{2} \cos\theta - 3\omega_{BAL} \frac{L}{2} \sin\theta j$$

$$\underline{V_c^2 = \omega_{BAL}^2 \frac{L^2}{4} \cos^2\theta + q_{ext}^2 \frac{L^2}{4} \sin^2\theta} \quad \sim \text{Hopefully no mistake}$$



The final equations:

$$2gL(m+A)(\cos\theta + 1) = \frac{1}{2} \left(\frac{1}{3} m L^2 \omega_{BA}^2 + \frac{1}{2} m L^2 \omega_{BA}^2 + m V_c^2 + M V_B^2 \right)$$

$$\omega_{Ax} = -\omega_{BA},$$

$$V_B = -2\omega_{BAL} L \sin\theta,$$

$$V_c^2 = \omega_{BAL}^2 \frac{L^2}{4} (\cos^2\theta + q \sin^2\theta)$$

Solve for ω_{BA}

$$2gL(m+A)(\cos\theta + 1) = \frac{1}{2} \left(\frac{1}{3} m L^2 \omega_{BA}^2 + m \omega_{BA}^2 \frac{L^2}{4} (\cos^2\theta + q \sin^2\theta) + A \cdot 4 \omega_{BA}^2 L^2 \sin^2\theta \right)$$

$$4qL(m+A)(\cos\theta + 1) = \left(\frac{1}{3} m + m \frac{L^2}{4} (\cos^2\theta + q \sin^2\theta) + 4A \sin^2\theta \right) \omega_{BA}^2$$

$$\omega_{BA} = \sqrt{\frac{4qL(m+A)\cos\theta}{\left(\frac{1}{3}m + m \frac{L^2}{4} (\cos^2\theta + q \sin^2\theta) + 4A \sin^2\theta\right)}}$$

Ans

$$V_B = -2\omega_{BAL} L \sin\theta$$