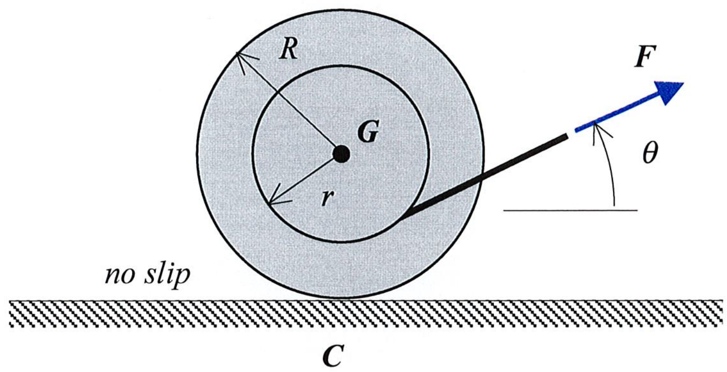
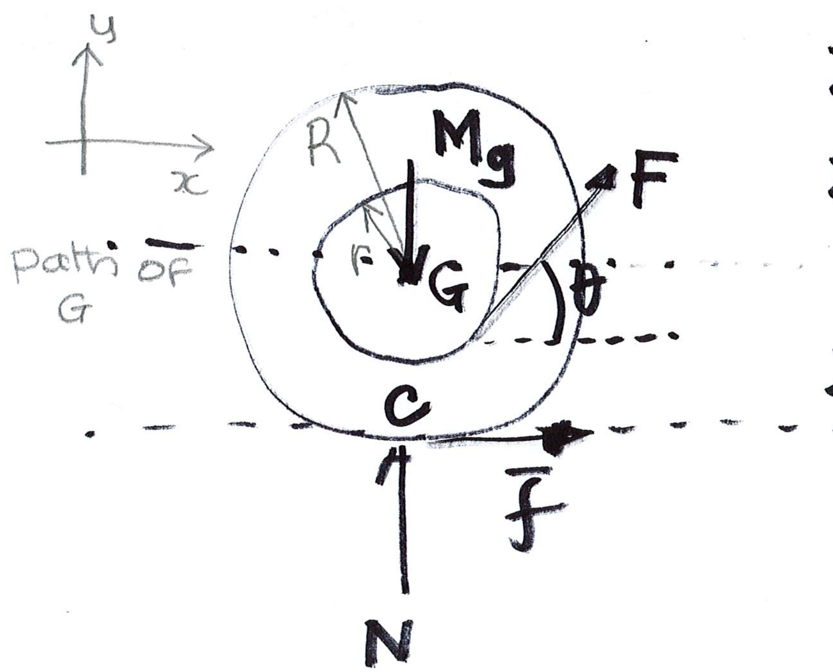
Example 5.A.12

Given: A spool has a mass of $m = 30$ kg, a centroidal radius of gyration $k_G = 0.25$ m, an outer radius of $R = 0.3$ m and an inner radius of $r = 0.1$ m. A constant force $F = 60$ N is applied at an angle of θ by a cord that is wrapped around the inner radius of the spool. The spool rolls without slipping on the rough horizontal surface.

Find: Determine the angular acceleration of the spool as a function of the angle θ.





$$\Sigma M_G = I_G \alpha \quad (1)$$

$$\Sigma F_x = F\cos\theta + f = m a_{Gx} \quad (2)$$

$$\Sigma F_y = -Mg + N + F\sin\theta = m a_{Gy} = 0 \quad (3)$$

....

Ans $\quad \vec{\alpha} = \dfrac{F(r - R\cos\theta)\,\hat{k}}{mR^2 + mk_G^2}$

Moment equation (ccw +ve)

Sum moments about G.

$$+F.r + fR = I_G \alpha \qquad (1a)$$

$$F\cos\theta + f = m a_{Gx} \qquad (2) \qquad \text{(from earlier)}$$

Kinematics

$$\bar{a}_G = \bar{a}_c + \bar{\alpha} \times \bar{r}_{G/C} - \omega^2 \bar{r}_{G/C}$$

$$a_{Gx}\hat{i} = a_c \hat{j} + \alpha\hat{k} \times (R\hat{j}) - \omega^2 R\hat{j}$$

moves on a straight line in $\pm x$ direction

rolling without slipping

$$a_G \hat{i} = a_c \hat{j} - \alpha R \hat{i} - \omega^2 R \hat{j}$$

$\hat{i}$ component:

$$a_{Gx} = -\alpha R \qquad (4)$$

Solve

(4) → (2) gives

$$F\cos\theta + f = -m\alpha R \qquad (2a)$$

(1a) − R (2a):

$$F(r - R\cos\theta) = (\underbrace{I_G}_{Mk_0^2} + MR^2)\alpha$$

$$\vec{\alpha} = \frac{F(r - R\cos\theta)}{(mk_0^2 + mR^2)}\hat{k} \quad \text{Ans}$$

$\cos\theta > r/R \longrightarrow$ Clockwise Rotation

$\cos\theta < r/R \longrightarrow$ Counter Clockwise

You can also do this problem by taking moments about C .

In this case, the <u>second</u> term in the moment about C equation is zero because the position vector of G with respect to C and the acceleration of C (rolling without slipping) are parallel: both are in the y direction and thus the cross product is zero.

The force **F** is the only force creating a moment about C; the line of action of all other forces go through C, so they do not result in any moments about C.

So the main challenge when we chose C as the point to take moments about is to do the geometry!

The moment about C created by the Force **F** is clockwise and =

$-F\cos(\theta)\ [R-(r/\cos(\theta))] = -F\ [R\cos(\theta)-r]$

Thus:

$-F\ [R\cos(\theta)-r] = I_C\ \alpha = [mk_0^2+mR^2]\ \alpha$

and thus: $\boldsymbol{\alpha} = F\ [r-R\cos(\theta)]/\ [mk_0^2+mR^2]\ \mathbf{k}$ rad/s^2