Visualization of coordinate syston rotating with 2 components.


- The $\tilde{I}, \hat{J} \underline{k}$ unit vectors are Fixed
- The $1,5, \frac{k}{l}$ unit vectors rotate with $\vec{w}=w_{1} \hat{\jmath}+w_{2} \hat{1}$

How do we interprete? We can visualize bo considering individual rotations around $\bar{j}$ and $a$ Lets pretend $W$, is applied then wi.
(1) Original Orientation

(3) Now, consider rotation about ax axis corves only

(2) Rotation about I axis on wi, only


This is just a visualization in reality $W_{c}$ and $W_{2}$ happen at the same tron


Find $V_{p}$ and $\overrightarrow{a_{p}}$.
IIIZ are global fixed axis.

Let's look are two ways we can have the coordinate system: 1) xyz fixed to rod OA, and 2) xyz fixed in distich
(1)

axyz-fixed in ot

xyz -foxed in disk
Define $\vec{\omega}$ and $\overline{2}$ of $x y z$ system

$$
\begin{aligned}
& \vec{w}=w_{1} \vec{j}+w_{2} \hat{k} \\
& \vec{\alpha}=\dot{w}_{1} \vec{j}+w_{1} \hat{j}+\dot{w}_{2} \hat{k}+w_{2} \vec{j} \vec{v} k \hat{c} \\
& \vec{\alpha}=w_{2}\left(w_{1} y_{\hat{j}}+w_{2} \hat{c}\right) \times \hat{c}=w_{1} w_{2} \eta \\
& \text { Find } \overrightarrow{v_{\rho}}
\end{aligned}
$$

$$
\vec{v}_{P}=\vec{V}_{A}+\left(V_{P} / A\right)_{r e l}+\vec{w} \times \overrightarrow{r_{P}}
$$

Now, we need $\vec{V}_{A}$, we can use the fired coordinate system

$$
\vec{V}_{A}=\vec{V}_{0}+\vec{W}_{O A} \times \vec{V}_{O / A}
$$

The observer doe not see point $P$ moving

Define $\vec{w}$ and $₹$ of $x y z$ system

$$
\begin{aligned}
& \vec{w}=w_{1} \hat{J} \\
& \vec{\alpha}=\frac{d \vec{w}}{d \tau}=\dot{w}_{1} \hat{\vec{T}}+w_{1} \vec{j}=0
\end{aligned}
$$

Find $\overrightarrow{V_{\rho}}$

$$
\vec{v}_{P}=\vec{V}_{A}+\left(V_{P} / A\right)_{r e l}+\vec{w} \times \overrightarrow{P_{P}}
$$

Now, we need $\overrightarrow{V_{A}}$, we can use the fired coordinate system

$$
\vec{V}_{A}=\vec{V}_{0}+\vec{W}_{O A} \times{\overrightarrow{V_{O}}}_{A}
$$

$$
\begin{aligned}
& \vec{v}_{0}=\hat{0}, \vec{\omega}_{\text {OAt }}=\omega_{1}, \vec{j}, \vec{r}_{O / A}=\angle \frac{a}{l}
\end{aligned}
$$

$$
\begin{aligned}
& \vec{r}_{p}=-L W_{i} \hat{\mathbb{Z}}
\end{aligned}
$$

The andlusis is the same up to this point.

Bade to

$$
\overrightarrow{V_{D}}=\vec{V}_{A}+\left(\vec{V}_{P / A}\right)_{r e l}+\vec{w} \times \overrightarrow{V_{P / A}}
$$

identify each term

$$
\begin{aligned}
& \vec{V}_{A}=-L W_{1} \hat{\pi} \\
& \vec{W}=W_{1} \hat{\jmath} \\
& \vec{V}_{D_{I A}}=R_{\hat{\jmath}}
\end{aligned}
$$

We need to find ( $\vec{V}_{\text {P }}$ ) rel.
Here we are going to fix point $A$ and treat the ditch as a rigid boding.


Find (VP/A)rel using a modified form of RB equations

$$
\left(\vec{V}_{P / A}\right)_{r e l}=\left(\vec{V}_{A}\right)_{\text {BYZ }}+\vec{W}_{2} \times \vec{V}_{P / A}
$$

$\left(\overrightarrow{V_{A}}\right)_{\text {byz }}=$ velocity of point $A$ in rum system, it is 'Oo".

$$
\begin{aligned}
& \vec{v}_{0}=\hat{0}, \overrightarrow{\omega_{O A t}}=\omega_{1}, \vec{j}, \vec{r}_{0 / A}=\angle \frac{a}{l}
\end{aligned}
$$

$$
\begin{aligned}
& \vec{r}_{p}=-L w_{1} \hat{\pi}
\end{aligned}
$$

The andlysis is the same up to this point.

Back to

$$
\overrightarrow{V_{D}}=\vec{V}_{A}+\left(\overrightarrow{V_{P} / A}\right)_{r e l}+\vec{w} \times \overrightarrow{r_{P / A}}
$$

identify each term

$$
\begin{aligned}
& \vec{V}_{A}=-L W_{1} \hat{\pi} \\
& \overrightarrow{w_{0}}=w_{1} \hat{J}+w_{2} \hat{k} \\
& {\overrightarrow{v_{D} / t}}^{n}=R_{j}
\end{aligned}
$$

we need to find $\left(\overrightarrow{v o l A}_{\text {A }}\right)$ rel. In this coordinate surstem (Vila )rel $=0$. The observer doesnit see polite $P$ move.


The observer doe not see point P moving
$x y z$ - foxed in disk
Plug in . . .

$$
\begin{aligned}
& \overrightarrow{V_{P}}=\overrightarrow{V_{A}}+\left(\overrightarrow{V_{D A}}\right)_{\text {rel }}+\vec{w} \times \overrightarrow{V_{D}} /{ }_{D} \\
& =-L w_{1} \hat{k}+\bar{O}+\left(w_{1} \hat{j}+w_{2} \hat{R}\right) \times R_{j}^{n} \\
& \left.\vec{v}_{n}=-L w_{1} \vec{k}-R w_{2}\right\rangle \\
& \overrightarrow{v_{p}}=-R_{w_{2}} \hat{I}-\angle w_{1} \vec{\pi} \quad \text { Ans }
\end{aligned}
$$

$$
\begin{aligned}
& \vec{w}_{2}=w_{2} \hat{k} \text { and } \overrightarrow{p / A}=R_{j} \\
& \left({\overrightarrow{v_{o} / A}}^{n}\right)_{n c l}=\overline{0}+w_{2} \hat{R} \times R_{j}^{n}=-R w_{2} \tilde{M}
\end{aligned}
$$

Plus in...

$$
\begin{aligned}
& =-L w_{1} \hat{k}-R w_{2} S^{I}+w_{j}^{I} J_{j}^{n} \times R \hat{\jmath}=0 \\
& \vec{r}_{p}=-R W_{2} \hat{\imath}-\angle w_{1} \hat{J} \text { Ans }
\end{aligned}
$$

The answers are the same ion the fixed coordinate system.

Find $\vec{a}_{p}$

We can find $\overrightarrow{a_{A}}$ using the global XYZ coordinate system

Go back to

$$
\begin{aligned}
\bar{a}_{0}=\vec{a}_{1}+\left(\overline{a_{p / A}^{A}}\right)_{r e l}+\bar{d} \times \overrightarrow{p_{0}}+ \\
\vec{w} \times\left(\vec{w} \times \vec{p}_{D / A}\right)+2 \vec{w} \times\left(\bar{p}_{p / A}\right)_{r d}
\end{aligned}
$$

identity each perm

$$
\begin{aligned}
& \overrightarrow{a n}_{n}=-L w_{n}^{2} \hat{I} \\
& \vec{w}=w_{1} \hat{J} \\
& \vec{\alpha}=0
\end{aligned}
$$

$$
(\text { Vol/ })_{r e l}=-R w_{2} n
$$

$$
\begin{aligned}
& \overrightarrow{w_{0}}=w_{1} \bar{\jmath} \text { then } \overrightarrow{\lambda_{0 A}}=\overline{0} \\
& \overrightarrow{a_{0}}=\overline{0}, \vec{v}_{0}=\angle \vec{I} \\
& \vec{a}_{1}=\overline{0}+\overline{0}+w_{1} \vec{J} \times\left(\vec{w}_{1}^{a} \bar{J} \times\langle\bar{I})\right. \\
& a_{A}=-L \omega_{r}^{2} \hat{I}
\end{aligned}
$$

$$
\begin{aligned}
& +\vec{w} \times(\vec{w} \times \sqrt{0} / \mathrm{m})+2 \vec{w} \times(\vec{v} / /)_{\mathrm{rel}}
\end{aligned}
$$

Find $\overrightarrow{a_{p}}$

$$
\begin{aligned}
& \bar{a}_{p}=\bar{a}_{t}+\left(\overrightarrow{a_{p}}\right) \text { rel }+\bar{\alpha} \times \overline{p_{n}} \\
& +\vec{w} \times(\vec{w} \times \vec{b} / \pi)+2 \vec{w} \times(\vec{r} / \pi)_{\mathrm{rel}}
\end{aligned}
$$

We can find $\vec{a}_{A}$ ustay the global XYZ coordinate system

$$
\begin{aligned}
& \vec{a}_{1}=\overrightarrow{a_{0}}+\overrightarrow{a_{01}} \times \overrightarrow{a_{10}}+\overrightarrow{u_{01}} \times\left(\overrightarrow{u_{01}} \times \overrightarrow{u_{40}}\right) \\
& \overrightarrow{w_{01}}=w_{1} \bar{\jmath} \text { then } \overrightarrow{\lambda_{\text {oh }}}=\overline{0} \\
& \vec{a}_{0}=\overline{0}, \vec{v}_{0}=\angle \vec{I} \\
& \vec{a}_{1}=\overrightarrow{0}+\overrightarrow{0}+w_{1} \vec{J} \times\left(w_{1} \vec{J} \times\left\langle\frac{a}{I}\right)\right. \\
& a_{f}=-L \omega_{1}^{2} \hat{I}
\end{aligned}
$$

Go back to

$$
\begin{aligned}
& \bar{a}_{0}= \vec{a}_{A}+\left(\overline{a_{p / A}}\right)_{r e l}+\bar{d} \times \overrightarrow{p_{p / A}}+ \\
& \overrightarrow{\vec{w}} \times\left(\overrightarrow{\vec{w}} \times \vec{p}_{p_{A}}\right)+2 \vec{w} \times\left(\bar{p}_{p / A}\right)_{r d}
\end{aligned}
$$

identity each perm

$$
\begin{aligned}
& \overrightarrow{a_{n}}=-\angle w_{1}^{2} \hat{I} \\
& \overrightarrow{w_{n}}=w_{1} \hat{J}+w_{z} \hat{E} \\
& \vec{\alpha}=w_{1} w_{z}{ }^{T} \\
& \left(v_{p / n}\right)_{r e l}=0
\end{aligned}
$$

we need to find ( $\overrightarrow{p D}_{\mathrm{A}}^{\mathrm{A}}$ ) rel.
Here we are goings to fix point $A$ and treat the dill as a rigid body,


$$
\left.\left(\overrightarrow{P_{P A}}\right)_{r l l}=a_{A A_{1}}\right)_{x+2}+\bar{\alpha}_{2} \times \overrightarrow{r P}_{P_{A}}+\vec{w}_{2} \times\left(\overline{w_{2}} \times \vec{p}_{P A}\right)
$$

$$
\left(a_{A}\right)_{\text {my }}=\text { accelcoution of "a "from }
$$ observer in xyz coordinate system $=11011$

$$
\begin{aligned}
& \hat{\alpha}_{2}=\overline{0}, \quad \vec{W}_{2}=W_{2} \hat{k}, \quad \bar{P}_{P_{A}}=R_{\tilde{u}} \\
& \left(\vec{a}_{01 t}\right)_{\text {rel }}=\overrightarrow{0}+\overrightarrow{0}+W_{\varepsilon} \hat{k} \times(\pi \hat{k} \hat{k} \times R \hat{\jmath}) \\
& =-\left\langle w_{2}^{2} \hat{\jmath}\right. \\
& \overrightarrow{a_{0}}=\vec{a}_{A}+\left(\overrightarrow{a_{D / A}}\right)_{\text {rel }}+\vec{\nabla} \times \vec{D} \vec{D} \overrightarrow{D N}_{A}+
\end{aligned}
$$

$$
\begin{aligned}
& \left.\overrightarrow{a_{D}}=\overrightarrow{a_{A}}+\left(\overrightarrow{a_{l}}\right)_{A}\right)_{v e l}+\vec{w} \times\left(\vec{w} \times \vec{v}_{A}\right) \\
& +2 \bar{w} \times(\overrightarrow{\mathrm{V} / \mathrm{A}}) \mathrm{rel} \\
& \vec{a}_{0}=-L w_{1}^{2} \hat{I}-R w_{i}^{2} \hat{\jmath}_{1} \\
& -\left(w, \frac{a}{j}\right) \times(w, \hat{j}) \times R \hat{j} \rightarrow 0 \\
& \left.+2\left(w_{1} \hat{\not}\right)_{j}\right)^{\prime} \times\left(-R w_{2} \hat{i}\right)
\end{aligned}
$$

We need to find (ap/A) rel. In this coordinate system $(a \stackrel{0}{p} / \mathrm{p})_{\mathrm{rel}}=0$. The observer doesnit see polite $P$ move.


The observer doe not see point P moving
$-L w_{7}^{2}{ }^{n}$



$$
\begin{aligned}
& \vec{a}_{0}=-\left(w_{1}^{2} \hat{I}+\overrightarrow{0}+w_{2} w_{1} \hat{\imath} \times R \hat{\jmath}\right. \\
& +\left(w_{1} \vec{j}^{\hat{j}}+w_{2} \hat{k}\right) \times\left(\left(w_{1} \hat{j}+w_{2}^{2} \hat{\imath}\right) \times R_{j}\right) \\
& +2\left(w_{1} \hat{J}+w_{2} \hat{k}\right) \times \overrightarrow{0}
\end{aligned}
$$

$$
\begin{aligned}
\overrightarrow{a_{p}}= & -L w_{1}^{2} \hat{\jmath}+R w_{2} w_{1} \hat{x} \\
& +\left(w_{1} \hat{\jmath}+w_{i} \hat{k}\right) \times\left(-R w_{2} \hat{\eta}\right) \\
\vec{a}_{p}= & -L w_{1}^{2} \hat{l}+R w_{2} w_{1} \hat{x} \\
& +R w_{2} w_{1} \hat{c}-R w_{2}^{2} g \\
\overrightarrow{a_{p}}= & -L w_{1}^{2} \hat{\eta}-R w_{2}^{2} \hat{J}+2 R w_{2} w_{1} \hat{x}
\end{aligned}
$$

ans

$$
\begin{aligned}
\overline{a p}_{p}= & -L w_{1}^{2} \hat{I}-R w_{2}^{2} \hat{\jmath} \\
& +2 k w_{2} w_{1} \hat{k} \\
\vec{a}_{p}= & -\angle w_{1}^{2} \hat{I}-R w_{2}^{2} \vec{\jmath} \\
& +2 R w_{2} w_{1} \hat{k}
\end{aligned}
$$

