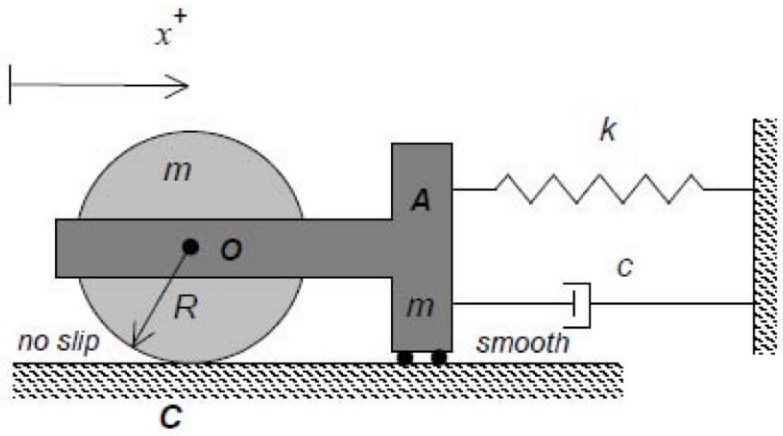
Homework 6.G

Given: A homogeneous wheel of mass m and outer radius R rolls without slipping on a horizontal surface. Block A (also having a mass of m) is pinned to the center O of the wheel and is able to slide without friction on the same horizontal surface. A spring (of stiffness k) and a dashpot (of damping constant c) are connected between block A and ground. Let x represent the motion of block A measured positively to the right. When $x = 0$ m, the spring is unstretched.

Find: For this problem:
(a) Draw individual free body diagrams for block A and the wheel;
(b) Derive the single differential equation of motion for the system in terms of the coordinate x, its time derivatives, and the following parameters: m, R, c, and k;
(c) Determine numerical values for: the undamped natural frequency ω_n, the damping ratio ζ, and the damped natural frequency ω_d; and
(d) Determine the response of the system $x(t)$ for $t > 0$, assuming the system is released when the springs are unstretched with $\dot{x}(0) = v_0$.



Use the following parameters in your analysis: $m = 4$ kg, $k = 2250$ N/m, $R = 0.1$ m, $c = 60$ kg/s, and $v_0 = 8$ m/s.