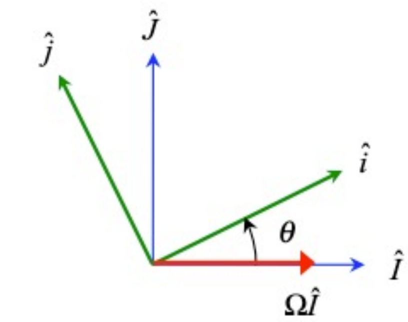
ME 274 – Spring 2023 **Name** SOLUTION
8:30 section – Quiz 4

Consider two sets of coordinate axes, having $\hat{i}\hat{j}\hat{k}-$unit vectors and $\hat{I}\hat{J}\hat{K}-$unit vectors , as shown in the figure. The angular velocity of the coordinate system for the $\hat{i}\hat{j}\hat{k}-$unit vectors is known to be:

$$\vec{\omega} = \dot{\theta}\hat{k} + \Omega\hat{I}$$



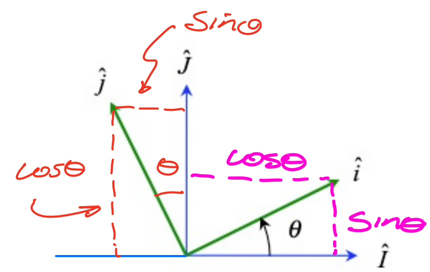
All answers below should be in terms of, at most: θ , $\dot{\theta}$, Ω and the unit vectors defined above.

Problem 1

Write down the $\hat{i}\hat{j}-$unit vectors in terms of the $\hat{I}\hat{J}-$unit vectors :

$\hat{i} =$ <u>$\cos\theta$</u> $\hat{I} +$ <u>$\sin\theta$</u> $\hat{J}$

$\hat{j} =$ <u>$-\sin\theta$</u> $\hat{I} +$ <u>$\cos\theta$</u> $\hat{J}$

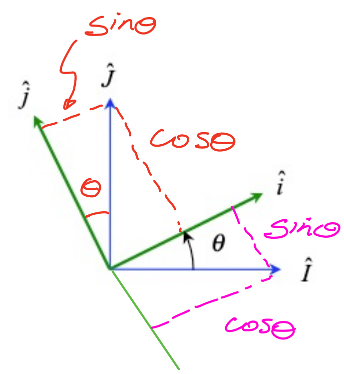


Problem 2

Write down the $\hat{I}\hat{J}-$unit vectors in terms of the $\hat{i}\hat{j}-$unit vectors :

$\hat{I} =$ <u>$\cos\theta$</u> $\hat{i} +$ <u>$(-\sin\theta)$</u> $\hat{j}$

$\hat{J} =$ <u>$\sin\theta$</u> $\hat{i} +$ <u>$\cos\theta$</u> $\hat{j}$



Problem 3

Write down the $\vec{\omega}$ vector in terms of the $\hat{i}\hat{j}\hat{k}-$unit vectors : $\vec{\omega} = \dot{\theta}\hat{k} + \Omega(\cos\theta\hat{i} - \sin\theta\hat{j})$

$\hat{\omega} =$ <u>$\Omega\cos\theta$</u> $\hat{i} +$ <u>$(-\Omega\sin\theta)$</u> $\hat{j} +$ <u>$\dot{\theta}$</u> $\hat{k}$

Problem 4 – BONUS question

If Ω and $\dot{\theta}$ are constant, write down the angular acceleration vector $\vec{\alpha}$ vector in terms of the $\hat{i}\hat{j}\hat{k}-$unit vectors :

$\vec{\alpha} =$ <u>$(-\dot{\theta}\Omega\sin\theta)$</u> $\hat{i} +$ <u>$(-\dot{\theta}\Omega\cos\theta)$</u> $\hat{j} +$ <u>0</u> $\hat{k}$

$$\vec{\alpha} = \cancel{\dot{\Omega}}\hat{I} + \Omega\cancel{\dot{\hat{I}}} + \cancel{\ddot{\theta}}\hat{k} + \dot{\theta}\dot{\hat{k}} = \dot{\theta}(\vec{\omega}\times\hat{k})$$

$$= \dot{\theta}\left[\Omega\cos\theta\hat{i} - \Omega\sin\theta\hat{j} + \dot{\theta}\hat{k}\right]\times\hat{k}$$

$$= \dot{\theta}\Omega(-\sin\theta\hat{i} - \cos\theta\hat{j})$$