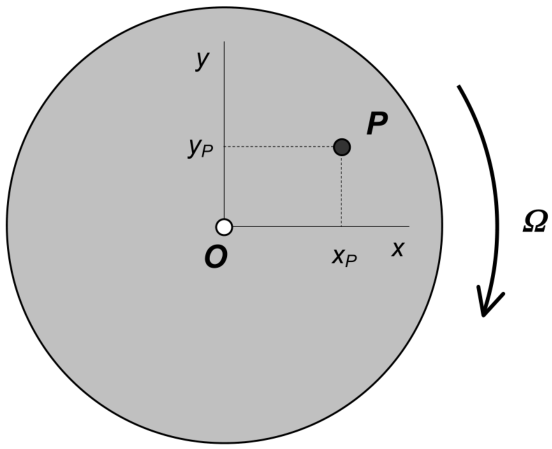
Question C3.1

Particle P moves on the top horizontal surface of a disk that is rotating in a clockwise sense about a vertical axis with a rate of $\Omega = 5$ rad/s. The position of P is described in terms of a set of Cartesian components x_P and y_P measured relative to the disk. When $(x_P, y_P) = (4, 3)$ ft, the velocity of P as seen by a stationary observer is: $\vec{v}_P = (10\hat{\imath} - 20\hat{\jmath})$ ft/s. Describe the velocity of P as seen by an observer on the disk.



TOP VIEW

$$\left(\vec{V}_{P/O}\right)_{rel} = \vec{V}_P - \cancel{\vec{V}_O}^{\,0} - \vec{\omega} \times \vec{r}_{P/O}$$

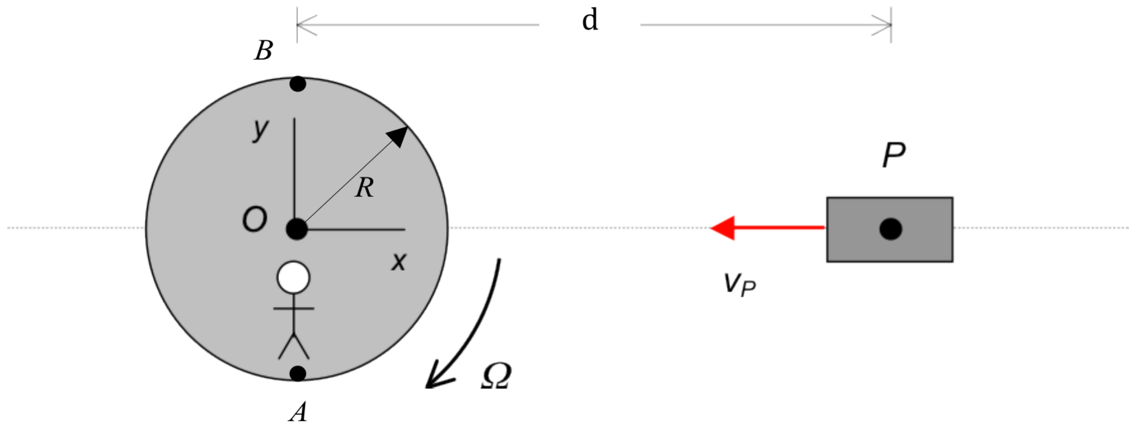
$\vec{r}_{P/O} = x_P\hat{\imath} + y_P\hat{\jmath}$

$\vec{\omega} = -\Omega\hat{k}$

$\vec{V}_P = (10\hat{\imath} - 20\hat{\jmath})$ ft/sec

Question C3.2

A disk is pinned to ground at its center O with the disk rotating clockwise at a constant rate of $\Omega = 3$ rad/s. Block P is traveling to the left along a straight path toward O with a constant speed of $v_P = 20$ ft/s. Determine the acceleration of P as seen by an observer on the disk when P is at a distance of 50 ft from O.



Suppose observer is on disk at point O.

$$(\vec{a}_{P/O})_{rel} = \cancelto{0}{\vec{a}_P} - \cancelto{0}{\vec{a}_O} - \cancelto{0}{\vec{\alpha}} \times \vec{r}_{P/O} - 2\vec{\omega} \times (\vec{v}_{P/O})_{rel} - \vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/O})$$

with $\vec{\omega} = -\Omega \hat{k}$

$$(\vec{v}_{P/O})_{rel} = \underbrace{\vec{v}_P}_{-v_P \hat{\imath}} - \cancelto{0}{\vec{v}_O} - \vec{\omega} \times \vec{r}_{P/O}$$

$$\vec{r}_{P/O} = d\hat{\imath}$$

[Same final answer if observer is at A or B or any point on the rotating disk.]

Question C3.3

Sprinkler arm OA is pinned to a cart at point O. The cart moves to the right with a speed of v_{cart} with $\dot{v}_{cart} = 2$ ft/s^2 = *constant*. Fluid flows through the sprinkler arm at a rate of $\dot{d}$ with $\ddot{d}$ = -3 ft/s^2 = *constant*. The sprinkler arm is being raised at a constant rate of $\dot{\theta} = 4$ rad/s. An observer and xyz coordinate system are attached to the sprinkler arm, as shown in the figure below. The following equation is to be used to find the acceleration of a pellet P that flows with the fluid in the arm:

$$\vec{a}_P = \vec{a}_O + \left(\vec{a}_{P/O}\right)_{rel} + \vec{\alpha} \times \vec{r}_{P/O} + 2\vec{\omega} \times \left(\vec{v}_{P/O}\right)_{rel} + \vec{\omega} \times \left[\vec{\omega} \times \vec{r}_{P/O}\right]$$

Provide numerical values for the following terms when: $d = 3$ ft, $v_{cart} = 3$ ft/s, $\dot{d} = 5$ ft/s and $\theta = 90°$.

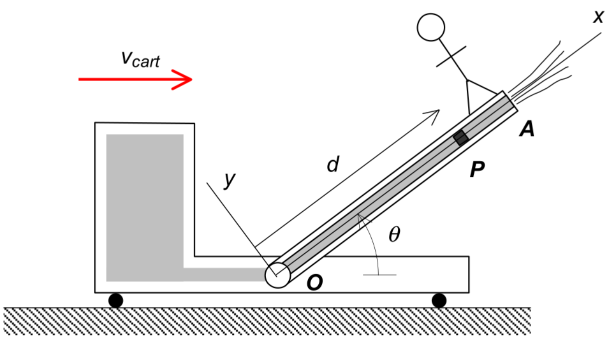
$\vec{a}_O =$ $\dot{V}_{cart}\,\hat{\imath}$

$\vec{\omega} =$ $\dot{\theta}\hat{k}$

$\vec{\alpha} =$ $\vec{0}$

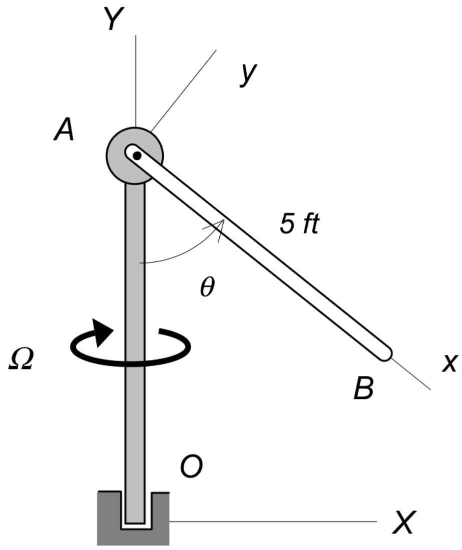
$\left(\vec{v}_{P/O}\right)_{rel} =$ $\dot{d}\,\hat{\imath}$

$\left(\vec{a}_{P/O}\right)_{rel} =$ $\ddot{d}\,\hat{\imath}$



Question C3.4

The vertical shaft OA rotates about a fixed axis with a constant rate of $\Omega = 8$ rad/s. The arm AB is pinned to OA and is being raised at a constant rate of $\dot{\theta} = 10$ rad/s. An observer and xyz axes are attached to AB. The XYZ axes are stationary. What is the angular acceleration vector for arm AB when $\theta = 90°$?



$$\vec{\omega} = -\Omega\hat{j} + \dot{\theta}\hat{k}$$

$$\vec{\alpha} = -\underbrace{\cancel{\dot{\Omega}}}_{0}\hat{j} - \Omega\underbrace{\cancel{\dot{\hat{j}}}}_{0} + \underbrace{\cancel{\ddot{\theta}}}_{0}\hat{k} + \dot{\theta}\dot{\hat{k}} \qquad = \vec{\omega}\times\hat{k}$$

Question C3.5

Arm AB rotates about a fixed vertical axis with a constant rate of ω_1. A ring, with its center at O and of radius r, rotates about arm AB with a constant rate of ω_2. A particle P moves along the ring with $\dot{\theta} = constant$. Let the XYZ axes be fixed, and the xyz axes be attached to the ring. At the position shown, $\theta = 90°$ and the xyz axes are aligned with the XYZ axes. It is desired to use the following equation to determine the acceleration of P for the position shown:

$$\vec{a}_P = \vec{a}_O + \left(\vec{a}_{P/O}\right)_{rel} + \vec{\alpha} \times \vec{r}_{P/O} + 2\vec{\omega} \times \left(\vec{v}_{P/O}\right)_{rel} + \vec{\omega} \times \left[\vec{\omega} \times \vec{r}_{P/O}\right]$$

Provide expressions for the following terms appearing in this equation.

$\vec{a}_O = -d\omega_1^2 \hat{\imath}$

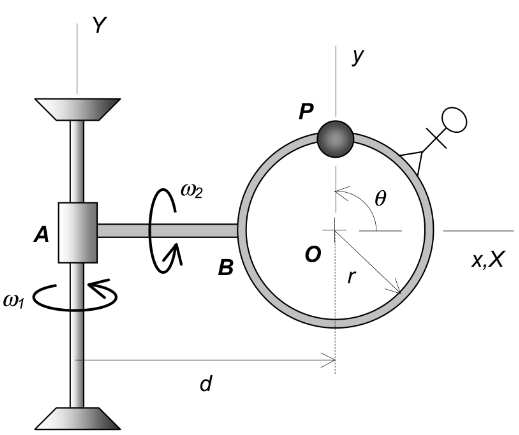
$\vec{\omega} = \omega_1 \hat{J} + \omega_2 \hat{\imath}$

$\vec{\alpha} = \cancel{\dot{\omega}_1}^{0} \hat{J} + \omega_1 \cancel{\dot{\hat{J}}}^{0} + \cancel{\dot{\omega}_2}^{0} \hat{\imath} + \omega_2 \dot{\hat{\imath}} \qquad \dot{\hat{\imath}} = \vec{\omega} \times \hat{\imath}$

$\left(\vec{v}_{P/O}\right)_{rel} = -r\dot{\theta}\hat{\imath}$

$\left(\vec{a}_{P/O}\right)_{rel} = -r\dot{\theta}^2 \hat{j}$

Observer attached to ring



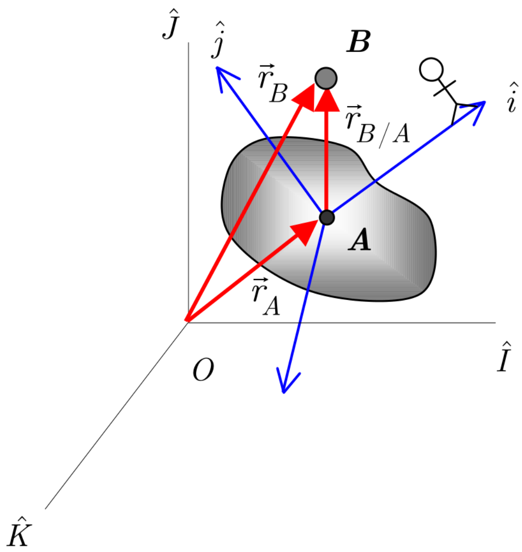
Question C3.6
Consider an observer who is riding along on a moving (translating and rotating) rigid body. We wish to use the observation of this person in describing the motion of some point B, which is not fixed to the body, in the following moving reference frame acceleration equation.

$$\begin{aligned}\vec{a}_B &= \vec{a}_A + \left(\vec{a}_{B/A}\right)_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \left[\vec{\omega} \times \vec{r}_{B/A}\right] \\ &= \vec{a}_A + \vec{a}_{B/A}\end{aligned}$$

Answer the following questions in words:

- What is the meaning of $\vec{\omega}$? ang. vel. of body
- What is the meaning of $\vec{\alpha}$? ang. acc of body
- What is the meaning of $\left(\vec{v}_{B/A}\right)_{rel}$? vel. of B as seen by observer
- What is the meaning of $\left(\vec{a}_{B/A}\right)_{rel}$? acc. of B " " " "
- What restrictions, if any, are on the choice of point A? Must be on same rigid body as observer
- What is the difference in meaning between $\vec{a}_{B/A}$ and $\left(\vec{a}_{B/A}\right)_{rel}$?

$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A \neq \left(\vec{a}_{B/A}\right)_{rel}$ = acc. of B as seen by observer



Chapter 3