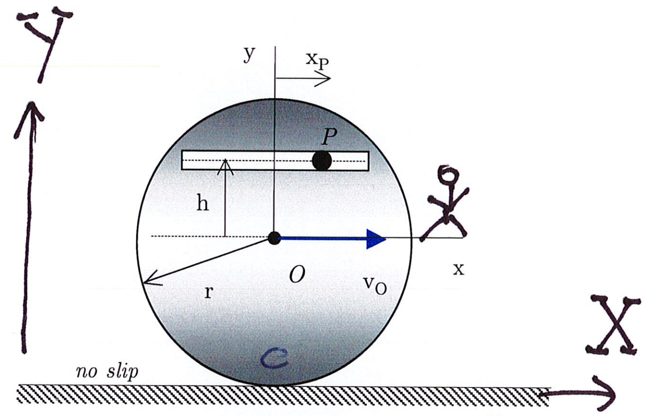
**Example 3.A.1**

**Given:** The disk shown below rolls without slipping on a horizontal surface. At the instant shown, the center O is moving to the right with a speed of $v_O = 5$ m/s with this speed decreasing at a rate of 2 m/s$^2$. Also for this instant, the particle P is at a position of $x_P = 0.2$ m with $\dot{x}_P = 2$ m/s $=$ *constant*, where $x_P$ is measured relative to the $xyz$ coordinate system that is attached to the disk.

**Find:** Determine:
(a) The velocity of particle P; and
(b) The acceleration of particle P.

Use the following parameters in your analysis: $h = 0.2$ m and $r = 0.6$ m.

Attach xyz to the disk




$\{\bar{v}_{P/O}\}_{rel} = \dot{x}_p \hat{i}$

$\{\bar{a}_{P/O}\}_{rel} = \ddot{x}_p \hat{i} = 0\hat{i}$

$\bar{v}_O = 5\,m/s\,\hat{I}$ $\quad \dot{v}_O = -2\,m/s^2\,\hat{I} = \bar{a}_O$

~~the~~ rigid body: $\quad v_O = -\omega r \quad | \quad a_O = -\alpha r$

relate C to O to get these $\quad \therefore \bar{\omega} = -\frac{v_O}{r}\hat{k} \quad | \quad \bar{\alpha} = -\frac{a_O}{r}\hat{k}$

velocity

$$\vec{v}_P = \vec{v}_O + \{\vec{v}_{P/O}\}_{rel} + \vec{\omega} \times \vec{r}_{P/O}$$

$$= v_O\hat{I} + \dot{x}_p\hat{i} + \left(-\frac{v_O}{r}\right)\hat{k} \times (x_p\hat{i} + h\hat{j})$$

At this instant $\hat{I} = \hat{i}$, $\hat{J} = \hat{j}$, $\hat{K} = \hat{k}$

$$\bar{v}_P = \left(v_O + \dot{x}_p + \frac{v_O}{r}h\right)\hat{i} + \left(-\frac{v_O x_p}{r}\right)\hat{j} \quad m/s$$

acceleration

$$\vec{a}_P = \vec{a}_O + \underbrace{\cancel{\{\bar{a}_{P/O}\}_{rel}}^{\,0}}_{\ddot{x}_p\hat{i} = 0} + \vec{\alpha} \times \vec{r}_{P/O} + 2\{\bar{\omega} \times \{\bar{v}_{P/O}\}_{rel}\} - \omega^2 \vec{r}_{P/O}$$

$$\bar{a}_p = a_o \hat{I} + \vec{0} + \left(-\frac{a_o}{r}\hat{k}\right) \times \left(x_p\hat{i} + h\hat{j}\right) + 2\left(\frac{-v_o}{r}\hat{k}\right) \times (\dot{x}_p)\hat{i}$$

$$- \left(\frac{v_o^2}{r^2}\right)\left(x_p\hat{i} + h\hat{j}\right)$$

At this instant $\hat{i} = \hat{I}$, $\hat{j} = \hat{J}$, $\hat{k} = \hat{K}$ (need to be in the same co-ordinate system to do x products.)

$$\vec{a}_p = \left(a_o + \frac{a_o}{r}h - \frac{v_o^2}{r^2}x_p\right)\hat{i} + \left[-\left(+\frac{a_o x_p}{r}\right) + \frac{2v_o}{r}\dot{x}_p + \frac{v_o^2}{r_o^2}h\right]\hat{j}$$

m/s$^2$

$\therefore$ $\bar{a}_p = \ldots\ldots$