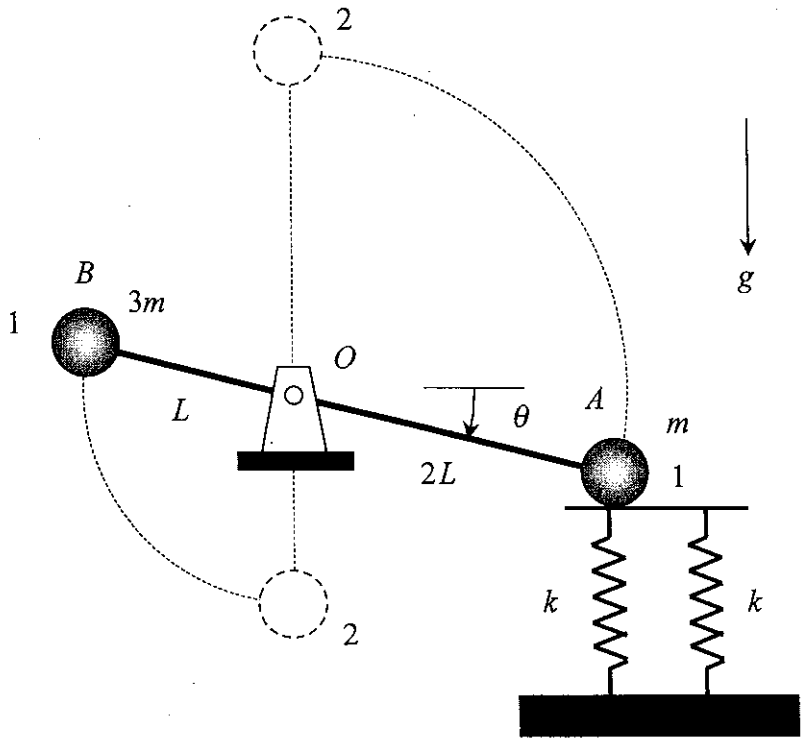
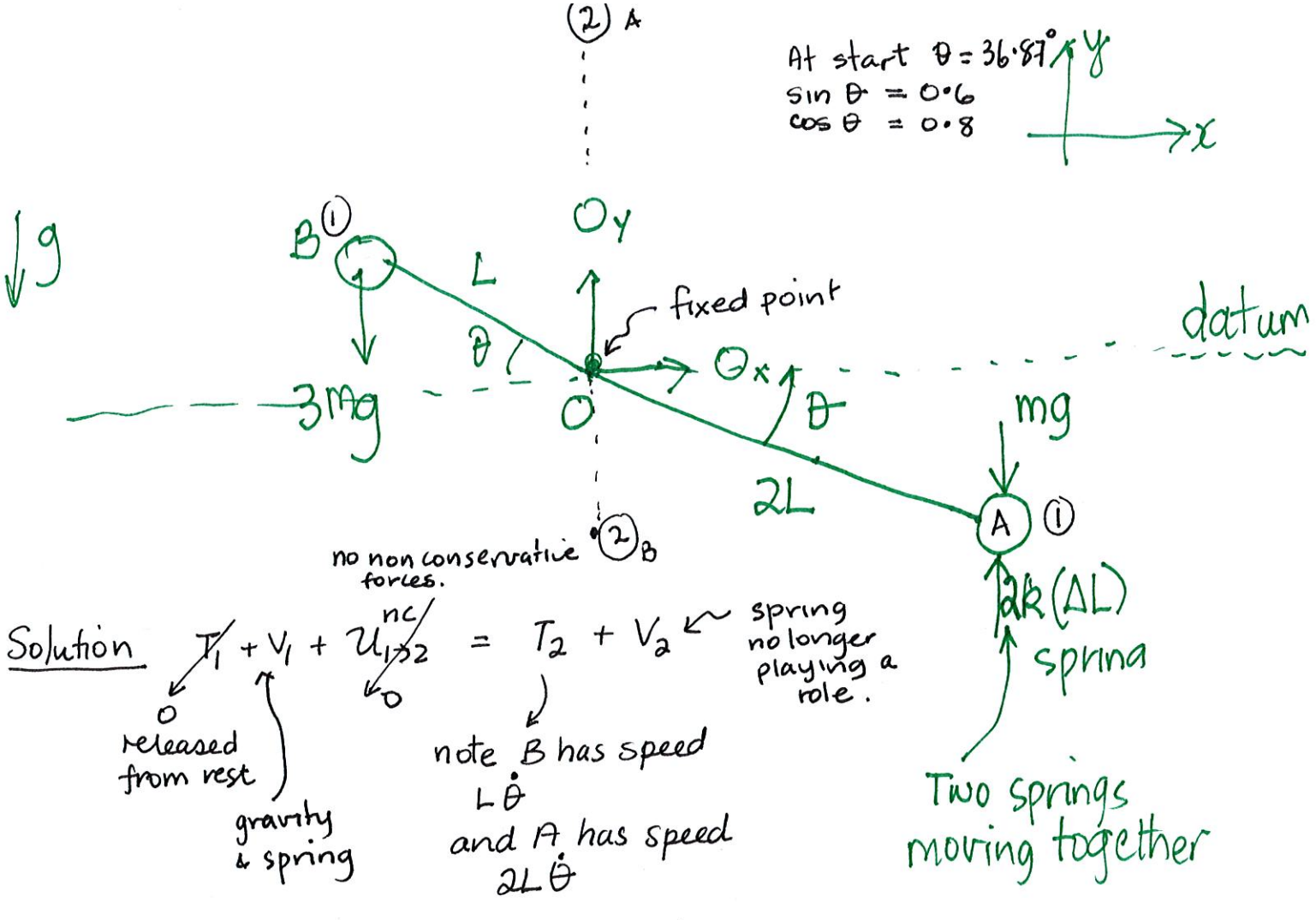
Example 4.B.7

Given: Particles A and B, having masses of m and $3m$, respectively, are connected by rigid bar AB, with AB having negligible mass. Bar AB is pinned to ground with a pin joint at O. This system is released from rest at position 1 with $\theta = \theta_1$, with A in contact with a pair of identical springs, as shown in the figure. Each spring has a stiffness of k, and the springs are unstretched when $\theta = 0$. Assume the dimensions of the particles to be negligible.

Find: Determine the speeds of particles A and B at position 2, where in position 2 particle A is directly above O.

Use the following parameters in your analysis: $\theta_1 = 36.87°$, $L = 0.1$ m, $m = 10$ kg and $k = 100$ N/m.





no non conservative forces.

Solution $T_1 + V_1 + \mathcal{U}_{1\to 2}^{nc} = T_2 + V_2$ ← spring no longer playing a role.

T_1: released from rest → 0

V_1: gravity & spring

$\mathcal{U}_{1\to 2}^{nc}$ → 0

T_2: note B has speed $L\dot{\theta}$ and A has speed $2L\dot{\theta}$

V_2

Two springs moving together

both are moving in circles: B of radius L, A of radius 2L

V_1 : Gravity and Springs:

$$= 3mgL\sin\theta - mg2L\sin\theta + \frac{1}{2}k_{sp}(2L\sin\theta)^2 \times 2 \quad (1)$$

↑ below datum

2 springs compressed by the same amount

~~extension~~ compression of spring at each start

$$V_1 = mgL\sin\theta + k_{sp}(4L^2\sin^2\theta)$$

$$T_2 = \frac{1}{2}(3m)(L\dot{\theta})^2 + \frac{1}{2}m(2L\dot{\theta})^2 = \frac{1}{2}m\dot{\theta}^2L^2\{3+4\} = \frac{7}{2}m\dot{\theta}^2L^2$$

V_2 = just gravity spring is not playing a role. stopped contributing when $\theta = 0$.

$$= -3mgL + 2mgL = -mgL$$

(below datum)

Substituting these expressions into equation (1) gives:

$$\therefore \quad mgL\sin\theta + 4k_{sp}L^2\sin^2(\theta) = \tfrac{7}{2}mL^2\dot{\theta}^2 - mgL$$

Solve for $\dot{\theta}$, then $v_A = 2L\dot{\theta}$ and $v_B = L\dot{\theta}$

$$\dot{\theta}^2 = \frac{mgL(1+\sin\theta) + 4k_{sp}L^2\sin^2\theta}{7/2\, mL^2} \longrightarrow$$

$$\dot{\theta}^{\bullet} = \sqrt{\frac{10(9.81)(0.1)(1+0.6) + 4(100)(0.1)^2(0.6)^2}{7/2(10)(0.1)^2}} \text{ rad/s}$$

$$= \sqrt{\frac{1.6\times 9.81 + 1.44}{0.35}} = 7.60 \text{ rad/s}^{\bullet}$$

$$\therefore \quad v_A = 2(0.1)(7.6) = 1.52 \text{ m/s}$$
$$v_B = (0.1)(7.6) = 0.76 \text{ m/s} \Big\} \longrightarrow \text{ANS}$$