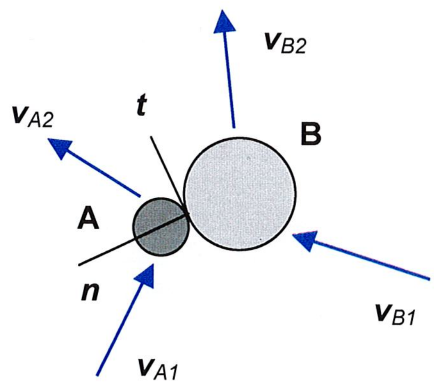
Discussion - Central Impact Subscript 1 JUST BEFORE IMPACT, SUBSCRIPT 2 JUST AFTER IMPACT

(1) $v_{At2} = v_{At1}$ — Block A TANGENT to line of impact

(2) $v_{Bt2} = v_{Bt1}$ — Block B TANGENT to line of impact

(3) $m_A v_{An2} + m_B v_{Bn2} = m_A v_{An1} + m_B v_{Bn1}$ — Blocks A and B: NORMAL to line of impact

(4) $e = -\dfrac{v_{Bn2} - v_{An2}}{v_{Bn1} - v_{An1}}$ — Coefficient of restitution equation NORMAL to line of impact



1. The above set of four equations can be used to analyze the central impact of a pair of particles A and B.

2. An *impulsive force* is a force with a large magnitude and very short duration in time. During impact, we will neglect the contributions of non-impulsive forces such as those due to weight and springs during the short time of impact Δt.

   - The impact force acting between particles A and B above is an impulsive force.
   - Reaction forces can be impulsive.
   - Gravitational and spring forces are not impulsive.

3. The application of the above four equations for A and B assumes that the impact force between A and B is the only impulsive forces acting on A and B during the impact.

4. In working problems using these four equations, be sure to clearly define the n-t coordinate axes prior to starting the problem.

5. The coefficient of restitution equation (the fourth equation above) is valid for only for velocity components along the line of impact (the normal direction, n). This equation is NOT valid for relating the total magnitudes of velocities or for the components of velocity along the plane of contact, t.

6. Mechanical energy in System AB is NOT conserved during impact, except for the special case of $e = 1$. (How do we know this?) You should NEVER attempt to use the work-energy equation during impact.

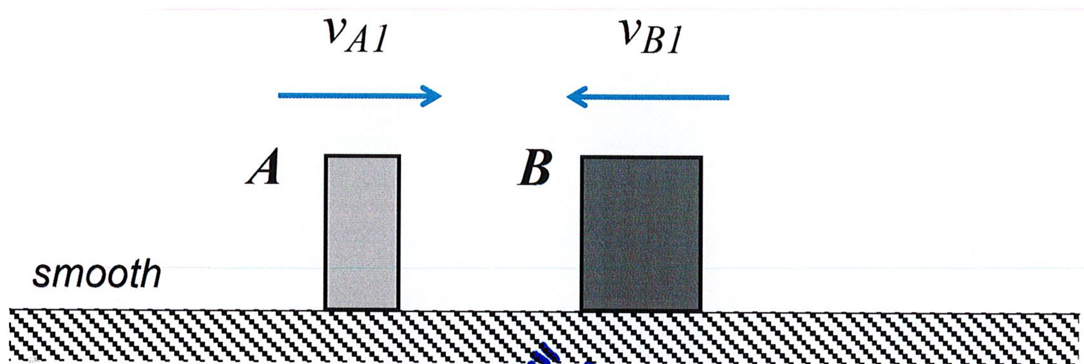
Chapter 4

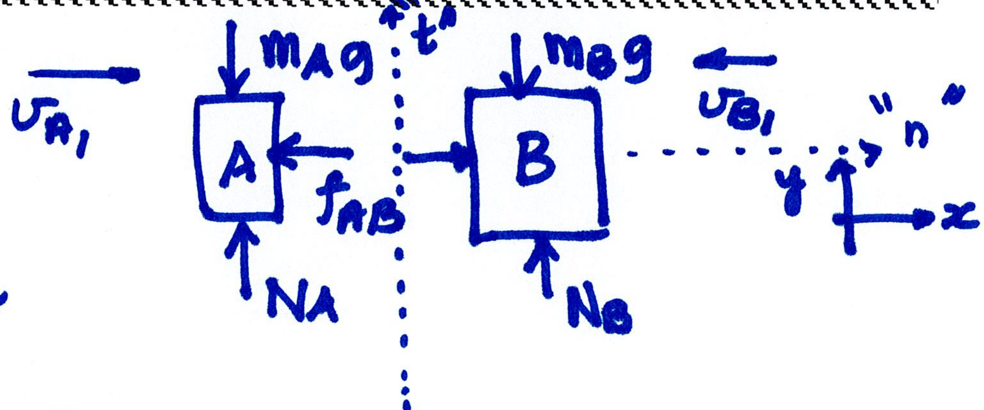
Example 4.C.10

Given: Blocks A and B (having masses of m_A and m_B, respectively) are initially moving to the right and left, respectively, on a smooth horizontal surface with speeds of v_{A1} and v_{B1}, respectively. At some instant in time, A strikes B. The coefficient of restitution of this impact is e. As a result of the impact, block B becomes stationary.

Find: Determine the initial speed v_{B1} of block B.

Use the following parameters in your analysis: $m_A = 2$ kg, $m_B = 3$ kg, $v_{A1} = 1.5$ m/s and $e = 0.4$.





"n" – x

"t" – y

no change in t direction

$v_{At_2} = v_{At_1} = 0 \quad (1)$

$v_{Bt_2} = v_{Bt_1} = 0 \quad (2)$

A&B together no forces in n-direction

conservation of linear momentum

$$m_A \underbrace{v_{An_2}}_{v_{A_2}} + m_B \cancel{v_{Bn_2}}^{\,0} = m_A \underbrace{v_{An_1}}_{v_{A_1}} + m_B \underbrace{v_{Bn_1}}_{\{= -v_{B_1}\}}$$

v_{A_2}: Assumed to the right. (expect a negative result.)

$v_{Bn_2} \to 0$: stops after impact

v_{A_1}: moving to the right

$-v_{B_1}$: moves to left.

<u>Rearrange</u>

$$m_A v_{A_2} + m_B v_{B_1} = m_A v_{A_1} \quad (*)$$

Coefficient of Restitution Equation

$$\varepsilon(v_{An_1} - \underbrace{v_{Bn}}_{(-v_{B_1})}) = -(\underbrace{v_{An_2}}_{v_{A_2}} - \cancel{v_{Bn_2}}^{\,0}) \quad (4)$$

$v_{An_1} \to v_{A_1}$

$(-v_{B_1})$: moves to left

v_{Bn_2}: Stops

<u>Rearrange</u>

$$v_{A_2} + \varepsilon v_{B_1} = -\varepsilon v_{A_1} \quad (**)$$

$m_A \times (**) - (*)$ removes v_{A_2} leaving

$$(\varepsilon m_A - m_B) v_{B_1} = (-m_A - \varepsilon m_A) v_{A_1}$$

$$v_{B_1} = \frac{+\, m_A(1+\varepsilon)\, v_{A_1}}{(m_B - \varepsilon m_A)} \text{ m/s}$$

Putting in the numbers:

$$v_{B1} = \frac{2(1+0.4)(1.5)}{3-(0.4)2} = \frac{3 \times \cancel{1.4}^{0.7}}{\cancel{2.2}_{1.1}}$$

$\boxed{v_{B_1} = \frac{21}{11} \text{ m/s};}$ $\bar{v}_{B1} = -\frac{21}{11}\hat{i}$ m/s to the left

Not asked for but what about v_{A_2}?

$\varepsilon(*) - m_B(**)$

$$(\varepsilon m_A - m_B)v_{A2} = (m_A \varepsilon + m_B \varepsilon)v_{A_1}$$

$$v_{A_2} = \frac{(m_A + m_B)\varepsilon\, v_{A_1}}{-(m_B - \varepsilon m_A)}$$

Putting in the numbers

$$v_{A2} = -\frac{(2+3)(0.4)(1.5)}{2.2} = -\frac{3.0}{2.2} = -\frac{15}{11}\text{ m/s}$$

$\bar{v}_{A2} = -15/11\,\hat{i}$ m/s to the left

Let's check that: $m_A v_{An_1} + m_B v_{Bn_1}$

$= m_A v_{An_2} + m_B v_{Bn_2}$

LHS: $$m_A v_{A1} + \frac{m_B m_A (1+e) v_{A_1}}{(m_B - \varepsilon m_A)}$$

$$m_A v_{A_1} - m_B v_{B_1} = m_A v_{A_2} + m_B \cancel{v_{B_2}}^{\,0}$$

LHS (Left Hand Side)

$$2(1.5) - \frac{3(21)}{11} = \frac{33 - 63}{11} = -\frac{30}{11}$$

RHS

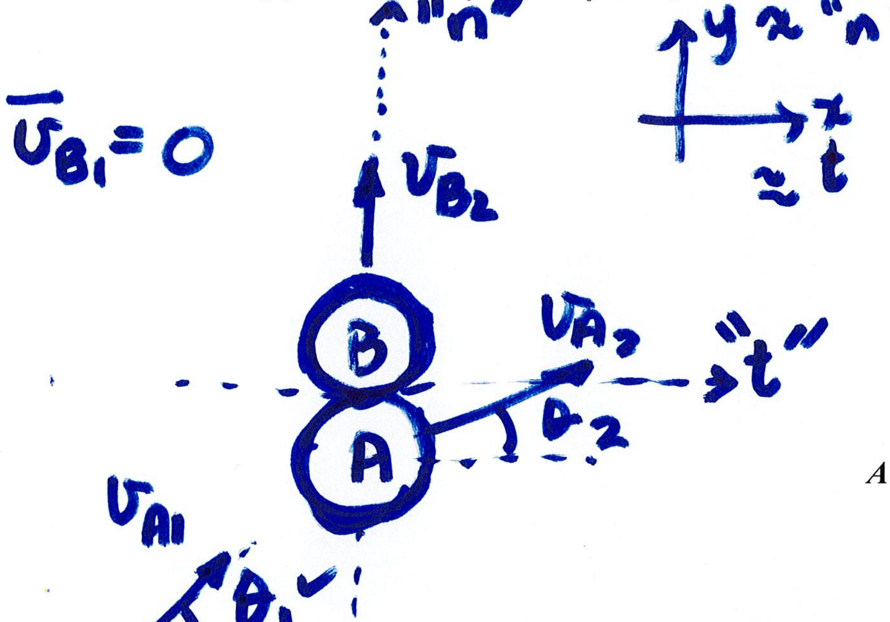
$$(2)\left(-\frac{15}{11}\right) = -\frac{30}{11} = LHS$$

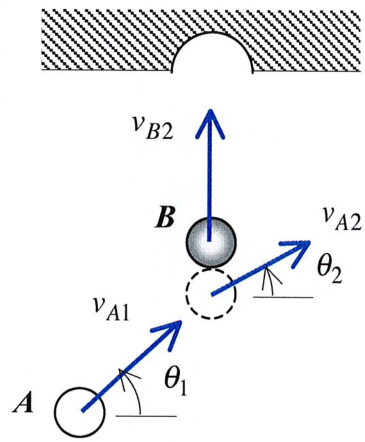
Example 4.C.12

Given: Cue ball A strikes a stationary object ball B, with a speed v_{A1} as shown in the figure below. The coefficient of restitution for this impact is e. After impact, A moves along a line defined by the angle θ_2, and B moves directly to the side pocket.

Find: Determine the numerical value of the rebound angle θ_2 of A, assuming the masses of A and B are the same.

Use the following parameters in your analysis: $\theta_1 = 45°$ and $e = 0.9$.





x direction
(t direction)

$$v_{A2_x} = v_{A1_x}$$

$$v_{A2x} = v_{A_1}\cos\theta_1 \quad \text{unchanged}$$

y-direction ("n direction) A&B together

$$m_A v_{Ay_2} + m_B v_{By_2} = m_A v_{Ay_1} + m_B \cancel{v_{By_1}}^{\,0}$$

$$\cancel{e = -\frac{(v_{By_2} - v_{Ay_2})}{(v_{By_1} - v_{Ay_1})}}$$

Ooops!

4.C.12 (cont.)

Rearrange:

$$\cancel{m_A}\, \underline{v_{Ay_2}} + \cancel{m_B}\, \underline{v_{By_2}} = \cancel{m_A}\, v_{A_1} \sin\theta_1 \quad (*)$$

$m_A = m_B$

$$\varepsilon\left(v_{Ay_2} - v_{By_2}\right) = -\left(v_{Ay_1} - \cancelto{0}{v_{By_1}}\right)$$

Rearrange:

$$\varepsilon\, \underline{v_{Ay_2}} - \varepsilon\, \underline{v_{By_2}} = -v_{A_1}\sin\theta \quad (**)$$

Want to find v_{Ay_2} so:

$\varepsilon(*) + (**)$

$$2\varepsilon v_{Ay_2} = (\varepsilon - 1)\, v_{A_1} \sin\theta_1$$

$$\therefore v_{Ay_2} = \frac{(\varepsilon - 1)}{2\varepsilon} v_{A_1} \sin\theta_1$$

$$\therefore \overline{v}_{A_2} = v_{A_1} \cos\theta_1\, \hat{i} + \frac{(\varepsilon - 1)}{2\varepsilon} v_{A_1} \sin\theta_1\, \hat{j}$$

$$\therefore \theta_2 = \tan^{-1} \frac{(\varepsilon - 1)\, \cancel{v_{A_1}} \sin\theta_1}{2\varepsilon\, \cancel{v_{A_1}} \cos\theta_1}$$

4.C.12 (cont.)

$$\theta_2 = -\tan^{-1}\left[\frac{(1-\varepsilon)}{2\varepsilon}\tan \underset{\substack{\uparrow \\ 45^\circ}}{\theta_1}\right]$$

$$= -\tan^{-1}\left[\frac{1-0.9}{1.8}\cdot(1)\right]$$

$$\theta_2 = -\tan^{-1}\left[\frac{1}{18}\right] \longrightarrow$$