Homework H.6.D

Given: A thin, homogeneous bar OA, having a mass of M and length L, is supported by a smooth pin joint at end O. A bullet B (mass m) strikes end A of the stationary bar with a speed of v_0, with the bullet sticking immediately after impact. Let $\theta(t)$ represent the angular motion of bar OA about pin O during the subsequent motion of the system. Assume that the motion of the bar is small, such that $\theta << 1$, and that the bullet can be modeled as a particle.

Find: For this problem:
(a) Determine the angular speed $\dot{\theta}(0)$ of the bar immediately after the bullet has stuck to the bar;
(b) Determine the dynamical equation of motion (EOM) of the bar/bullet system in terms of θ that will describe its motion immediately after the bullet has stuck to the bar;
(c) The EOM that you found above will be "nonlinear" in the dependent variable θ due to the presence of the $sin\theta$ term. Note that for small angles θ, we can approximate $sin\theta$ by θ; that is, $sin\theta \approx \theta$. Make this replacement in your EOM to produce its "linearized" form valid for small angles of oscillations.
(d) Determine undamped natural frequency ω_n for the system; and,
(e) Determine an expression for $\theta(t)$ describing the motion of the system for $t > 0$.