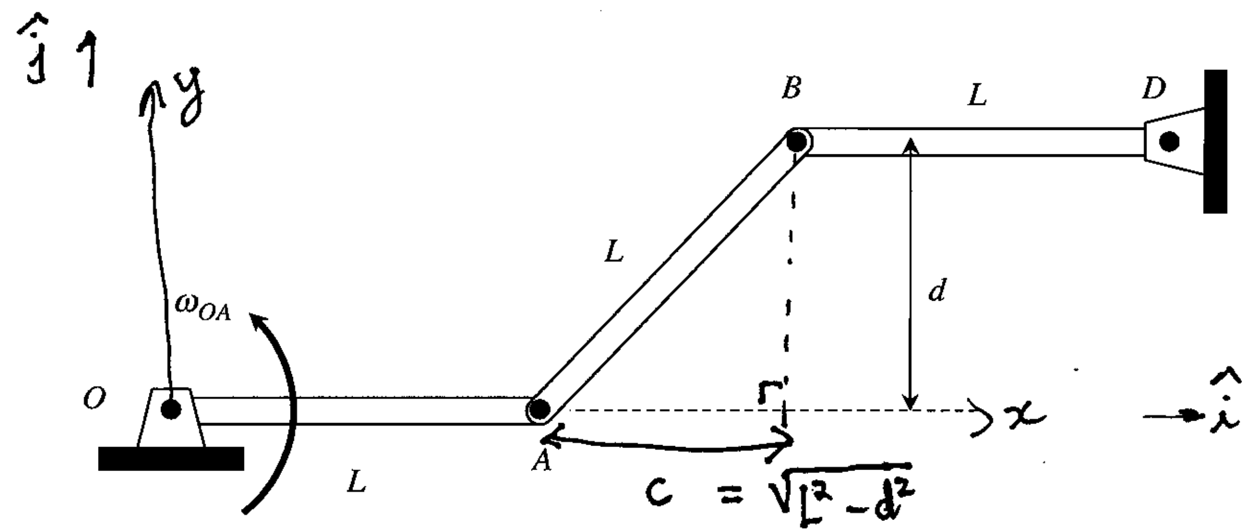
(1)

Example 2.A.10

Given: At the instant shown, link OA rotates counterclockwise about pin O with a constant angular speed of ω_{OA}. At the instant shown, links OA and BD are horizontal.

Find: Determine at this instant:
(a) The angular acceleration of link AB; and
(b) The angular acceleration of link BD.

Use the following parameters in your analysis: $\omega_{OA} = 3$ rad/s, $L = 0.5$ m and $d = 0.4$ m. Also, be sure to express your answers as vectors.



$\bar{\omega}_{OA} = \omega_{OA}\hat{k}$ constant

$\bar{\alpha}_{OA} = \bar{0}$

$$\vec{v}_A = \cancelto{\vec{0}}{\vec{v}_O} + \vec{\omega}_{OA} \times \vec{r}_{A/O} = \omega_{OA}\hat{k} \times (L\hat{i}) = L\omega_{OA}\hat{j} \qquad (1)$$

fixed point

$$\vec{a}_A = \cancelto{\bar{0}}{\vec{a}_O} + \cancelto{0}{\vec{\alpha}_{OA}} \times \vec{r}_{A/O} - \omega_{OA}^2 \vec{r}_{A/O} = -\omega_{OA}^2 L\hat{i} \qquad (2)$$

ω_{OA} constant

$$\vec{v}_B = \cancelto{0}{\vec{v}_D} + \vec{\omega}_{BD} \times \bar{r}_{B/D} = \omega_{BD}\hat{k} \times (-L\hat{i}) = -\omega_{BD}L\hat{j} \qquad (3)$$

fixed point.

$$\vec{a}_B = \cancelto{0}{\vec{a}_D} + \bar{\alpha}_{BD} \times \bar{r}_{B/D} - \omega_{BD}^2 \vec{r}_{B/A} = \alpha_{BD}\hat{k} \times (-L\hat{i}) - \omega_{BD}^2(-L\hat{i})$$

$$\vec{a}_B = -\alpha_{BD}L\hat{j} + L\omega_{BD}^2\hat{i} \qquad (4)$$

Relate B to A velocities

3 $\bar{v}_B = \bar{v}_A + \bar{\omega}_{AB} \times \bar{r}_{B/A}$ 1

$-\omega_{BD} L \hat{j} = L\omega_{OA}\hat{j} + \omega_{AB}\hat{k} \times (c\hat{i} + d\hat{j})$

$-\omega_{BD} L \hat{j} = L\omega_{OA}\hat{j} + c\omega_{AB}\hat{j} - d\omega_{AB}\hat{i}$

$\hat{i}$: $0 = -d\omega_{AB}$ $\therefore$ $\omega_{AB} = 0$ at this instant

$\hat{j}$: $-\omega_{BD} L = L\omega_{OA} + c\cancel{\omega_{AB}}^{0}$ $\rightarrow$ $\omega_{BD} = \omega_{OA}$ at this instant

Relate B to A acceleration

4 $\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega^2_{AB}\vec{r}_{B/A}$

2

$-\alpha_{BD} L\hat{j} + L\omega^2_{BD}\hat{i} = -\omega^2_{OA} L\hat{i} + \alpha_{AB}\hat{k} \times (c\hat{i} + d\hat{j}) - \cancel{\omega^2_{AB}}^{0}(c\hat{i} + d\hat{j})$

$-\alpha_{BD} L\hat{j} + L\omega^2_{BD}\hat{i} = -\omega^2_{OA} L\hat{i} + c\alpha_{AB}\hat{j} - d\alpha_{AB}\hat{i}$

i: $L\underbrace{\omega^2_{BD}}_{=\omega^2_{OA}} = -\omega^2_{OA} L - \alpha_{AB} d$ $\rightarrow$ $\vec{\alpha}_{AB} = -2\omega^2_{OA}\frac{L}{d}\hat{k}$ rad/s

j: $-\alpha_{BD} L = c\alpha_{AB}$ $\rightarrow$ $\vec{\alpha}_{BD} = -\frac{c\alpha_{AB}}{L}\hat{k} = 2c\omega^2_{OA}/d\,\hat{k}$ rad/s

$\omega_{OA} = 3\ rad/s \qquad \alpha_{OA} = 0\ rad/s^2$

$L = 0.5m\ ; \quad d = 0.4m \quad \therefore \quad c = \sqrt{(0.5)^2 - (0.4)^2} = 0.3m$

$$\vec{\alpha}_{AB} = \frac{-2(3)^2(0.5)}{(0.4)} = \frac{-9}{0.4} = -22.5\hat{k}\ rad/s^2 \longrightarrow$$

$$\vec{\alpha}_{BO} = \frac{c\ \alpha_{AB}}{L} = \frac{0.3 \times 22.5}{0.5} = 13.5\hat{k}\ rad/s^2 \longrightarrow$$