ME 274 – Summer 2022
Study resources
Final Exam

Working sample exam questions is a good practice to follow in your exam preparation. However, as you prepare for the first midterm exam in ME 274, consider the fact that starting out with a solid understanding of the fundamental principles will take you much further in doing well in the course than simply getting practice with old exams.

The following is a suggested path for you to take in your exam preparation:

- *<u>Review the fundamentals</u>*. If you have not yet read the lecture book material for the course, you need to do that now. If you have done so, you do not need to go back and reread everything. Review the one-page summary sheets provided to you every class period on the Daily Schedule page of the blog. For your convenience, I am providing you with PDFs for all of these summary pages from Section 3.B onward through Section 5.A of the lecture book. Make note of anything on these summary pages with which you are not comfortable. Read more of those topics in the lecture book and/or ask your instructor (me) for help.

- *<u>Review your homework</u>*. Go back over your submitted and graded homework on Gradescope. Make note of things that you did not do well on with those homeworks and of any unresolved issues.

- *<u>Review old sample exams</u>*. Weekly Joys has many sample exams from previous semesters, including solutions. Focus on problems that cover topics with which you are having problems.

- *<u>Save back a sample exam</u>*. Work this sample exam under the pressure of one hour. Quit at the end of 150 minutes. Do an honest appraisal of your work. Where did you have difficulties? Where did you spend most of your time? From this, determine where you might have weaknesses and work on those topics as the exam approaches. Do not wait until the exam to find out your weaknesses. It is too late then. Talk to your instructor or visit the tutorial room for help.

Study resources
Included in this packet is the following material:

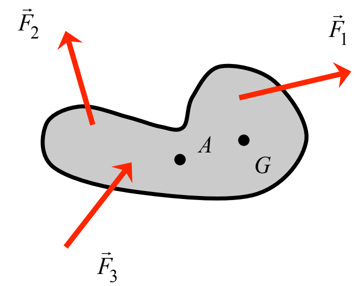
- The set of PDFs of the daily course summaries for the course since Exam 2 (these are also found on the course website).
- A sample ME 274 final exam from a previous term.

Feel free to use this material in your exam preparation.

Summary: Work/Energy Equation 1

FUNDAMENTAL equation: $T_2 + V_2 = T_1 + V_1 + U_{1\to 2}^{(nc)}$
with:

$$T = \frac{1}{2} m v_A^2 + \frac{1}{2} I_A \omega^2 + m \vec{v}_A \bullet \left(\vec{\omega} \times \vec{r}_{G/A} \right)$$



SPECIAL CHOICES FOR POINT "A": If A is *EITHER* the c.m. *OR* a fixed point, then the kinetic energy equation reduces to:

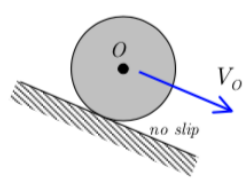
$$T = \boxed{\frac{1}{2} m v_A^2} + \boxed{\frac{1}{2} I_A \omega^2}$$

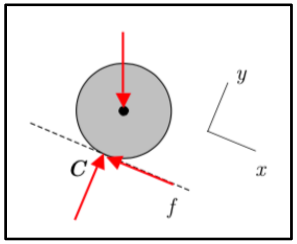
translation *rotation*

PARALLEL AXIS THEOREM: As with the Newton/Euler equation, you will need to use the PAT if you choose A to be anything other than the c.m.

SYSTEM CHOICE: Make your system BIG! Include as many components within system to make workless forces INTERNAL (no work on system). Different choice than for Newton/Euler.

ROLLING WITHOUT SLIPPING: The friction force at a no-slip point does not work. Why? Recall that the no-slip point is stationary – no work is done on a stationary point.





Summary: Work/Energy Equation 2

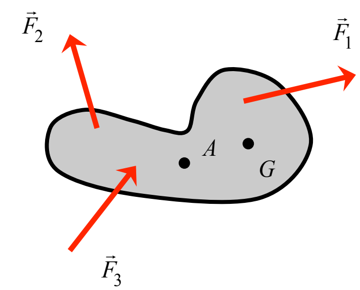
FUNDAMENTAL equations: $T_2 + V_2 = T_1 + V_1 + U_{1\to 2}^{(nc)}$

with:

$$T = \frac{1}{2} m v_A^2 + \frac{1}{2} I_A \omega^2$$

with A = ***EITHER*** c.m. ***OR*** fixed point

SOLUTION METHOD: the four-step plan



Kinetics: Four-Step Problem Solving Method

The suggested plan of action for solving kinetics problems:

1. ***Free body diagram(s)***. Draw appropriate free body diagrams (FBDs) for the problem. Your choice of FBDs is problem dependent. For some problems, you will draw an FBD for each body; for others, you will draw an FBD for the entire system. An integral part of your FBDs is your choice of coordinate systems. For each FBD, draw the unit vectors corresponding to your coordinate choice.

2. ***Kinetics equations***. At this point, you will need to choose what solution method(s) that you will need to use for the particular problem at hand. In this section of the course we will study four basic methods: Newton/Euler, work/energy, linear impulse/momentum and angular impulse/momentum. Based on your choice of method(s), write down the appropriate equations from your FBD(s) from Step 1.

3. ***Kinematics***. Perform the needed kinematic analysis. A study of the equations in Step 2 above will guide you in deciding what kinematics are needed to find a solution to the problem.

4. ***Solve***. Count the number of unknowns and the number of equations from above. If you do not have enough equations to solve for your unknowns, then you either: (i) need to draw more FBDs, OR (ii) need to do more kinematic analysis. When you have sufficient equations for the number of unknowns, solve for the desired unknowns from the above equations.

Draw free-body diagram of entire system for work/energy.

Be sure to use the correct mass moment of inertia for your choice of point "A". Use PAT if necessary.

Typically the most difficult step. Recall the rigid body kinematics from Chapter 2.

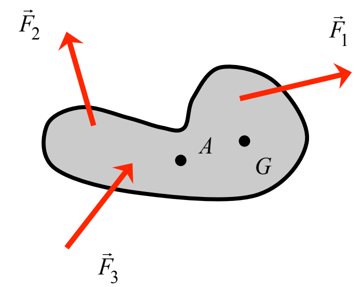
If you are short equations, go back to Step 3 − Kinematics.

Summary: Impulse/Momentum Equations 1

FUNDAMENTAL equations (for your chosen "system"):

$$m\vec{v}_{G2} = m\vec{v}_{G1} + \int_1^2 \vec{F}\,dt$$

$$\vec{H}_{A2} = \vec{H}_{A1} + \int_1^2 \vec{M}_A\,dt$$



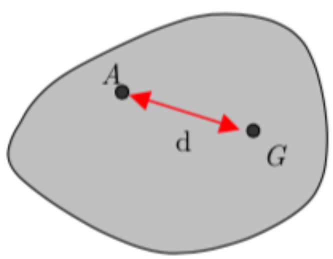
for A = c.m. *OR* fixed point, with:

$$\vec{H}_A = I_A\vec{\omega} \quad ; \quad \textit{for a RIGID BODY}$$
$$= m\vec{r}_{P/A} \times \vec{v}_P \quad ; \quad \textit{for a particle P}$$

SYSTEM CHOICE: Make your system big (make as many forces *internal* as possible to take advantage of conservation).

PARALLEL AXIS THEOREM: You will need to use the PAT if you choose A to be anything other than the c.m.: $I_A = I_G + md^2$

cm



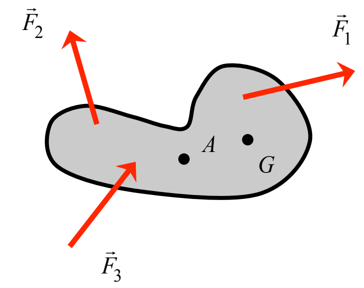
Summary: Impulse/Momentum Equations 2

FUNDAMENTAL equations (for A = cm *OR* fixed point):

$$m\vec{v}_{G2} = m\vec{v}_{G1} + \int_1^2 \vec{F}\,dt$$

$$\vec{H}_{A2} = \vec{H}_{A1} + \int_1^2 \vec{M}_A\,dt$$

$$\vec{H}_A = I_A\vec{\omega} \quad ; \quad \text{for a RIGID BODY}$$
$$= m\vec{r}_{P/A} \times \vec{v}_P \quad ; \quad \text{for a particle } P$$



SOLUTION METHOD: the four-step plan

Kinetics: Four-Step Problem Solving Method

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Draw free-body diagram of entire system for impulse momentum.

Be sure to use the correct mass moment of inertia for your choice of point "A". Use PAT if necessary.

Typically the most difficult step. Recall the rigid body kinematics from Chapter 2.

If you are short equations, go back to Step 3 – Kinematics.

Summary: Particle and Planar Rigid Body Kinetics

WHICH TOOL(s) TO USE?
Put effort up front deciding on which method(s) to use: *Newton-Newton/Euler*, *work/energy*, *linear impulse momentum* or *angular impulse momentum*. Use the Kinetics Table in Section 5.D of the lecture book as a guide.

PARTICLE or RIGID BODY?
How is a particle distinguished from a rigid body? For a *particle*, we have:

$$\sum \vec{M}_G = I_G\vec{\alpha} = \vec{0} \quad \text{(EITHER } I_G = 0 \text{ OR } \vec{\alpha} = \vec{0}\text{)}$$

THE FOUR-STEP PLAN: Follow it…it is your friend!

Kinetics Table

Method	Body model	Fundamental equations
Newton-Euler *(relating forces to accelerations)*	**particle**	$\sum \vec{F} = m\vec{a}$
	rigid body *(G = c.m. and A = any point on body)*	$\sum \vec{F} = m\vec{a}_G$ $\sum \vec{M}_A = I_A\vec{\alpha} + m\vec{r}_{G/A}\times\vec{a}_A$
Work-energy *(relating change in speed to change in position)*	**particle**	$T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv^2$
	rigid body *(G = c.m. and A = any point on body)*	$T_1 + V_1 + U_{1\to2}^{(nc)} = T_2 + V_2$ where $T = \frac{1}{2}mv_A^2 + \frac{1}{2}I_A\omega^2 + m\vec{v}_A\cdot(\vec{\omega}\times\vec{r}_{G/A})$
Linear impulse-momentum *(relating change in velocity to change in time)*	**particle**	$\int_{t_1}^{t_2}\sum\vec{F}\,dt = m\vec{v}_2 - m\vec{v}_1$
	rigid body *(G = c.m.)*	$\int_{t_1}^{t_2}\sum\vec{F}\,dt = m\vec{v}_{G2} - m\vec{v}_{G1}$
Angular impulse-momentum *(relating change in angular velocity to change in time)*	**particle** *(O = fixed point)*	$\int_{t_1}^{t_2}\sum\vec{M}_O\,dt = \vec{H}_{O2} - \vec{H}_{O1}$ where $\vec{H}_O = m\vec{r}_{P/O}\times\vec{v}_P$
	rigid body *(A = fixed point or c.m.)*	$\int_{t_1}^{t_2}\sum\vec{M}_A\,dt = \vec{H}_{A2} - \vec{H}_{A1}$ where $\vec{H}_A = I_A\vec{\omega}$

Summary: Vibrations - EOMs

GOAL? To derive the differential equation of motion (EOM):

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

HOW? Good news: You already know how to do this! The four-step plan

1) *FBD*
2) *Newton/Euler*
3) *Kinematics*
4) *Solve* (here "solve" means combing together into a single EOM)

THE CATCH: You *EOM must describe motion for all time*, not just a single instant in time. Define your coordinate(s) at the start, and stick with them throughout. If you do, you are all set.

WHAT's NEXT? We will spend the rest of the semester solving our differential EOMs. You already know how to do that also! Woo Hoo!

Summary: Vibrations - Free Undamped Response

EOM: For free response of undamped system:

$$m\ddot{x} + kx = 0$$

STANDARD FORM OF EOM: Divide EOM by "m" to get:

$$\ddot{x} + \omega_n^2 x = 0$$

where $\omega_n = \sqrt{\dfrac{k}{m}} = natural\ \ frequency$

SOLUTION OF EOM:

$$x(t) = C\,cos\,\omega_n t + S\,sin\,\omega_n t$$

HOW TO FIND THE RESPONSE COEFFICIENTS, C AND S?
Enforce the initial conditions on the solution:

$$x(0) = x_0 = C\,cos\,0 + S\,sin\,0 \quad \Rightarrow \quad C = x_0$$

$$\dot{x}(0) = \dot{x}_0 = -C\omega_n sin0 + S\omega_n cos0 \quad \Rightarrow \quad S = \frac{\dot{x}_0}{\omega_n}$$

Summary: Vibrations - damped free response

EOM: For free response of damped system:

$$m\ddot{x} + c\dot{x} + kx = 0$$

STANDARD FORM OF EOM: Divide EOM by “m” to get:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$$

where

$$2\zeta\omega_n = c/m \;\;\Rightarrow\;\; \zeta = c/2\sqrt{km} = damping\ ratio$$

$$\omega_n = \sqrt{k/m} = natural\ \ frequency$$

SOLUTION OF EOM: For $0 \le \zeta < 1\,(UNDERdamped)$

$$x(t) = e^{-\zeta\omega_n t}\left(C\cos\omega_d t + S\sin\omega_d t\right) \quad ; \quad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

HOW TO FIND THE RESPONSE COEFFICIENTS, C AND S?
Enforce the initial conditions on the solution:

$$x(0) = x_0 = e^0\left(C\cos 0 + S\sin 0\right) \;\;\Rightarrow\;\; C = x_0$$

$$\dot{x}(0) = \dot{x}_0 = -\zeta\omega_n e^0\left(C\cos 0 + S\sin 0\right) + \omega_d e^0\left(-C\sin 0 + S\cos 0\right) \;\;\Rightarrow\;\; S = \frac{\dot{x}_0 + \zeta\omega_n x_0}{\omega_d}$$

Summary: *Vibrations - Forced Response 1*

EOM: For forced response:

$$M\ddot{x} + C\dot{x} + Kx = F_0 sin\Omega t \quad \Rightarrow \quad \ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{F_0}{M} sin\Omega t$$

TOTAL RESPONSE: Since this is a linear EOM, we can write:

$$x(t) = x_C(t) + x_P(t)$$

where $x_C(t) = e^{-\zeta\omega_n t}\left(C\cos\omega_d t + S\sin\omega_d t\right)$

HOW TO FIND PARTICULAR SOLUTION?

$$x_P(t) = Asin\Omega t + Bcos\Omega t$$

Substitute into EOM and solve for *A* and *B*. If the system is <u>undamped</u>, then you will find that $B = 0$.

ENFORCING IC's: The initial conditions are enforced on the TOTAL solution *x(t)*, NOT on the complementary solution $x_C(t)$!

Summary: *Vibrations - Forced Response 2*

EOM: For forced response:

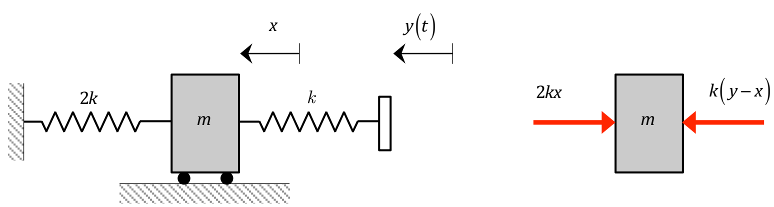
$$M\ddot{x} + C\dot{x} + Kx = F_0 sin\Omega t \quad \Rightarrow \quad \ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = \frac{F_0}{M} sin\Omega t$$

PARTICULAR SOLUTION

$$x_P(t) = Asin\Omega t + Bcos\Omega t$$

Substitute into EOM and solve for *A* and *B*. If the system is <u>undamped</u>, then you will find that *B = 0*.

BASE EXCITATION: For many problems, the energy input is through a moving support (prescribed motion, y(t)), rather than an applied force. <u>*Example*</u>:



$$\sum F_x = -2kx + k(y - x) = m\ddot{x} \quad \Rightarrow \quad m\ddot{x} + 3kx = \boxed{ky(t)}$$ *base excitation*

Summary: Vibrations - Forced Response 3

EOM for harmonic excitation: Derive using the four-step plan (*FBD*, *Newton/Euler*, *kinematics* and *reducing to a single equation*)

$$M\ddot{x} + C\dot{x} + Kx = F_0 sin\Omega t$$

STANDARD FORM OF EOM (identify natural frequency and damping ratio):

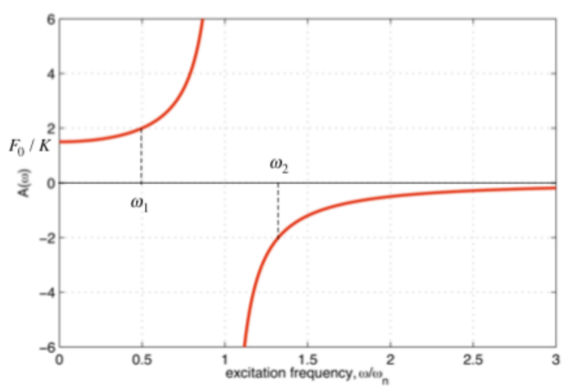
$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{F_0}{M} sin\Omega t \quad \Rightarrow \quad \boxed{\omega_n = \sqrt{K / M}\ ;\ 2\zeta\omega_n = C / M}$$

COMPLEMENTARY SOLUTION (underdamped):

$$x_C(t) = e^{-\zeta\omega_n t}\left(C\cos\omega_d t + S\sin\omega_d t\right)$$ *(solve for C and S by imposing ICs on TOTAL solution)*

PARTICULAR SOLUTION (solve for A and B): $x_P(t) = A sin\Omega t + B cos\Omega t$

INTERPRETATION OF PARTICULAR SOLUTION (undamped):



- For $\Omega < \omega_n$: A > 0 (in phase)
- For $\Omega = \omega_n$: A → ∞ (resonance)
- For $\Omega < \omega_n$: A < 0 (180° out of phase)

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(Last) *(First)*

Final Examination (AM)

August 6, 2020

INSTRUCTIONS

Begin each problem on a separate sheet.

All problems are of equal value and will be graded on the basis of 20 points maximum for each problem.

Please remember that in order for you to obtain maximum credit for a problem, the solution must be clearly presented, i. e.:

- coordinate systems used must be clearly identified.
- where ever appropriate, free body diagrams must be drawn. These should be drawn separately from the given figures.
- units must be clearly stated as part of the answer.
- vectors must be clearly identified with proper vector notation (e.g., $\bar{v}$, $\underline{v}$ or $\vec{v}$)

If the solution does not follow a logical thought process, it will be assumed in error.

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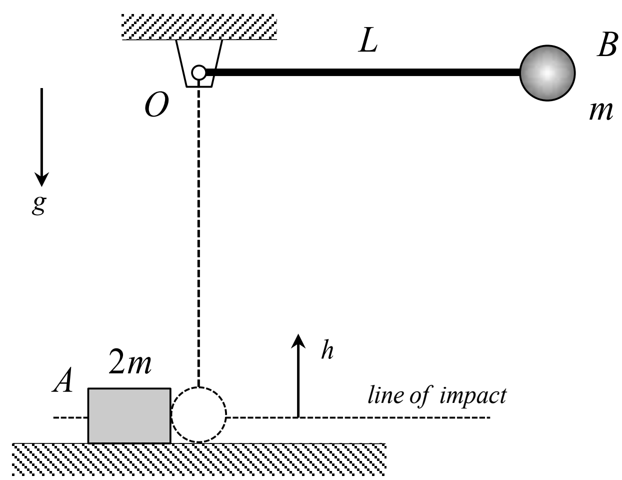
Final Examination (AM)

Problem No. 1

Given: Particle B (having a mass of m) is connected to ground at O with a link OB (having a length of L and negligible mass). OB is released from rest with a horizontal orientation. At its bottom-most position, B strikes stationary block A, which has a mass of $2m$. The coefficient of restitution for this impact is $e = 0.8$.

Find: Determine the maximum height h above the line of impact that B reaches after impacting A. Leave your answer in terms of, at most, m, g and L.

You must clearly show your ***four steps*** in your solution, including appropriate FBD(s).



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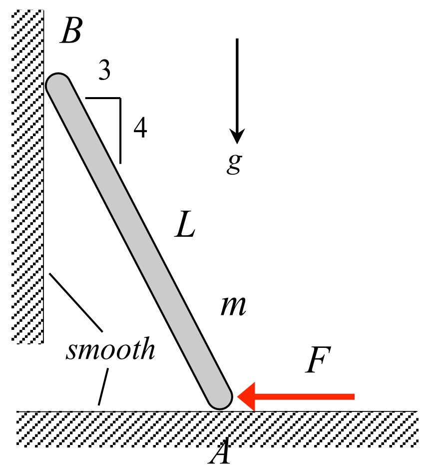
Final Examination (AM)
Problem No. 2

Given: A thin, homogeneous bar AB has a length of L and a mass of m. The bar is held in place against a smooth horizontal surface at A and a smooth vertical surface at B, with the bar having an orientation as shown in the figure. With a horizontal force F acting to the left at end A of the bar, the bar is released from rest.

Find: Determine the angular acceleration of the bar on release. Express your answers in terms of, at most, m, L, F and g.

You must clearly show your ***four steps*** in your solution, including appropriate FBD(s).



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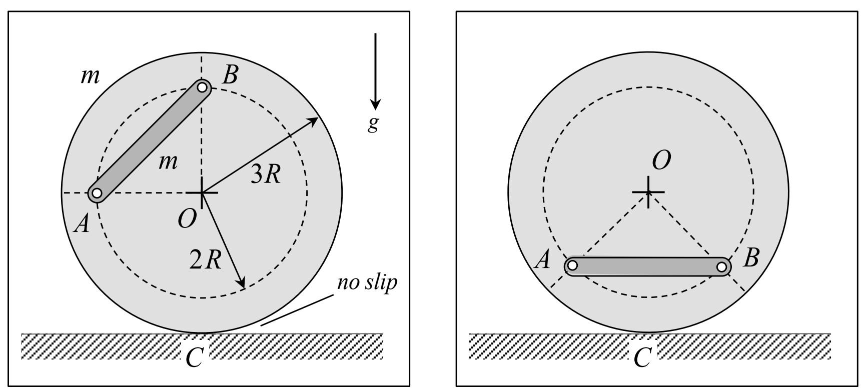
Final Examination (AM)
Problem No. 3

***Given*:** A thin, homogeneous bar AB (having a mass of m) is pinned to a homogeneous disk at ends A and B, with A and B at a radial distance of $2R$ from the disk's center C. The disk has a mass of m and an outer radius of $3R$. The system is released from rest at a position where A is on the same horizontal line as O and B located directly above O. The disk is able to roll without slipping on a horizontal surface, as shown in the figure.

***Find*:** Determine the angular speed of the disk when the disk has rotated to a position where AB is horizontal under the disk center O. Express your answer in terms of, at most, m, R and g.

You must clearly show your ***four steps*** in your solution, including appropriate FBD(s).

HINT: With bar AB pinned to the disk described, the angular speed of AB is the same as that of the disk.



Position 1 (released from rest) **Position 2 (AB horizontal)**

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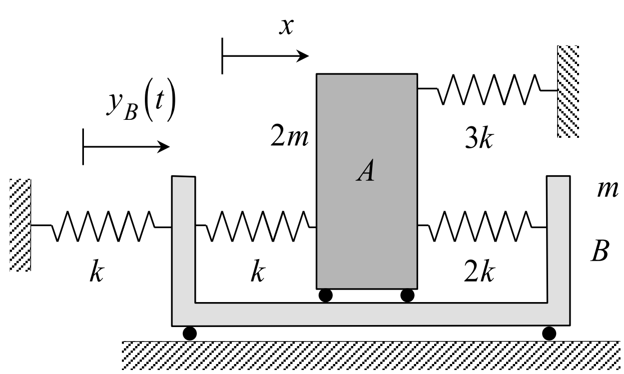
Problem No. 4

Given: A system is made up of a cart B (having a mass of m) and block A (having a mass of $2m$). The system is interconnected by a set of springs, as shown in the figure. Cart B is given a prescribed motion of $y_B(t) = b\sin\Omega t$. The springs are all unstretched when $x = y_B = 0$.

Find: For this problem:

a) Derive the dynamical equation of motion (EOM) for this system in terms of the coordinate x. You must clearly show your ***four steps*** in your derivation, including appropriate FBD(s).
b) What is the numerical value for the natural frequency for the system?
c) Determine the particular solution of your EOM derived in a) above. Leave your answer in terms of b.
d) Does the response found in c) above show A moving in-phase or out-of-phase with B?

Use the following parameter values: $m = 10\ kg$, $k = 4000\ N/m$ and $\Omega = 20\ rad/s$.



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Final Examination (AM)
Problem No. 5

PART A – 4 points

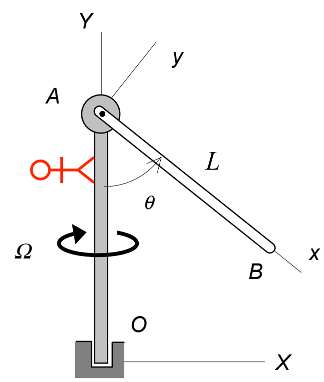
Shaft OA is rotating about the fixed Y-axis with a constant rotational speed of Ω. Arm AB is being raised at a constant rate of $\dot{\theta}$. It is desired to use the following moving reference frame kinematics to describe the acceleration of end B:

$$\vec{a}_B = \vec{a}_A + \left(\vec{a}_{B/A}\right)_{rel} + \vec{\alpha}\times\vec{r}_{B/A} + 2\vec{\omega}\times\left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}_{B/A}\right)$$

(i) If an <u>*observer attached to OA*</u> is used for this equation, what are the following two terms in the above equation?

$\left(\vec{v}_{B/A}\right)_{rel} =$

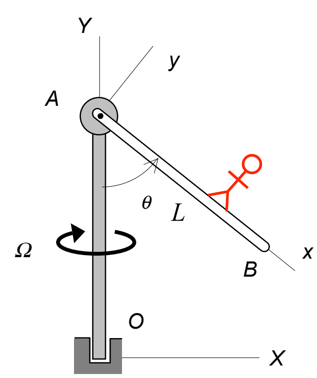
$\left(\vec{a}_{B/A}\right)_{rel} =$



(ii) If an <u>*observer attached to AB*</u> is used for this equation, what are the following two terms in the above equation?

$\left(\vec{v}_{B/A}\right)_{rel} =$

$\left(\vec{a}_{B/A}\right)_{rel} =$



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Final Examination (AM)

Problem No. 5

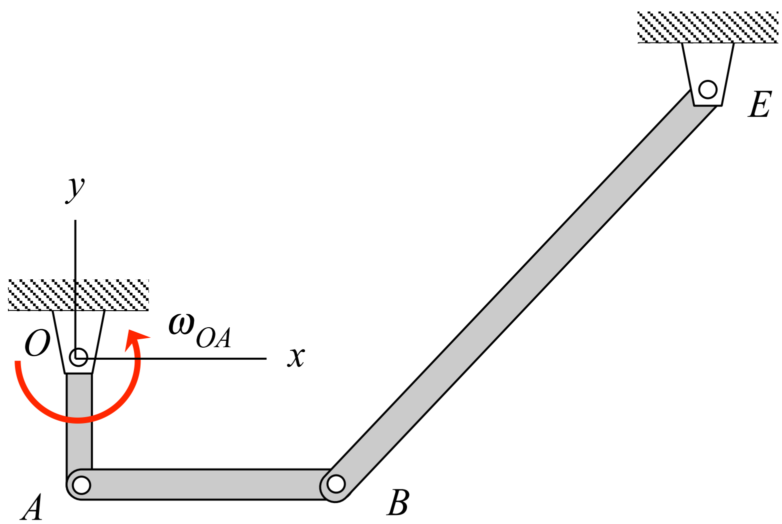
PART B – 4 points

The four-bar mechanism shown below is made up of links OA, AB and BE. Let ω_{OA}, ω_{AB} and ω_{BE} represent the angular speeds of these links. Assume that the figure of the mechanism below has been drawn to scale.

i. Which of the three speeds ω_{OA}, ω_{AB} and ω_{BE} is the LARGEST?

ii. Which of the three speeds ω_{OA}, ω_{AB} and ω_{BE} is the SMALLEST?

You must provide justification for your answers above.



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Problem No. 5

PART C – 4 points

An L-shaped bar is pinned to ground at end O. At an instant when the bar is rotating CCW with an angular speed of ω, particle P impacts end B of the bar, as shown below. The coefficient of restitution for this impact is: $0 \le e < 1$. All motion occurs in a horizontal plane.

Circle *all* correct statements below in regard to the kinetics of the ***system of bar+P*** during the time of impact:

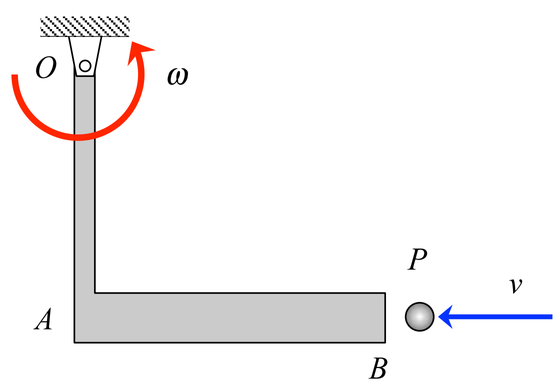
a) *Linear momentum* is conserved for the system.

b) *Angular momentum* about point O is conserved for the system.

c) *Energy* is conserved for the system.

d) None of the above.

You must provide justification for your answer above.



HORIZONTAL plane

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(Last) (First)

Final Examination (AM)

Problem No. 5

PART D – 4 points

Blocks A and B (having masses of m and $2m$, respectively) are connected by a lightweight, rigid rod. In System 1, a force F acts to the left on block A. In System 2, the same force acts to the left on block B. Let F_1 and F_2 represent the magnitude of the load carried by the rod in Systems 1 and 2, respectively.

Circle the answer below that most accurately describes the magnitudes of F_1 and F_2:

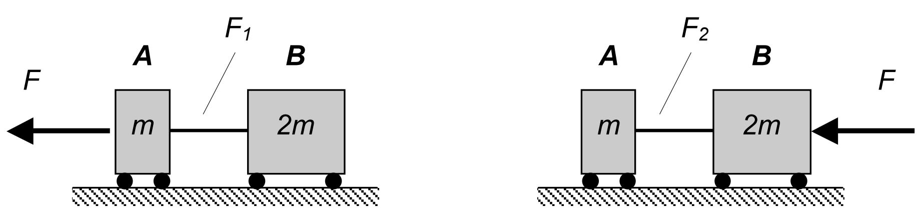
a) $F_1 > F_2$

b) $F_1 = F_2$

c) $F_1 < F_2$

d) More information is needed to answer this question.

You must provide justification for your answer above.



System 1 System 2

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PART E – 4 points

Particle P, having a mass of m, is able to slide on a smooth homogeneous arm of mass m, with the arm pinned to ground at end O. At Position 1, P has a speed of v_{P1}, and is at a radial position of R_1. At Position 2, P has slid outward on the arm to a position of R_2 (where $R_2 > R_1$), and the bar is rotating about O with an angular speed of ω_2, with $\dot{R}_2 > 0$. All motion is in a horizontal plane.

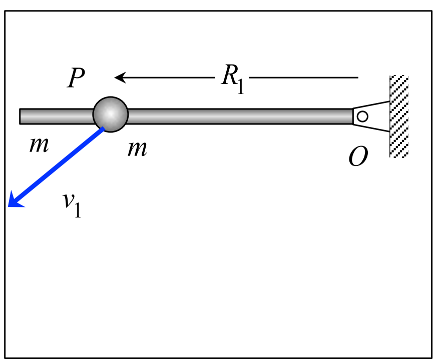
Choose the response below that most accurately describes the speed of P at Position 2, v_{P2}:

a) $v_{P2} < R_2\omega_2$

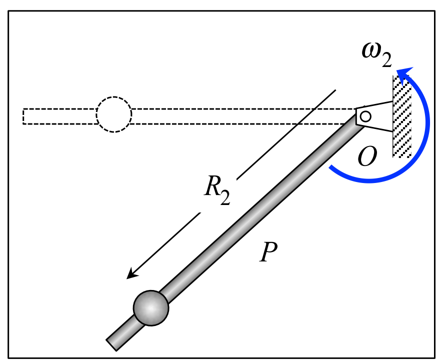
b) $v_{P2} = R_2\omega_2$

c) $v_{P2} > R_2\omega_2$

You must provide justification for your answer above.



Position 1



Position 2