Problem 1 (20 points):

Given: A particle P is traveling in the xy-plane on a path given by:

$$y = cx^2 + b, \tag{1}$$

where x and y are given in meters and c and b are constants with units 1/ meters and meters, respectively, and in such a way that

$$\dot{x} = 2 \text{ m/s} \quad \text{and} \quad \ddot{x} = -4 \text{ m/s}^2. \tag{2}$$

Find: For the position of $x = -0.5$ m: assume $c > 0$, $b > 0$. Note that your final answers should be in terms of at most, the known variables c, b.

(a) Determine the Cartesian components of the velocity and acceleration of P.

(b) Make a sketch of the velocity and acceleration vectors found in (a), as well as the path unit vectors $\hat{e}_t$ and $\hat{e}_n$ on the figure provided below.

(c) Determine the rate of change of speed.

(d) Is the speed of P increasing, decreasing or constant? Provide an explanation for your response.

Solution:

(a) Determine the Cartesian components of the velocity and acceleration of P.

First find $\dot{y}$ and $\ddot{y}$

$\dot{y} = dy/dt = dy/dx \; dx/dt = 2cx\dot{x} = v_y$

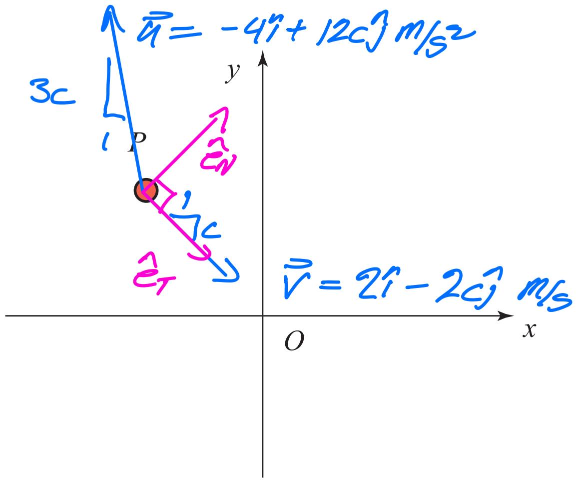
$\ddot{y} = dv_y/dt = \frac{dv_y}{dx}\frac{dx}{dt} + \frac{dv_y}{dv_x}\frac{dv_x}{dt} = 2c\dot{x}\dot{x} + 2cx\ddot{x}$

$\hookrightarrow$ Note you could use implicit differentiation of v_y realizing $v_y \sim f(x, \dot{x}, t)$

$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} = \dot{x}\hat{i} + 2cx\dot{x}\hat{j} = \underline{2\hat{i} - 2c\hat{j}}$ m/s $\sim$ Ans

$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} = \ddot{x}\hat{i} + 2c(\dot{x}^2 + x\ddot{x})\hat{j} = \underline{-4\hat{i} + 12c\hat{j}}$ m/s²

(b) Make a sketch of the velocity and acceleration vectors found in (a), as well as the path unit vectors $\hat{e}_t$ and $\hat{e}_n$ on the figure provided below.



~ Ans

Figure 1 -Please draw on this figure.

(c) Determine the rate of change of speed.

First find $\hat{e}_T = \dfrac{\vec{v}}{|v|} = \dfrac{2\hat{i} - 2c\hat{j}}{\sqrt{2^2 + 2^2c^2}} = \dfrac{\hat{i} - c\hat{j}}{\sqrt{1+c^2}}$

Now, $\vec{a} = \dot{v}\hat{e}_T + \dfrac{v^2}{\rho}\hat{e}_N$ $\qquad \hat{e}_T \cdot \hat{e}_T = 1$, $\hat{e}_T \cdot \hat{e}_N = 0$

$$\hat{e}_T \cdot \vec{a} = \dot{v}\cancelto{1}{\hat{e}_T \cdot \hat{e}_T} + \frac{v^2}{\rho}\cancelto{0}{\hat{e}_N \cdot \hat{e}_T}$$

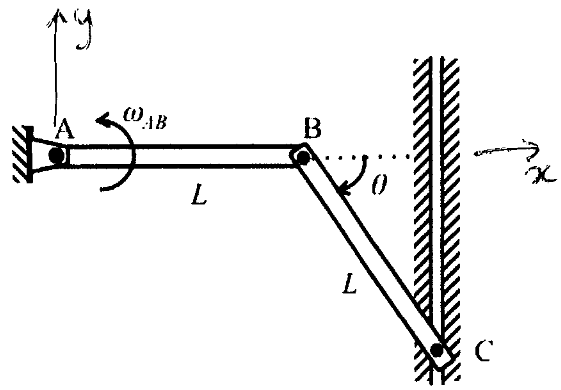
$$\dot{v} = \vec{a} \cdot \hat{e}_T = \left(-4\hat{i} + 12c\hat{j}\right) \cdot \left(\frac{\hat{i} - c\hat{j}}{\sqrt{1+c^2}}\right) = \frac{-4}{\sqrt{1+c^2}}\left(1 + 3c^2\right) \text{ m/s}^2$$

(d) Is the speed of P increasing, decreasing or constant? Provide an explanation for your response.

From the figure in (b) the tangential component of $\vec{a}$ points in the opposite direction of $\vec{v}$ and this agrees with the computation from (c) where $\dot{v} < 0$ for all values of c. Thus the speed of P is decreasing.

Problem 2 (20 points)

Given: Two links AB and BC are connected at B by a pin joint. Point A is a fixed point, and Point C is free to move in the straight slot shown.



Link AB is rotating at a counter clock wise constant rate at ω_{AB} rad/s.
The links are both of length L meters.

The slot is perpendicular to link AB at the instant.

Find:
(a) the velocity and acceleration of point B,
(b) the angular velocity of bar BC,
(c) the velocity of point C,
(d) the angular acceleration of bar BC,
(e) the acceleration of point C.

Note that your final answers should be in terms of (at most) the known variables: L, θ, and ω_{AB}.
Express your answers as vectors in cartesian coordinates.

Solution:

(a) the velocity and acceleration of point B

Point B moves on a circle of radius L.

$$\vec{v}_B = \underbrace{\cancel{\vec{v}_A}}_{0} + \vec{\omega}_{AB}\times\vec{r}_{B/A} = +\omega_{AB}\hat{k}\times L\hat{i} = \underline{+\omega_{AB}L\hat{j}\ m/s} \rightarrow \text{Ans}$$

$$\vec{a}_B = \underbrace{\cancel{\vec{a}_A}}_{0} + \underbrace{\cancel{\vec{\alpha}_{AB}}}_{0}\times\vec{r}_{B/A} - \omega_{AB}^2\vec{r}_{B/A} = \underline{-\omega_{AB}^2 L\hat{i}\ m/s^2} \rightarrow \text{Ans}$$

ω_{AB} constant

(b) the angular velocity of bar BC

From C: $\vec{v}_B = \vec{v}_C + \vec{\omega}_{BC}\times\vec{r}_{B/C}$

$$= \omega_{AB}L\hat{j} = v_C\hat{j} + \omega_{BC}\hat{k}\times(-L\cos\theta\hat{i} + L\sin\theta\hat{j})$$

$$\omega_{AB}L\hat{j} = (v_C - L\cos\theta\,\omega_{BC})\hat{j} - L\omega_{BC}\sin\theta\,\hat{i}$$

$$\hat{i}:\quad 0 = -L\omega_{BC}\sin\theta \longrightarrow \underline{\vec{\omega}_{BC} = 0\hat{k}\ rad/s}$$

(c) the velocity of point C

$\hat{j}$: $\omega_{AB} L = v_C - L\cos\theta\, \cancel{\omega_{BC}}$ $\rightarrow 0$ at this instant

$\vec{v}_C = \underline{\omega_{AB} L\, \hat{j}\ m/s}$ → Ans(c)

(d) the angular acceleration of bar BC

$\bar{a}_B = \bar{a}_C + \bar{\alpha}_{BC} \times \bar{r}_{B/C} - \cancel{\omega_{BC}^2}\, \vec{r}_{B/C}$ $\rightarrow 0$ at this instant

$\bar{a}_B = a_C\hat{j} + \alpha_{BC}\hat{k} \times (-L\cos\theta\,\hat{i} + L\sin\theta\,\hat{j})$

$\bar{a}_B = a_C\hat{j} - \alpha_{BC} L\cos\theta\,\hat{j} - \alpha_{BC} L\sin\theta\,\hat{i}$ (3)

From part (a) we have $\bar{a}_B = -\omega_{AB}^2 L\,\hat{i}$ (4)

$\therefore \hat{i}$ of (3)(4): $+\cancel{L}\omega_{AB}^2 = +\alpha_{BC}\cancel{L}\sin\theta$

$\therefore \alpha_{BC} = \dfrac{\omega_{AB}^2}{\sin\theta}$ $\bar{\alpha}_{BC} = \dfrac{\omega_{AB}^2}{\sin\theta}\hat{k}\ rad/s^2$ → Ans(d)

(e) the acceleration of point C

j: components of (3) and (4)

$a_C - \alpha_{BC} L\cos\theta = 0 \quad \therefore a_C = \alpha_{BC} L\cos\theta$

$\vec{a}_C = \alpha_{BC} L\cos\theta\,\hat{j}$

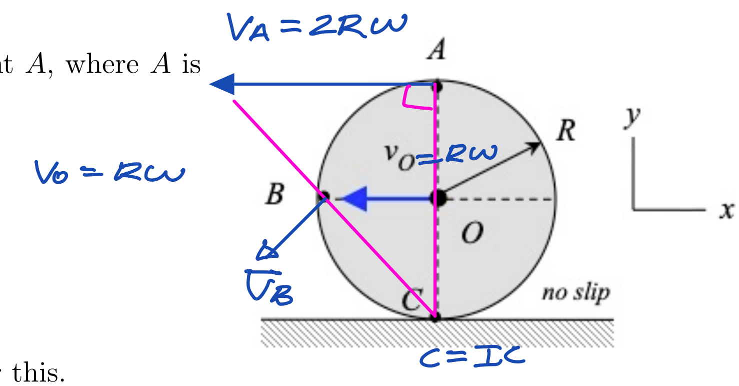
$\bar{a}_C = \dfrac{\omega_{AB}^2}{\sin\theta} \cdot L\cos\theta\,\hat{j}\ \underline{m/s^2}$ → ANS (e)

NOTE: *You are NOT asked to provide justification for your answers here in Problem 3. A correct response will receive full credit. Any work provided will not be graded, only the final answer.*

Problem 3A(8 points): ______________________

Given: A disk rolls without slipping to the left, with its center O having a constant speed of v_o.

Circle the correct answer or each of the three questions that follow.



(1) **2 points** - Let $\vec{v}_A$ be the velocity of point A, where A is on the circumference of the disk:

- (a) $|\vec{v}_A| > |\vec{v}_O|$ (See IC)
- (b) $|\vec{v}_A| = |\vec{v}_O|$
- (c) $|\vec{v}_A| < |\vec{v}_O|$
- (d) more information is needed to answer this.

(2) **3 points** - Let $\vec{a}_B$ and $\vec{a}_C$ be the accelerations of points B and C, respectively, where B and C are on the circumference of the disk. For the position shown:

$$\vec{a}_B = \cancel{\vec{a}_O}^{\,0} + \cancel{\vec{\alpha}}^{\,0} \times \vec{r}_{B/O} - \omega^2 r_{B/O}$$
$$\vec{a}_C = \cancel{\vec{a}_O}^{\,0} + \cancel{\vec{\alpha}}^{\,0} \times \vec{r}_{B/O} - \omega^2 \vec{r}_{C/O}$$

- (a) $|\vec{a}_B| > |\vec{a}_C|$
- (b) $|\vec{a}_B| = |\vec{a}_C|$
- (c) $|\vec{a}_B| < |\vec{a}_C|$
- (d) more information is needed to answer this.

(3) **3 points** - For the position shown:

- (a) the velocity of point B has only a y-component.
- (b) the velocity of point B has only an x-component.
- (c) the velocity of point B has both non-zero x- and y-components. (See IC)
- (d) the velocity of point B is zero.
- (e) more information is needed to answer this.

Problem 3B(6 points): ______________________

Given: For the position shown, point P is moving in the xy-plane with a velocity and an acceleration (in polar coordinates) of: $\vec{v} = (-30\hat{e}_r - 40\hat{e}_\theta)$ in/s and $\vec{a} = -10\hat{e}_r$ in/s^2. θ

Circle the correct answer for each of the two questions that follow.

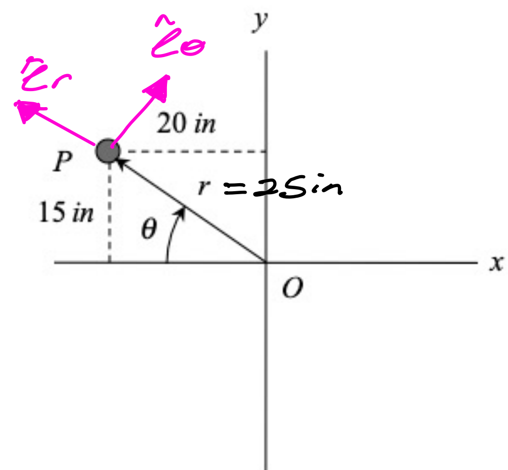
(1) **3 points** - Let ρ be the radius of curvature of the path of P. Then:

$\boxed{\text{(a) } \rho > r}$

(b) $\rho = r$

(c) $\rho < r$

(d) more information is needed to answer this.



$$\dot{v} = (-10\hat{e}_r)\cdot\left[\frac{-30\hat{e}_r - 40\hat{e}_\theta}{50}\right]$$
$$= 6 \text{ in/s}^2$$
$$\rho = \frac{v^2}{\sqrt{|\vec{a}|^2 - \dot{v}^2}} = \frac{50^2}{\sqrt{10^2 - 6^2}}$$
$$= \frac{625}{2} > r$$

(2) **3 points** - For the position shown:

$\boxed{\text{(a) } \ddot{r} > 0}$

(b) $\ddot{r} = 0$

(c) $\ddot{r} < 0$

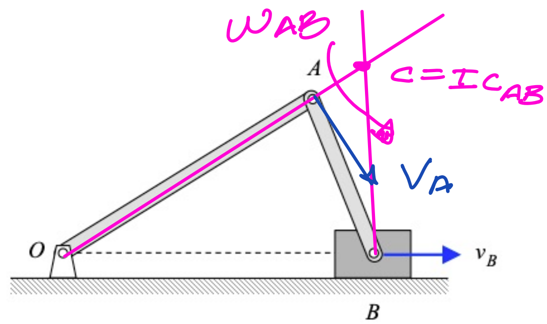
(d) more information is needed to answer this.

$$\ddot{r} - r\dot{\theta}^2 = 0 \Rightarrow \ddot{r} = r\dot{\theta}^2 > 0$$

Problem 3C (6 points): ______

Given: The slider-crank mechanism shown below is made up of links OA and AB, with end O of OA pinned to ground. Links OA andAB are pinned together at A. A slider pinned to AB at B moves along a flat surface to the right with a speed of v_B. The figure provided is drawn to scale.

Circle the correct answer for each of the two questions that follow.



(1) **3 points** - Let v_A represent the speed of A for the position shown. For this position:

(a) $v_A > v_B$

$\overline{AC} < \overline{BC}$

(b) $v_A = v_B$

(c) $v_A < v_B$

(d) more information is needed to answer this.

(2) **3 points** - Let ω_{OA} and ω_{AB} represent the angular speed of OA and AB, respectively, for the position shown. For this position:

(a) $\omega_{OA} > \omega_{AB}$

$\overline{OA} > \overline{AC}$

(b) $\omega_{OA} = \omega_{AB}$

(c) $\omega_{OA} < \omega_{AB}$

(d) more information is needed to answer this.