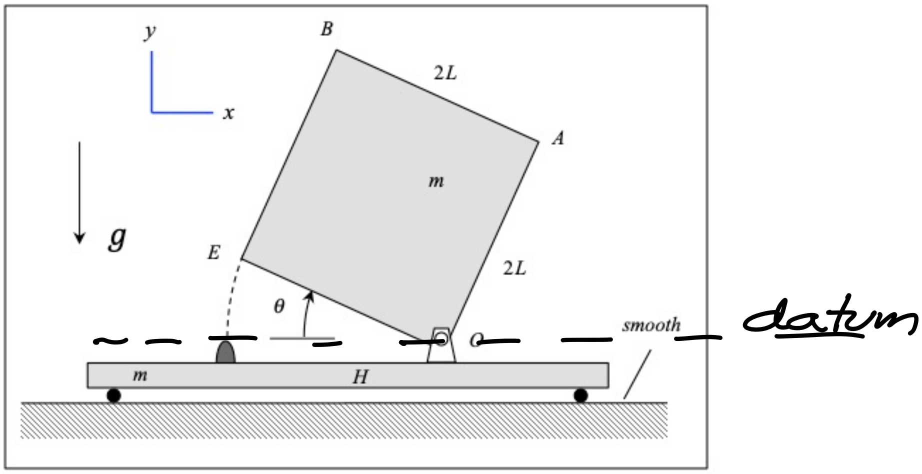
ME 274 – Summer 2022 **Name** SOLUTION

Final Examination (ALTERNATE)

PROBLEM NO. 1 – 20 points

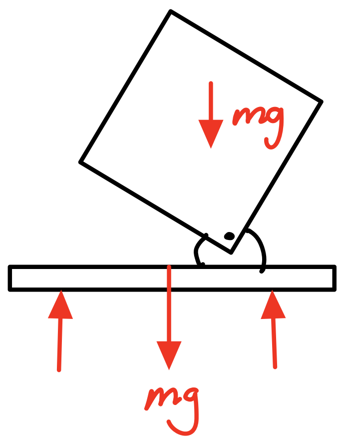


Given: A homogeneous square plate OABE (having a mass of *m*) is pinned to cart H (which has a mass of m). The cart is able to slide along a smooth, horizontal surface. The system is released from rest from a position where corner B is displaced slightly to the left of being directly above O.

Find: It is desired to know the angular velocity of the plate when $\theta = 0$, immediately *before* the plate strikes the bumper at E. Please follow the four steps provided below, and present your work within the appropriate steps.

Solution:

<u>SETP 1</u>: Choose your system and draw an appropriate free body diagram for your system.



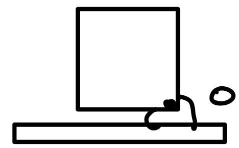
<u>STEP 2</u>: Kinetics

- $\sum F_x = 0 \Rightarrow \cancel{m} v_{O2} + \cancel{m} v_{Gx2} = m \cancel{v_{O1}}^{0} + m \cancel{v_{Gx1}}^{0} = 0$ (1)

- $\cancel{T_1}^{0} + V_1 + \cancel{U_{1\to 2}^{(nc)}}^{0} = T_2 + V_2$

$$mg\sqrt{2}L = \frac{1}{2} m v_O^2 + \frac{1}{2} m v_{G2}^2 + \frac{1}{2} I_G \omega_2^2 + mgL \qquad (2)$$

w/ $I_G = \frac{1}{12} m \left[(2L)^2 + (2L)^2\right] = \frac{2}{3} m L^2$



<u>*STEP 3*</u>: Kinematics

$$\vec{V}_{G2} = \vec{V}_{O2} + \vec{\omega}_2 \times \vec{r}_{G/O}$$
$$= V_{O2}\hat{\imath} + (\omega_2\hat{k}) \times (-L\hat{\imath} + L\hat{\jmath})$$
$$= (V_{O2} - L\omega_2)\hat{\imath} + (-L\omega_2\hat{\jmath})$$

$\therefore$ $V_{Gx2} = V_{O2} - L\omega_2$ (3)

$V_{Gy2} = -L\omega_2$ (4)

& $V_{G2}^2 = V_{Gx2}^2 + V_{Gy2}^2$ (5)

<u>*STEP 4*</u>: Solve for the angular velocity of the plate. Write your answer as a vector. Leave your answer in terms of, at most: *m*, *g*, *b*, *h* and θ.

(1) $\Rightarrow$ $V_{O2} = -V_{Gx2}$

(3) $\Rightarrow$ $V_{Gx2} = -V_{Gx2} - L\omega_2 \Rightarrow V_{Gx2} = \frac{L}{2}\omega_2$

(5) $\Rightarrow$ $V_{G2}^2 = \left(\frac{L}{2}\omega_2\right)^2 + (L\omega_2)^2 = \frac{5}{4}L^2\omega_2^2$

(2) $\Rightarrow$ $(\sqrt{2}-1)\cancel{m}gL = \frac{1}{2}\cancel{m}\left(-\frac{L}{2}\omega_2\right)^2 + \frac{1}{2}\cancel{m}\left(\frac{5}{4}L^2\omega_2^2\right) + \frac{1}{2}\left(\frac{2}{3}\cancel{m}L^2\right)\omega_2^2$

$\hookrightarrow$ $\vec{\omega}_2 = \sqrt{\dfrac{(\sqrt{2}-1)gL}{\left(\frac{1}{8} + \frac{5}{8} + \frac{1}{3}\right)L^2}}\,\hat{k} = \sqrt{\frac{12}{13}(\sqrt{2}-1)}\sqrt{\frac{g}{L}}\,\hat{k}$

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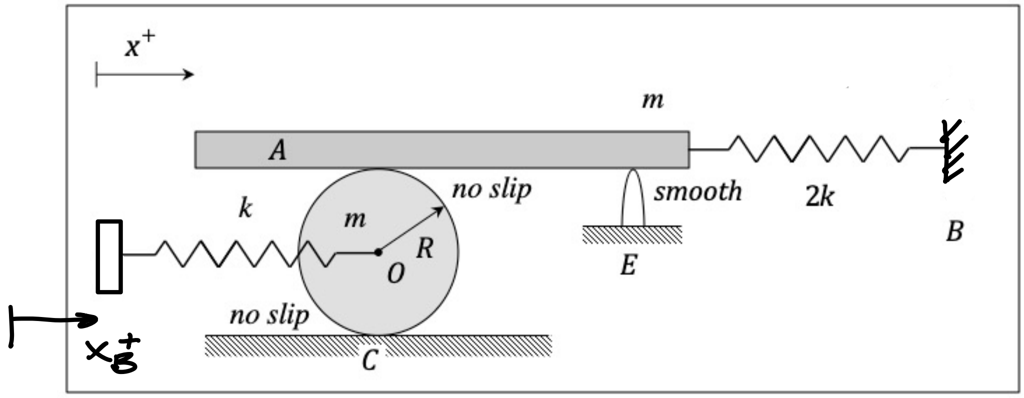
PROBLEM NO. 2 – 20 points

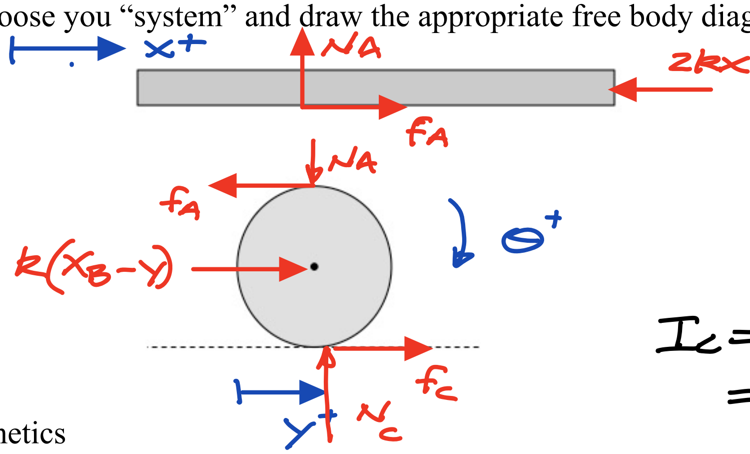


Given: Consider the system above that is made up of a homogeneous disk (with a mass of m and outer radius R), block A (having a mass of m), two springs (of stiffnesses k and $2k$) and a moveable base B. The disk rolls without slipping on a fixed horizontal surface, with block A translating without slipping on the top surface of the disk. Base B moves with a prescribed horizontal motion of $x_B(t) = b\sin\Omega t$. Let the coordinate x measure the motion of block A. The springs are unstretched when $x = x_B = 0$.

Find: It is desired to know the differential equation of motion (EOM) for the system in terms of the x coordinate, and the particular solution for the EOM. Please follow the steps provided below, and present your work within the appropriate steps.

Solution:

<u>SETP 1</u>: Choose you "system" and draw the appropriate free body diagram(s) for your system.



$$I_C = I_O + mR^2 = \frac{3}{2}mR^2$$

<u>STEP 2</u>: Kinetics

Disk: $\sum M_C = [k(x_B - y)]R - f_A(2R) = I_C\ddot{\theta}$ (1)

Bar: $\sum F_x = f_A - 2kx = m\ddot{x}$ (2)

(1) $\Rightarrow$ $f_A = [-I_C\ddot{\theta} + Rk(x_B - y)]\frac{1}{2R}$

(2) $\Rightarrow$ $f_A = m\ddot{x} + 2kx$

$\therefore$ $m\ddot{x} + 2kx = \frac{1}{2R}[-I_C\ddot{\theta} + Rk(x_B - y)]$ (3)

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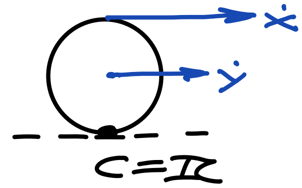
PROBLEM NO. 2 – continued



<u>*STEP 3*</u>: Kinematics

Since $C = IC \Rightarrow$

- $\dot{x} = 2R\dot{\theta} \Rightarrow \dot{\theta} = \dfrac{\dot{x}}{2R} \Rightarrow \ddot{\theta} = \dfrac{\ddot{x}}{2R}$ (4)
- $\dot{y} = R\dot{\theta} = \dfrac{\dot{x}}{2} \Rightarrow y = \frac{1}{2}x$ (5)

<u>*STEP 4*</u>: EOM. Leave your answer as a differential equation in terms of, at most: *m, k, R, x* and time derivatives of *x*.

(3), (4), (5) $\Rightarrow$

$$m\ddot{x} + 2kx = \frac{1}{2\cancel{R}}\left[-\left(\frac{3}{2}mR^2\right)\frac{\ddot{x}}{2R} + \cancel{R}k\left(x_B - \frac{x}{2}\right)\right]$$

$$\frac{11}{8}m\ddot{x} + \frac{9}{4}kx = \frac{1}{2}kb\sin\Omega t$$

$$\ddot{x} + \underbrace{\frac{18}{11}\frac{k}{m}}_{\omega_n^2}x = \frac{4}{11}\frac{k}{m}b\sin\Omega t \longleftarrow \text{EOM}$$

<u>*STEP 5*</u>*:* DERIVE the particular solution of the EOM starting with the general form of a linear differential equation with sinusoidal excitation.

$$\begin{cases} x_p(t) = A\cos\Omega t + B\sin\Omega t \\ \ddot{x}_p(t) = -A\Omega^2\cos\Omega t - B\Omega^2\sin\Omega t \end{cases}$$

$$\therefore (-\Omega^2 + \omega_n^2)A\cos\Omega t + (-\Omega^2 + \omega_n^2)B\sin\Omega t = \frac{4}{11}\frac{k}{m}b\sin\Omega t$$

<u>$\cos\Omega t$</u>: $(-\Omega^2 + \omega_n^2)A = 0 \Rightarrow A = 0$

<u>$\sin\Omega t$</u>: $(-\Omega^2 + \omega_n^2)B = \frac{4}{11}\frac{k}{m}b \Rightarrow B = \dfrac{\frac{4}{11}\frac{k}{m}b}{-\Omega^2 + \omega_n^2}$

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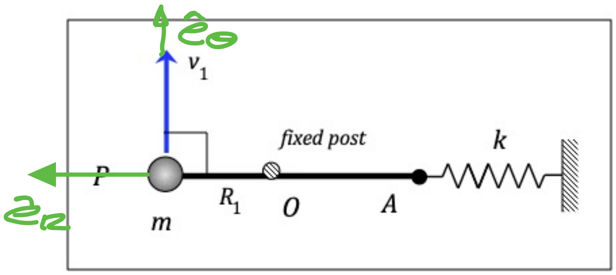
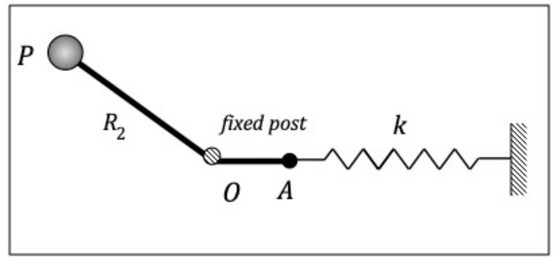
PROBLEM NO. 3 – 20 points



Position 1



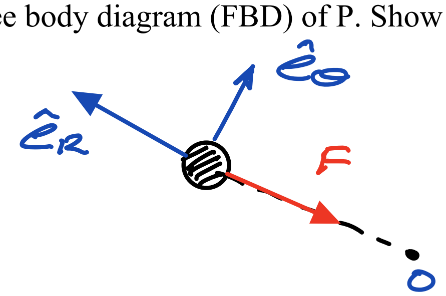
Position 2

Given: Particle P (having a mass of m) is able to slide on a smooth, HORIZONTAL surface. A cable is attached to P, with the cable being in contact with a smooth, fixed post at O, and with a spring of stiffness k attached to the cable at end A of the cable. At Position 1, P is at a distance of R_1 from post O and is moving with a speed of v_1 in a direction that is perpendicular to OP. The spring is unstretched at Position 1. At Position 2, P has moved outward with the radial distance from O to P being $R_2 = 2R_1$.

Find: It is desired to know the velocity vector of P at Position 2.

Solution:

SETP 1: Draw a free body diagram (FBD) of P. Show the polar unit vectors $\hat{e}_R$ and $\hat{e}_\theta$ in your FBD.



STEP 2: Kinetics

- $\Sigma M_O = 0 \Rightarrow \vec{H}_O = \text{const}$

$$\begin{cases} \vec{H}_{O1} = m\vec{r}_{P_1/O} \times \vec{v}_{P1} = m(R_1\hat{e}_R) \times (v_1\hat{e}_\theta) = mR_1v_1\hat{k} \\ \vec{H}_{O2} = m\vec{r}_{P/O} \times \vec{v}_{P2} = m(R_2\hat{e}_R) \times (\dot{R}_2\hat{e}_R + R_2\omega_2\hat{e}_\theta) \end{cases}$$

$$= mR_2^2\omega_2\hat{k}$$

$$\vec{H}_{O1} = \vec{H}_{O2} \Rightarrow mR_1v_1 = mR_2^2\omega_2 \Rightarrow \omega_2 = \frac{R_1}{R_2^2}v_1 = \frac{v_1}{4R_1} \qquad (1)$$

- $T_1 + \cancel{V_1}^{\,0} + \cancel{U_{1\to 2}^{(nc)}}^{\,0} = T_2 + V_2$

$$\cancel{\tfrac{1}{2}}mv_1^2 = \cancel{\tfrac{1}{2}}mv_2^2 + \cancel{\tfrac{1}{2}}kR_1^2 \Rightarrow v_2^2 = v_1^2 - \frac{k}{m}R_1^2 \qquad (2)$$

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PROBLEM NO. 3 – continued

STEP 3: Kinematics: Write down the velocity of P in terms of its polar coordinates and using the polar unit vectors $\hat{e}_R$ and $\hat{e}_\theta$.

$$\vec{V}_2 = \dot{R}_2 \hat{e}_R + R_2 \omega_2 \hat{e}_\theta$$

$$\hookrightarrow V_2^2 = \dot{R}_2^2 + R_2^2 \omega_2^2 \qquad (3)$$

STEP 4: Find the velocity of P. Write your answer as a vector, and in terms of, at most: m, R_1, k and v_1.

$$(1), (2), (3): \quad V_1^2 - \frac{k}{m} R_1^2 = \dot{R}_2^2 + \overbrace{(2R_1)^2 \left[\frac{R_1}{(2R_1)^2}\right]^2}^{1/4} V_1^2$$

$$\hookrightarrow \dot{R}_2 = \sqrt{\frac{3}{4} V_1^2 - \frac{k}{m} R_1^2}$$

$$\therefore \quad \vec{V}_2 = \sqrt{\frac{3}{4} V_1^2 - \frac{k}{m} R_1^2}\, \hat{e}_R + \frac{V_1}{2} \hat{e}_\theta \longleftarrow \vec{V}_2$$

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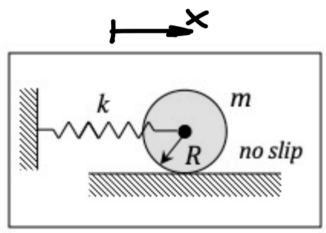
PROBLEM NO. 4 – 20 points TOTAL

NOTE: You are not required to show your work on Problem 3. There is no partial credit awarded for the different parts of the problem.

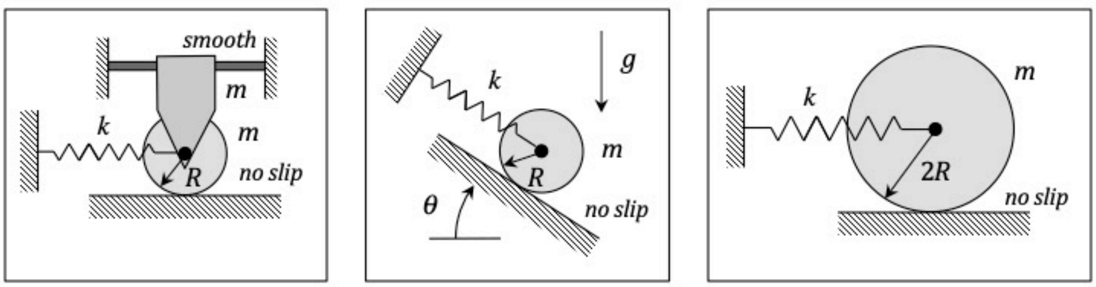


System 0

EOM:

$\frac{3}{2}m\ddot{x}+kx=0$

$\hookrightarrow \omega_n=\sqrt{\frac{2}{3}}\sqrt{\frac{k}{m}}$



System 1 System 2 System 3

Let ω_{n0}, ω_{n1}, ω_{n2} and ω_{n3} represent the natural frequencies for Systems 0, 1, 2 and 3 shown above. For all four systems, the disks are homogeneous and have a mass of m.

PART A.1 – 2 pts. – choose the correct response

$\boxed{\text{a) } \omega_{n0} > \omega_{n1}}$

b) $\omega_{n0} = \omega_{n1}$

c) $\omega_{n0} < \omega_{n1}$

d) More information is needed to answer this question.

System 1: $\frac{5}{2}m\ddot{x}+kx=0$

$\hookrightarrow \omega_{n1}=\sqrt{\frac{2}{5}}\sqrt{\frac{k}{m}}$

PART A.2 – 2 pts. – choose the correct response

a) $\omega_{n0} > \omega_{n2}$

$\boxed{\text{b) } \omega_{n0} = \omega_{n2}}$

c) $\omega_{n0} < \omega_{n2}$

d) More information is needed to answer this question.

ω_n not influenced by gravity

PART A.3 – 2 pts. – choose the correct response

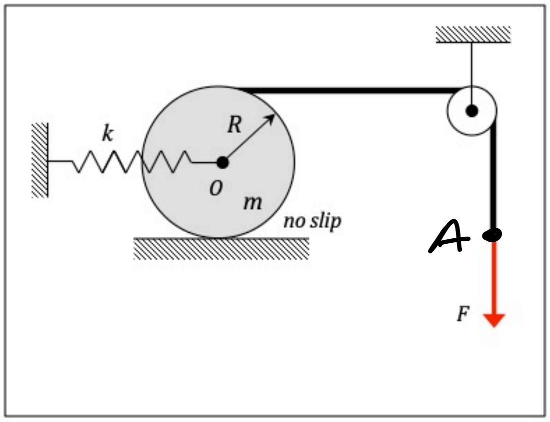
ω_n not influenced by R

a) $\omega_{n0} > \omega_{n3}$

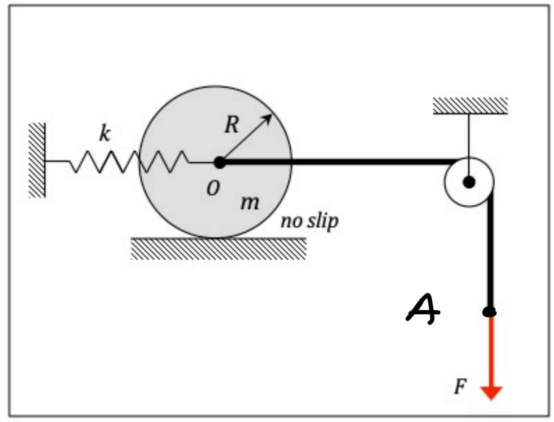
$\boxed{\text{b) } \omega_{n0} = \omega_{n3}}$

c) $\omega_{n0} < \omega_{n3}$

d) More information is needed to answer this question.



System I



System II

PART B – 1 pt.

The force F for both Systems I and II pulls the center O of the disk to the right through a distance of d. Let $U_{1\to2}^{(I)}$ and $U_{1\to2}^{(II)}$ represent the work done for F for Systems I and II, respectively.

a) $\boxed{U_{1\to2}^{(I)} > U_{1\to2}^{(II)}}$

b) $U_{1\to2}^{(I)} = U_{1\to2}^{(II)}$

c) $U_{1\to2}^{(I)} < U_{1\to2}^{(II)}$

d) More information is needed in order to answer this question.

A in System I moves twice as far as does A in System II

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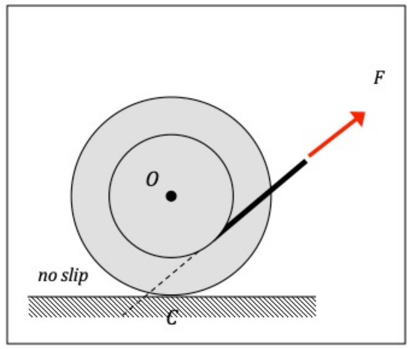
PROBLEM NO. 4 (continued)



PART C – 1 pt.

As a result of the applied force F, the center of the drum O will:

a) Move to the right.
b) Will not move.
c) Move to the left.
d) More information is needed in order to answer this question.

$\sum M_C = CW \Rightarrow$ O moves to the right

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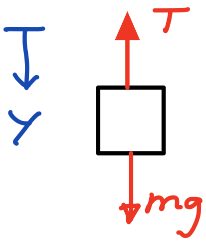
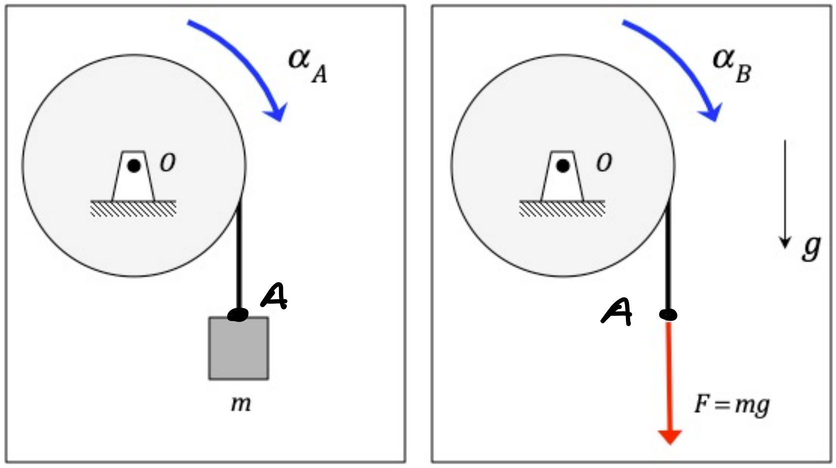
PROBLEM NO. 4 (continued)





System A **System B**

PART D – 1 pt.

Consider Systems A and B above containing identical disks pinned to ground at center O. In System A, a block of mass *m* is attached to the end of the cable, and in System B a force $F = mg$ is attached to the end of the cable. For both systems, the cables do not slip on the disks. Let α_A and α_B represent the resulting clockwise angular acceleration of the disks in Systems A and B, respectively. Choose the correct response below:

a) $\alpha_A > \alpha_B$

b) $\alpha_A = \alpha_B$

c) $\boxed{\alpha_A < \alpha_B}$

d) More information is needed to answer this question.

A: $\sum F_Y = -T + mg = m a_A \; (>0)$

$\hookrightarrow T = m(g - a_A) < F$

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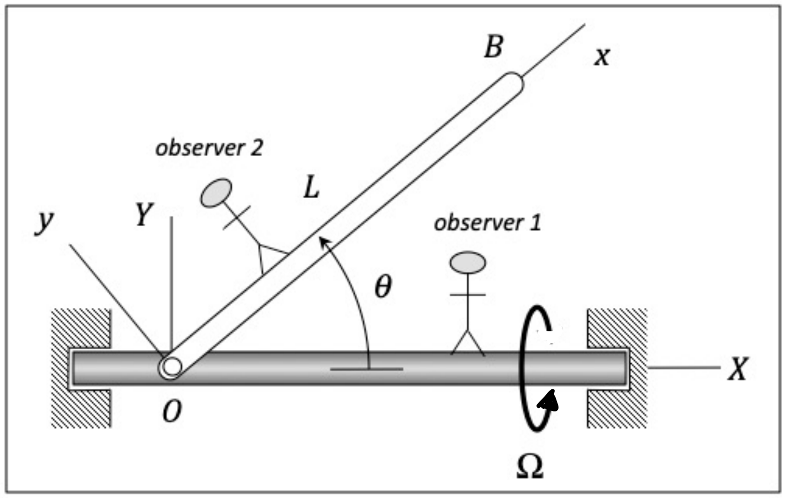
PROBLEM NO. 4 (continued)



PART D

The horizontal shaft above is rotating about a fixed axis with a *constant* rate of Ω. Bar OB is pinned to the horizontal shaft, with the elevation angle θ increasing at a *constant* rate of $\dot{\theta}$. The following moving reference frame kinematics equation is to be used to describe the acceleration of point B for $0<\theta<90°$:

$$\vec{a}_B = \vec{a}_O + \left(\vec{a}_{B/O}\right)_{rel} + \vec{\alpha}\times\vec{r}_{B/O} + 2\vec{\omega}\times\left(\vec{v}_{B/O}\right)_{rel} + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}_{B/O}\right)$$

D.1 – 2 pts. Using an <u>*observer 2 (attached to OB)*</u>, fill in the following terms below for this equation (*in terms of their xyz-coordinates*):

$\vec{\omega} = \Omega\hat{I} + \dot{\theta}\hat{k} = \Omega(\cos\theta\,\hat{i} - \sin\theta\,\hat{j}) + \dot{\theta}\vec{k}$

$\vec{\alpha} = \dot{\theta}\dot{\hat{k}} = \dot{\theta}(\vec{\omega}\times\hat{k}) = \dot{\theta}\,[\Omega\cos\theta\,\hat{i} - \Omega\sin\theta\,\hat{j} + \dot{\theta}\hat{k}]\times\vec{k}$

$= -\dot{\theta}\Omega\sin\theta\,\hat{i} - \dot{\theta}\Omega\cos\theta\,\hat{j}$

D.2 – 2 pts. Using an <u>*observer 1 (attached to the horizontal shaft)*</u>, fill in the following terms below for this equation (*in terms of their xyz-coordinates*):

$\left(\vec{v}_{B/O}\right)_{rel} = L\dot{\theta}\hat{j}$

$\left(\vec{a}_{B/O}\right)_{rel} = -L\dot{\theta}^2\hat{i}$

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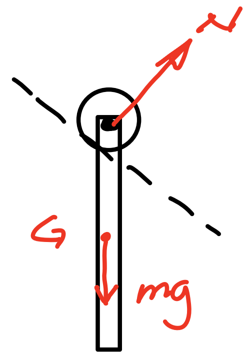
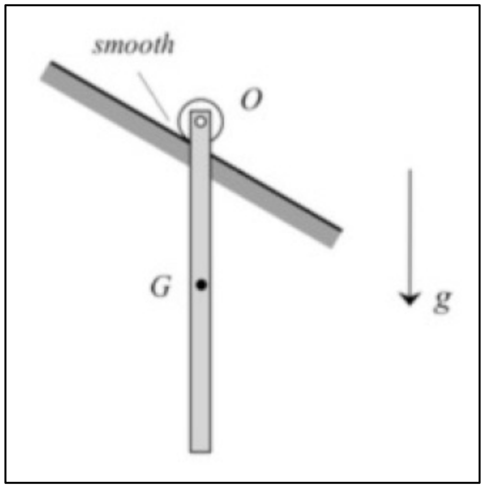
PROBLEM NO. 4 (continued)





PART E – 1 pt.

A thin, homogeneous bar is attached to a roller at end O. The roller is able to roll along a smooth incline, as shown. The bar is released from rest. On release, the *angular acceleration* of the bar is:

$\sum M_G = CW \Rightarrow \alpha = CW$

a) $\boxed{\text{clockwise.}}$

b) counterclockwise.

c) zero.

d) More information is needed to answer this question.

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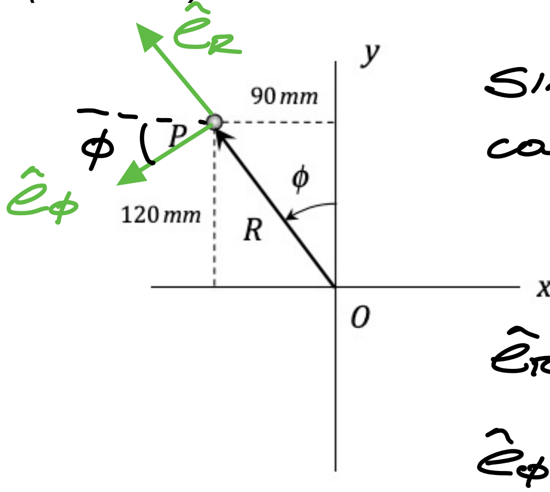
PROBLEM NO. 4 (continued)



$$\sin\phi = \frac{90}{150} = 0.6$$
$$\cos\phi = \frac{120}{150} = 0.8$$

$$\hat{e}_R = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j}$$
$$= -0.6\hat{i} + 0.8\hat{j}$$
$$\hat{e}_\phi = -\cos\phi\,\hat{i} - \sin\phi\,\hat{j}$$
$$= -0.8\hat{i} - 0.6\hat{j}$$

PART F
The velocity and acceleration of particle P are known in terms of their Cartesian components:

$$\vec{v} = (400\,\hat{i} + 300\,\hat{j})\,mm/s$$
$$\vec{a} = (-50\,\hat{i} + 20\,\hat{j})\,mm/s^2$$

For this motion, choose the correct responses:

F.1 – 2 pts.

a) $\dot{R} > 0$

$\boxed{\text{b) } \dot{R} = 0}$

c) $\dot{R} < 0$

$$\dot{R} = \hat{e}_R \cdot \vec{v} = (-0.6\hat{i} + 0.8\hat{j}) \cdot (400\hat{i} + 300\hat{j})$$
$$= (-0.6)(400) + (0.8)(300) = 0$$

F.2 – 2 pts.

a) $\dot{\phi} > 0$

b) $\dot{\phi} = 0$

$\boxed{\text{c) } \dot{\phi} < 0}$

$$R\dot{\phi} = \hat{e}_\phi \cdot \vec{v}$$
$$= (-0.8\hat{i} - 0.6\hat{j}) \cdot (400\hat{i} + 300\hat{j})$$
$$= (-0.8)(400) + (-0.6)(300) = -500$$
$$\hookrightarrow \dot{\phi} < 0$$

F.3 – 2 pts.

$\boxed{\text{a) } \ddot{\phi} > 0}$

b) $\ddot{\phi} = 0$

a) $\ddot{\phi} < 0$

$$R\ddot{\phi} + 2\cancel{\dot{R}}^{\,0}\dot{\phi} = \hat{e}_\phi \cdot \vec{a}$$
$$= (-0.8\hat{i} - 0.6\hat{j}) \cdot (-50\hat{i} + 20\hat{j})$$
$$= (+0.8)(50) + (-0.6)(20)$$
$$= 28$$
$$\hookrightarrow \ddot{\phi} > 0$$