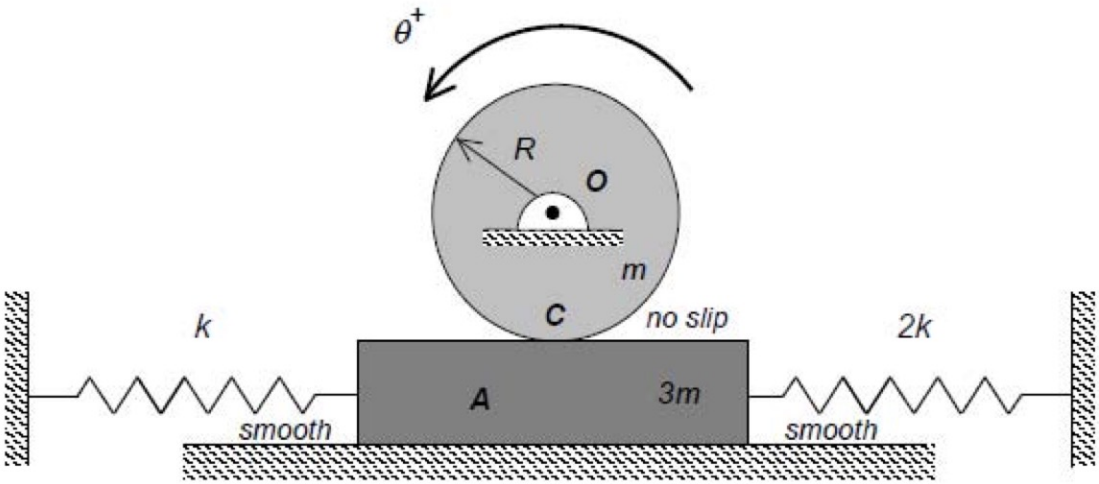
Homework 6.C

Given: A homogeneous drum having a mass of m and outer radius R is pinned to ground at its center O. This drum is in geared contact with block A. Block A, having a mass of $3m$, is able to slide along a smooth horizontal surface and in such a way that the block does not slip in its contact with drum. Two springs, having stiffnesses of k and $2k$, are attached between block A and ground, as shown in the figure below. Let θ represent the rotation of the drum with θ being measured positive counterclockwise. When $\theta = 0$ rad the springs are unstretched.

Find: For this problem:
(a) Draw individual free body diagrams of the drum and block;
(b) Derive the single differential equation of motion (EOM) for the system in terms of the coordinate θ, its time derivatives, and, at most, the following parameters: m, R, and k;
(c) Based on the EOM derived above, determine the natural frequency of the system. Express the answer in both rad/s and Hz; and
(d) Assuming the system is released when the springs are unstretched with $\dot{\theta}(0) = \omega_0$ (CCW), determine the response of the system $\theta(t)$, for $t > 0$.



Use the following parameters in your analysis: $m = 0.75$ kg, $k = 5500$ N/m, $R = 0.25$ m, and $\omega_0 = 0.5$ rad/s.