Example 3.B.10

Given: $\dot{L} = 0.06$ m/s $=$ *constant*, $\omega_1 = 1.2$ rad/s $=$ *constant* and $\omega_2 = 1.5$ rad/s $=$ *constant*. At the position shown, AC is aligned with the fixed Y-axis, $L = 0.12$ m, and $d = 0.02$ m.

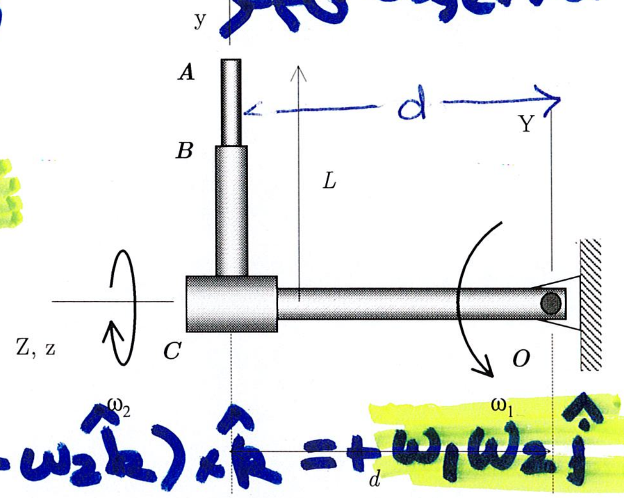
$\ddot{L} = 0, \ \dot{\omega}_1 = 0$ and $\dot{\omega}_2 = 0$

Find: Determine:

(a) The velocity of end A of the telescoping rod AC; and

(b) The acceleration of the same point.

Attach xyz to arm CB origin at C



$$\bar{\omega} = \omega_1 \hat{I} - \omega_2 \hat{k}$$

$$\frac{d\bar{\omega}}{dt} = -\omega_2 \dot{\hat{k}}$$

$$= -\omega_2 (\bar{\omega} \times \hat{k})$$

$$= -\omega_2 (\omega_1 \hat{i} - \omega_2 \hat{k}) \times \hat{k} = +\omega_1 \omega_2 \hat{j}$$

$$\bar{v}_A = \bar{v}_C + \{\bar{v}_{A/C}\}_{rel} + \bar{\omega} \times \bar{r}_{A/C}$$

C moves on a circle of radius d

$\bar{v}_C = -\omega_1 d \hat{j}$ at this instant (1)

$\{\bar{v}_{A/C}\}_{rel} = \dot{L} \hat{j}$ (2)

$$(\bar{\omega} \times \bar{r}_{A/C}) = (\omega_1 \hat{I} - \omega_2 \hat{k}) \times (L \hat{j})$$

$\hat{I} = \hat{i}$ at this instant

$\bar{v}_A = \omega_1 L \hat{k} + \omega_2 L \hat{i}$ (3)

from (1), (2) & (3)

$$\bar{v}_A = -\omega_1 d \hat{j} + \dot{L} \hat{j} + \omega_1 L \hat{k} + \omega_2 L \hat{i}$$

$$= \omega_2 L \hat{i} + (\dot{L} - \omega_1 d) \hat{j} + \omega_1 L \hat{k}$$ (4) ANS (a)

$$\bar{a}_A = \underset{\substack{\uparrow \\ \text{accel. of} \\ \text{origin of} \\ xyz}}{\bar{a}_C} + \{\bar{a}_{A|C}\}_{rel} + \bar{\alpha} \times \bar{r}_{A|C} + 2\bar{\omega} \times \{\bar{v}_{A|C}\}_{rel} + \bar{\omega} \times (\bar{\omega} \times \bar{r}_{A|C})$$

$\bar{a}_C$: C moves on a circle of radius d, constant angular velocity

$$\bar{a}_C = -\omega_1^2 d\,\hat{k} \quad (5)$$

accel. points towards center of curvature. When ω_1 is constant – no $\dot{\omega}_1 d$ component in $\hat{j}$ direction

$$\{\bar{a}_{A|C}\}_{rel} = \ddot{L}\hat{j} = \bar{0} \quad (6)$$

$\dot{L}$ is constant.

$$\bar{\alpha} \times \bar{r}_{A|C} = \omega_1\omega_2\hat{k} \times L\hat{j} = -\omega_1\omega_2 L\hat{i} \quad (7)$$

$$2\bar{\omega} \times \{\bar{v}_{A|C}\} = 2(\omega_1\hat{I} - \omega_2\hat{k}) \times \dot{L}\hat{j} = 2(\omega_1\hat{i} - \omega_2\hat{k}) \times \dot{L}\hat{j}$$ at this instant.

$$= 2\dot{L}\omega_1\hat{k} + 2\dot{L}\omega_2\hat{i} \text{ (at this instant)} \quad (8)$$

$$\bar{\omega} \times (\bar{\omega} \times \bar{r}_{A|C}) = (\omega_1\hat{i} - \omega_2\hat{k}) \times (\omega_1 L\hat{k} + \omega_2 L\hat{i})$$ ← from (3)

$$= -\omega_1^2 L\hat{j} + \omega_2^2 L\hat{j} = (\omega_2^2 - \omega_1^2)L\hat{j} \quad (9)$$

Pull all the terms together from equations (5) → (9)

$$\therefore \bar{a}_A = (-\omega_1\omega_2 L + 2\dot{L}\omega_2)\hat{i} + (\omega_2^2 - \omega_1^2)L\hat{j} + (2\dot{L}\omega_1 - \omega_1^2 d)\hat{k} \quad m/s^2$$

ANS (b)