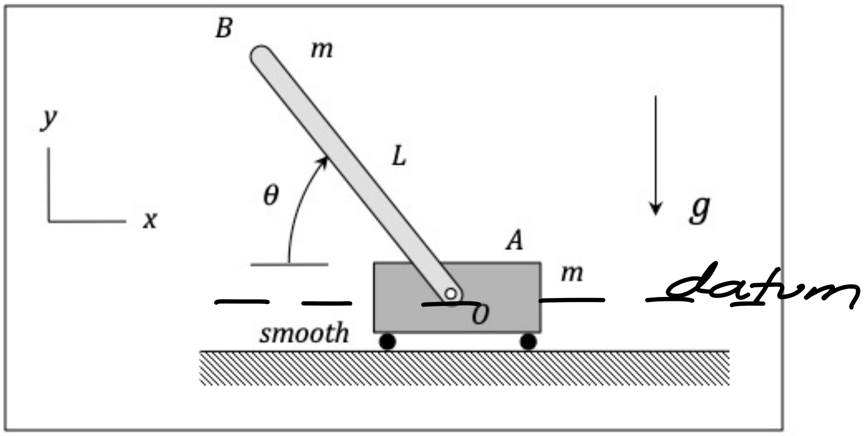
ME 274 – Summer 2022 **Name** SOLUTION
Final Examination (REGULAR)
PROBLEM NO. 1 – 20 points

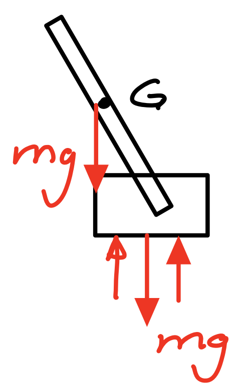


Given: A thin, homogeneous bar BO (having a mass of m and length L) is pinned to block A (which has a mass of m). Block A is able to slide along a smooth, horizontal surface. The system is released from rest with bar BO being at an angle of θ, where $0 < \theta < \pi/2$.

Find: It is desired to know the angular velocity of bar BO when $\theta = 0$. Please follow the four steps provided below, and present your work within the appropriate steps.

Solution:

SETP 1: Choose your system and draw an appropriate free body diagram for your system.



STEP 2: Kinetics

- $\sum F_x = 0 \Rightarrow \cancel{m} v_{A2} + \cancel{m} v_{Gx2} = m \cancel{v_{A1}}^{0} + m \cancel{v_{Gx1}}^{0} = 0 \quad (1)$
- $\cancel{T_1}^{0} + V_1 + \cancel{U_{1\to2}^{(nc)}}^{0} = T_2 + \cancel{V_2}^{0}$

$$mg\frac{L}{2}\sin\theta = \frac{1}{2} m v_{A2}^2 + \frac{1}{2} m v_{G2}^2 + \frac{1}{2} I_G \omega_2^2 \quad (2)$$

w/ $I_G = \frac{1}{12} mL^2$

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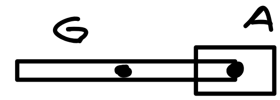
PROBLEM NO. 1 – continued



STEP 3: Kinematics

$$\vec{V}_{G_2} = \vec{V}_{A_2} + \vec{\omega}_2 \times \vec{r}_{G/A}$$

$$= v_{A_2}\hat{i} + (\omega_2 \hat{k}) \times \left(-\frac{L}{2}\hat{i}\right)$$

$$= v_{A2}\hat{i} - \frac{\omega_2 L}{2}\hat{j}$$

$\therefore$ $V_{Gx2} = V_{A2}$ (3)

$V_{Gy2} = -\frac{\omega_2 L}{2}$ (4)

& $V_{G2}^2 = V_{A2}^2 + \left(-\frac{\omega_2 L}{2}\right)^2$ (5)

STEP 4: Solve for the angular velocity of bar BO. Write your answer as a vector. Leave your answer in terms of, at most: *m*, *g*, *L* and θ.

(1) & (3): $\left.\begin{array}{l} V_{A2} = -V_{Gx2} \\ V_{A2} = V_{Gx2} \end{array}\right\} \Rightarrow V_{A2} = V_{Gx2} = 0$ (6)

(2),(5) & (6):

$$\cancel{m}g\frac{L}{\cancel{2}}\sin\theta = \frac{1}{\cancel{2}}\cancel{m}\left(\frac{\omega_2^2 L^2}{4}\right)^2 + \frac{1}{\cancel{2}}\left(\frac{1}{12}\cancel{m}L^2\right)\omega_2^2$$

$$\hookrightarrow \vec{\omega}_2 = \sqrt{\frac{gL\sin\theta}{\frac{L^2}{4} + \frac{L^2}{12}}}\,\hat{k}$$

$$= \sqrt{\frac{3g}{L}\sin\theta}\,\hat{k}$$

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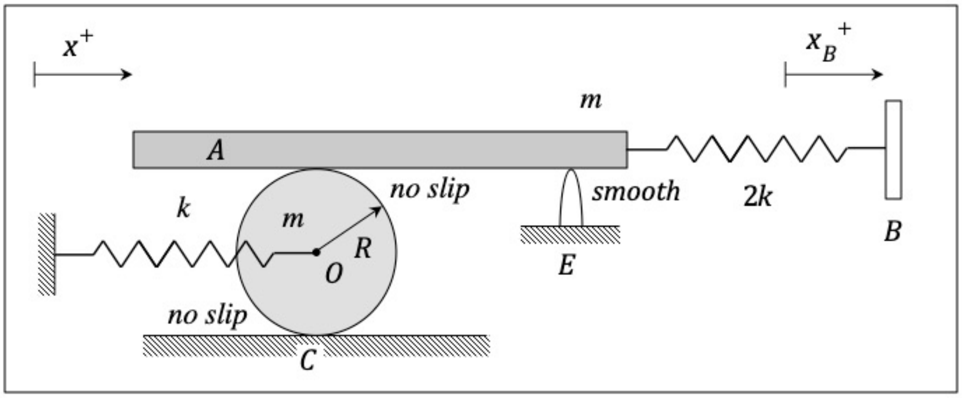
PROBLEM NO. 2 – 20 points

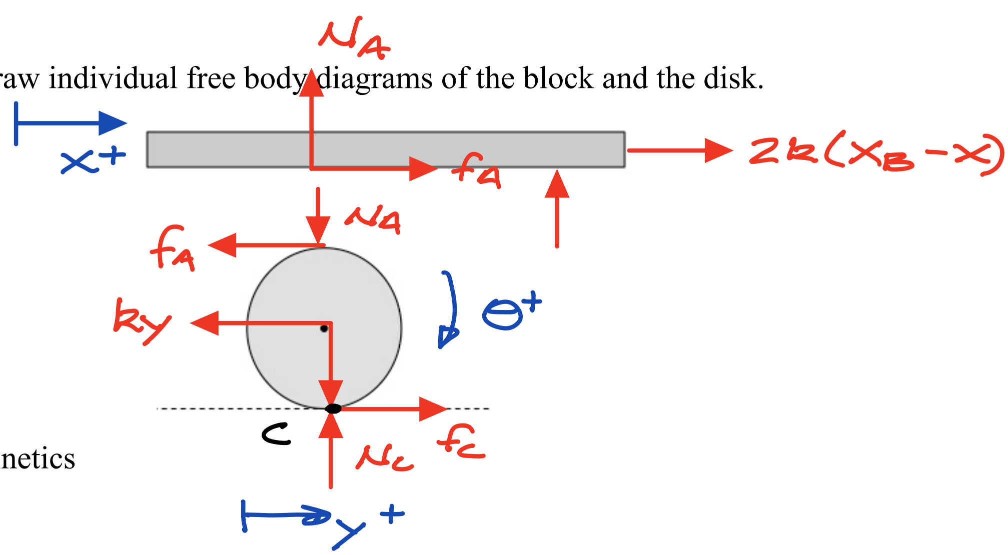


Given: Consider the system above that is made up of a homogeneous disk (with a mass of m and outer radius R), block A (having a mass of m), two springs (of stiffnesses k and $2k$) and a moveable base B. The disk rolls without slipping on a fixed horizontal surface, with block A translating without slipping on the top surface of the disk. Base B moves with a prescribed horizontal surface of $x_B(t) = b\sin\Omega t$. Let the coordinate x measure the motion of block A. The springs are unstretched when $x = x_B = 0$.

Find: It is desired to know the differential equation of motion (EOM) for the system in terms of the x coordinate, and the particular solution for the EOM. Please follow the steps provided below, and present your work within the appropriate steps.

Solution:

<u>SETP 1</u>: Draw individual free body diagrams of the block and the disk.



<u>STEP 2</u>: Kinetics

Disk: $\sum M_C = -(ky)R - f_A(2R) = I_C\ddot{\theta}$ (1)

Bar: $\sum F_x = 2k(x_B - x) + f_A = m\ddot{x}$ (2)

w/ $I_C = I_O + mR^2 = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2$

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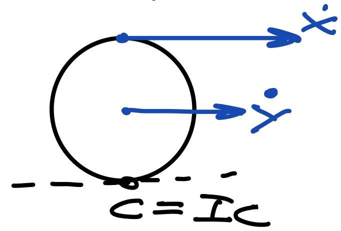
PROBLEM NO. 2 – continued

STEP 3: Kinematics

Since $C = I_C \Rightarrow$

- $\dot{x} = 2R\dot{\theta} \Rightarrow \dot{\theta} = \dfrac{\dot{x}}{2R} \Rightarrow \ddot{\theta} = \dfrac{\ddot{x}}{2R}$ (3)
- $\dot{y} = R\dot{\theta} = \frac{1}{2}\dot{x} \Rightarrow y = \frac{1}{2}x$ (4)



STEP 4: EOM. Leave your answer as a differential equation in terms of, at most: *m, k, R, x* and time derivatives of *x*.

$$(1) \Rightarrow f_A = \frac{1}{2R}\left[-\left(\frac{3}{2}mR^2\right)\ddot{\theta} - kRy\right]$$

$$(2) \Rightarrow f_A = m\ddot{x} - 2k(x_B - x)$$

$$\frac{1}{2R}\left[-\frac{3}{2}mR^2\ddot{\theta} - kRy\right] = m\ddot{x} - 2k(x_B - x)$$

(3) & $(4) \Rightarrow$

$$\frac{1}{2\cancel{R}}\left[-\frac{3}{2}m\cancel{R^2}\left(\frac{\ddot{x}}{2\cancel{R}}\right) - k\cancel{R}\left(\frac{1}{2}x\right)\right] = m\ddot{x} - 2k(x_B - x)$$

$$\frac{11}{4}m\ddot{x} + \frac{9}{2}kx = 4kb\sin\Omega t$$

$$\ddot{x} + \underbrace{\frac{18}{11}\frac{k}{m}}_{\omega_n^2}x = \frac{16}{11}\frac{k}{m}b\sin\Omega t \longleftarrow \text{EOM}$$

STEP 5: DERIVE the particular solution of the EOM starting with the general form of a linear differential equation with sinusoidal excitation.

$$\begin{cases} x_p(t) = A\cos\Omega t + B\sin\Omega t \\ \ddot{x}_p(t) = -A\Omega^2\cos\Omega t - B\Omega^2\sin\Omega t \end{cases}$$

$$\therefore (-\Omega^2 + \omega_n^2)A\cos\Omega t + (-\Omega^2 + \omega_n^2)B\sin\Omega t = \frac{16}{11}\frac{k}{m}b\sin\Omega t$$

$\underline{\cos\Omega t}$: $(-\Omega^2 + \omega_n^2)A = 0 \Rightarrow A = 0$

$\underline{\sin\Omega t}$: $(-\Omega^2 + \omega_n^2)B = \frac{16}{11}\frac{k}{m}b \Rightarrow B = \dfrac{\frac{16}{11}\frac{k}{m}b}{-\Omega^2 + \omega_n^2}$

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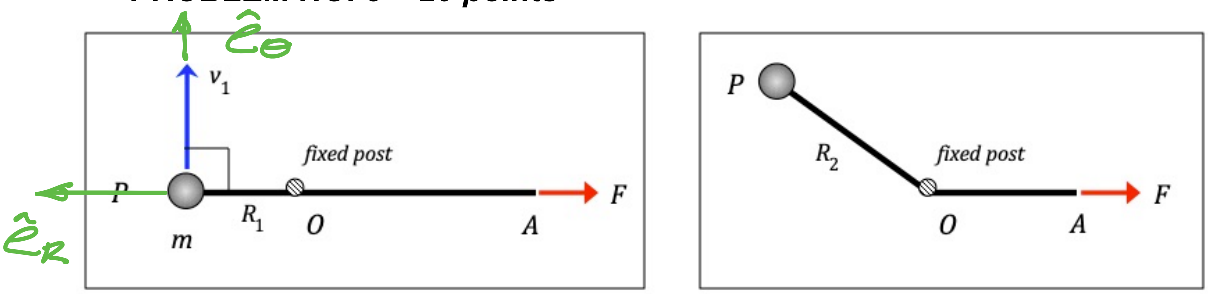
PROBLEM NO. 3 – 20 points



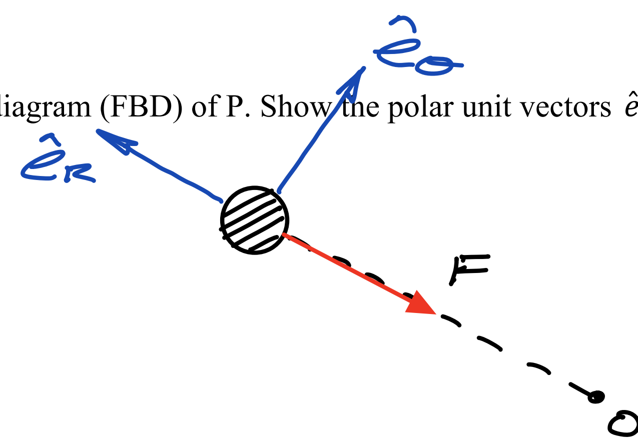
Position 1 Position 2

Given: Particle P (having a mass of m) is able to slide on a smooth, HORIZONTAL surface. A cable is attached to P, with the cable being in contact with a smooth, fixed post at O, and with a constant force F acting at end A of the cable. At Position 1 P is at a distance of R_1 from post O and is moving with a speed of v_1 in a direction that is perpendicular to OP. At Position 2, P has moved outward with the radial distance from O to P being $R_2 = 2R_1$.

Find: It is desired to know the velocity vector of P at Position 2.

Solution:

SETP 1: Draw a free body diagram (FBD) of P. Show the polar unit vectors $\hat{e}_R$ and $\hat{e}_\theta$ in your FBD.



STEP 2: Kinetics

- $\Sigma M_O = 0 \Rightarrow \vec{H}_O = \text{const.}$

$$\begin{cases} \vec{H}_{O1} = m\vec{r}_{P/O} \times \vec{v}_1 = m(R_1\hat{e}_R)\times(v_1\hat{e}_\theta) = mR_1v_1\hat{k} \\ \vec{H}_{O_2} = m\vec{r}_{P/O} \times \vec{v}_P = m(R_2\hat{e}_R)\times(\dot{R}_2\hat{e}_R + R_2\omega_2\hat{e}_\theta) \\ \qquad = mR_2^2\omega_2\hat{k} \end{cases}$$

$$\vec{H}_{O1} = \vec{H}_{O2} \Rightarrow \cancel{m}R_1v_1 = \cancel{m}R_2^2\omega_2 \Rightarrow \omega_2 = \frac{R_1}{R_2^2}v_1 = \frac{v_1}{4R_1} \quad (1)$$

- $T_1 + \cancel{V_1} + U_{1\to 2}^{(nc)} = T_2 + \cancel{V_2} \Rightarrow$

$$-FR_1 + \frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 \Rightarrow v_2^2 = v_1^2 - \frac{2FR_1}{m} \quad (2)$$

<u>STEP 3</u>: Kinematics: Write down the velocity of P in terms of its polar coordinates and using the polar unit vectors $\hat{e}_R$ and $\hat{e}_\theta$.

$$\vec{V}_2 = \dot{R}_2 \hat{e}_R + R_2 \omega_2 \hat{e}_\theta$$

$$\hookrightarrow V_2^2 = \dot{R}_2^2 + R_2^2 \omega_2^2 \qquad (3)$$

<u>STEP 4</u>: Find the velocity of P. Write your answer as a vector, and in terms of, at most: m, R_1, F and v_1.

$$(1), (2), (3) \Rightarrow V_1^2 - \frac{2FR_1}{m} = \dot{R}_2^2 + \overbrace{(2R_1)^2 \left[\frac{R_1}{(2R_1)^2}\right]^2}^{1/4} V_1^2$$

$$\hookrightarrow \dot{R}_2 = \sqrt{\frac{3}{4} V_1^2 - \frac{2FR_1}{m}}$$

$$\therefore \quad \vec{V}_2 = \sqrt{\frac{3}{4} V_1^2 - \frac{2FR_1}{m}}\ \hat{e}_R + \underbrace{(2R_1)\left(\frac{V_1}{4R_1}\right)}_{\frac{V_1}{2}} \hat{e}_\theta$$

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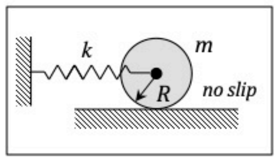
PROBLEM NO. 4 – 20 points TOTAL

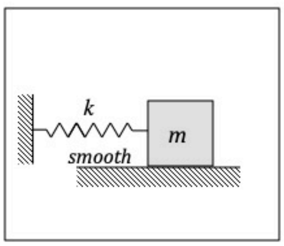
NOTE: You are not required to show your work on Problem 4. There is no partial credit awarded for the different parts of the problem.



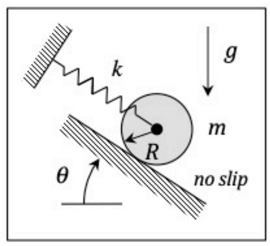
System 0

EOM: $\frac{3}{2}m\ddot{x}+kx=0$

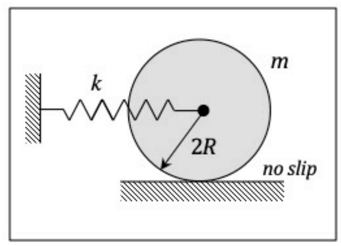
↳ $\omega_{n0}=\sqrt{\frac{2}{3}}\sqrt{\frac{k}{m}}$



System 1



System 2



System 3

Let ω_{n0}, ω_{n1}, ω_{n2} and ω_{n3} represent the natural frequencies for Systems 0, 1, 2 and 3 shown above, respectively. For all four systems, the disks are homogeneous and have a mass of m.

PART A.1 – 2 pts. – choose the correct response

System 1: $m\ddot{x}+kx=0$

↳ $\omega_{n1}=\sqrt{\frac{k}{m}}$

a) $\omega_{n0} > \omega_{n1}$

b) $\omega_{n0} = \omega_{n1}$

$\boxed{\text{c) } \omega_{n0} < \omega_{n1}}$

d) More information is needed to answer this question.

PART A.2 – 2 pts. – choose the correct response

System 2: ω_n not influenced by gravity

a) $\omega_{n0} > \omega_{n2}$

$\boxed{\text{b) } \omega_{n0} = \omega_{n2}}$

c) $\omega_{n0} < \omega_{n2}$

d) More information is needed to answer this question.

PART A.3 – 2 pts. – choose the correct response

ω_n not influenced by R

a) $\omega_{n0} > \omega_{n3}$

b) $\boxed{\omega_{n0} = \omega_{n3}}$

c) $\omega_{n0} < \omega_{n3}$

d) More information is needed to answer this question.

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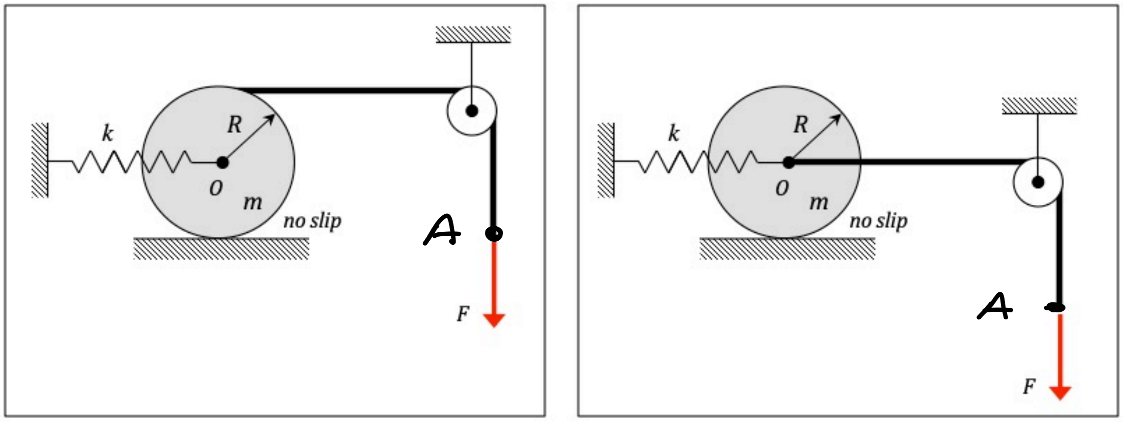
PROBLEM NO. 4 (continued)



System I System II

PART B – 1 pt.

The force F for both Systems I and II pulls the center O of the disk to the right through a distance of d. Let $U_{1\to2}^{(I)}$ and $U_{1\to2}^{(II)}$ represent the work done for F for Systems I and II, respectively.

a) $\boxed{U_{1\to2}^{(I)} > U_{1\to2}^{(II)}}$

b) $U_{1\to2}^{(I)} = U_{1\to2}^{(II)}$

c) $U_{1\to2}^{(I)} < U_{1\to2}^{(II)}$

d) More information is needed in order to answer this question.

A in System I moves twice as far as A in System II.

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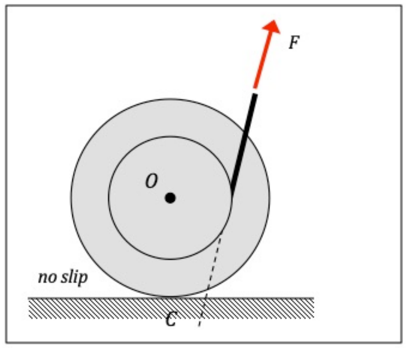
PROBLEM NO. 4 (continued)

PART C – 1 pt.



As a result of the applied force F, the center of the drum O will:

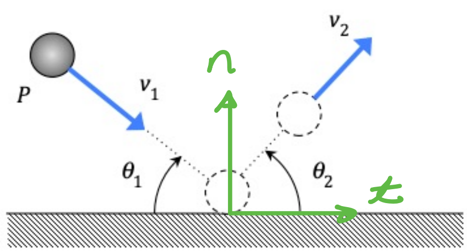
a) Move to the right. F creates a CCW moment about C

b) Will not move.

c) $\boxed{\text{Move to the left.}}$

d) More information is needed in order to answer this question.

PART D – 1 pt.



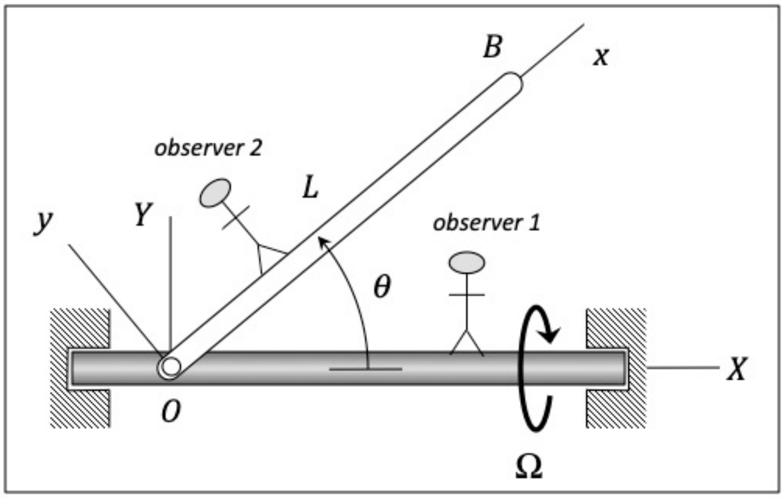
Particle P strikes a stationary wall with a speed of v_1 and an angle θ_1. For a coefficient of restitution of $0 < e < 1$, the rebound angle θ_2 is such that:

a) $\theta_2 > \theta_1$ $V_{2t} = V_{1t}$

b) $\theta_2 = \theta_1$ $V_{2n} < V_{1n}$

c) $\boxed{\theta_2 < \theta_1}$

d) More information is needed in order to answer this question.



PART D

The horizontal shaft above is rotating about a fixed axis with a *constant* rate of Ω. Bar OB is pinned to the horizontal shaft, with the elevation angle θ increasing at a *constant* rate of $\dot{\theta}$. The following moving reference frame kinematics equation is to be used to describe the acceleration of point B for $0<\theta<90°$:

$$\vec{a}_B = \vec{a}_O + \left(\vec{a}_{B/O}\right)_{rel} + \vec{\alpha}\times\vec{r}_{B/O} + 2\vec{\omega}\times\left(\vec{v}_{B/O}\right)_{rel} + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}_{B/O}\right)$$

D.1 – 2 pts. Using an <u>*observer 1 (attached to the horizontal shaft),*</u> fill in the following terms below for this equation (*in terms of their xyz-coordinates*):

$\vec{\omega} = -\Omega \hat{I} = -\Omega(\cos\theta\,\hat{i} - \sin\theta\,\hat{j})$

$\vec{\alpha} = \vec{0}$

D.2 – 2 pts. Using an <u>*observer 2 (attached to OB),*</u> fill in the following terms below for this equation (*in terms of their xyz-coordinates*):

$\left(\vec{v}_{B/O}\right)_{rel} = \vec{0}$

$\left(\vec{a}_{B/O}\right)_{rel} = \vec{0}$

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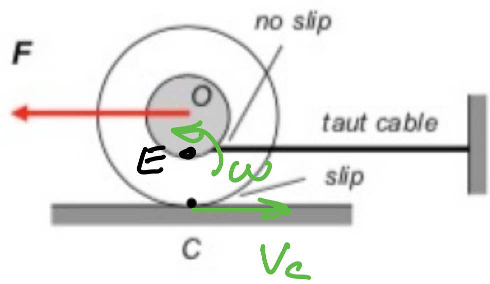
PROBLEM NO. 4 (continued)



PART E – 1 pt.
A force F is applied to the center O of a stepped drum. A cable is wrapped around the inner radius of the drum, with the other end of the cable being attached to a fixed wall. The drum *slips* on the fixed horizontal surface that supports the drum. Choose the correct response below regarding the direction of motion of the contact point C.

a) $\boxed{\text{C moves to the right.}}$
b) C remains stationary.
c) C moves to the left.
d) More information is needed to answer this question.

E = IC for drum ⇒ drum will rotate CCW ⇒ C moves to right

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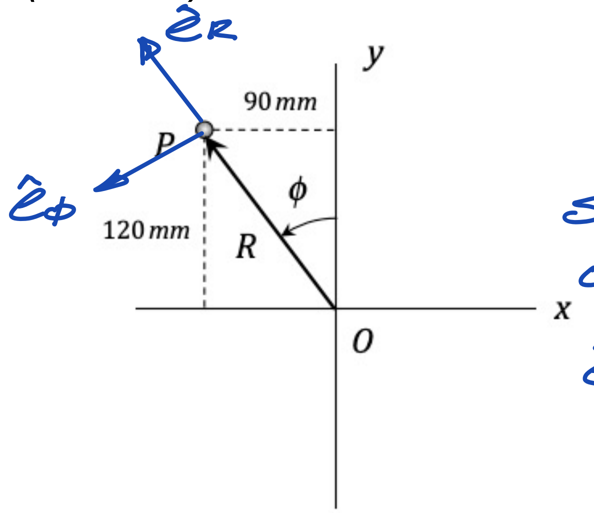
PROBLEM NO. 4 (continued)



$$\sin\phi = \frac{90}{160} = 0.6$$
$$\cos\phi = \frac{120}{160} = 0.8$$
$$\hat{e}_r = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j}$$
$$= -0.6\hat{i} + 0.8\hat{j}$$

PART F

The velocity and acceleration of particle P are known in terms of their Cartesian components:

$$\vec{v} = (400\,\hat{i} + 300\,\hat{j})\,mm/s$$
$$\vec{a} = (20\,\hat{j})\,mm/s^2$$

For this motion:

F.1 – 2 pts.

a) $\dot{R} > 0$

$\boxed{\text{b) } \dot{R} = 0}$

c) $\dot{R} < 0$

$$\dot{R} = \vec{V}\cdot\hat{e}_R = (400\hat{i} + 300\hat{j})\cdot(-0.6\hat{i} + 0.8\hat{j})$$
$$= -400(0.6) + 300(0.8) = 0$$

F.2 – 2 pts.

$\boxed{\text{a) } \ddot{R} > 0}$

b) $\ddot{R} = 0$

c) $\ddot{R} < 0$

$$\ddot{R} - R\dot{\theta}^2 = \vec{a}\cdot\hat{e}_R$$
$$= (20\hat{j})\cdot(-0.6\hat{i} + 0.8\hat{j})$$
$$= 16$$
$$\rightarrow \ddot{R} = 16 + R\dot{\theta}^2 > 0$$

F.3 – 2 pts.

If v is the speed of P, then:

$\boxed{\text{a) } \dot{v} > 0}$

b) $\dot{v} = 0$

c) $\dot{v} < 0$

$$\dot{v} = \vec{a}\cdot\frac{\vec{V}}{|\vec{V}|} = (20\hat{j})\cdot\left[\frac{400\hat{i} + 300\hat{j}}{500}\right]$$
$$= (20)(0.6) = 12 > 0$$