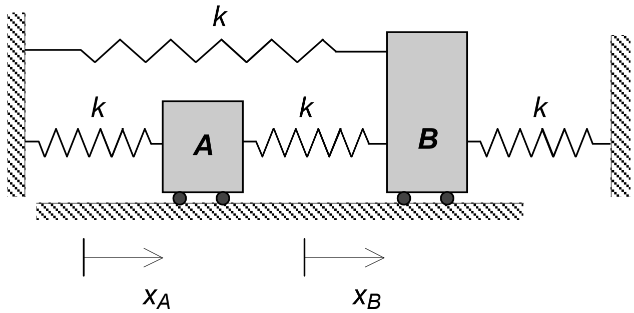
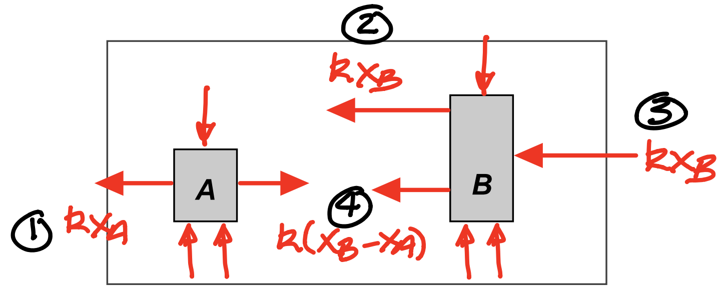
Question C6.1

The positions of particles A and B are described by the coordinates x_A and x_B shown below. All springs are unstretched when $x_A = x_B = 0$. In the free body diagrams shown below, draw and label (in terms of k, x_A and x_B) the spring force vectors on particles A and B.





FBD's of particles A and B

① For $x_A > 0$, spring stretched

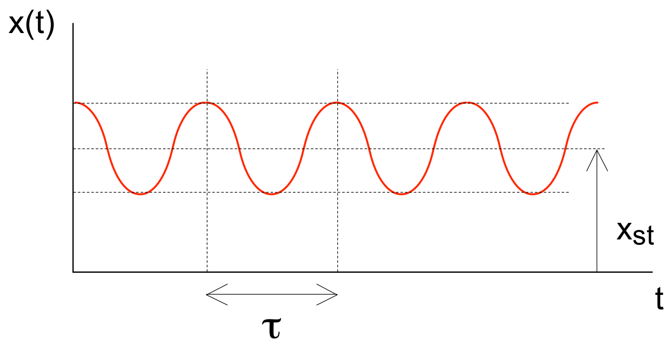
② For $x_B > 0$, spring stretched

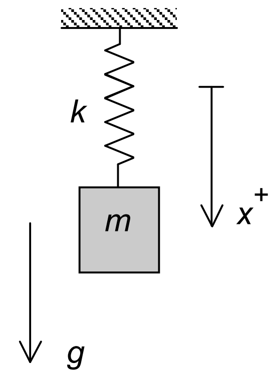
③ For $x_B > 0$, spring compressed

④ For $x_B - x_A > 0$, spring stretched

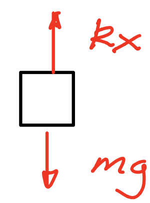
Question C6.2

The time history for $x(t)$ for the free response of the undamped single-DOF system is shown below. Here, $x_{st} = 0.05$ m. Determine the natural period of free response τ for this system.





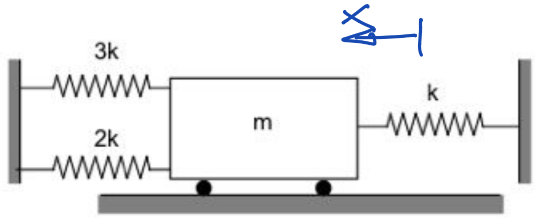
$$\tau = \frac{2\pi}{\omega_n}$$

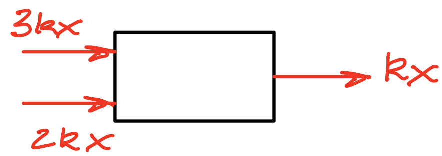


$$\Sigma F_x = mg - kx = m\ddot{x}$$

Question C6.3

Consider the spring-mass system shown below. Determine the natural frequency for the system.

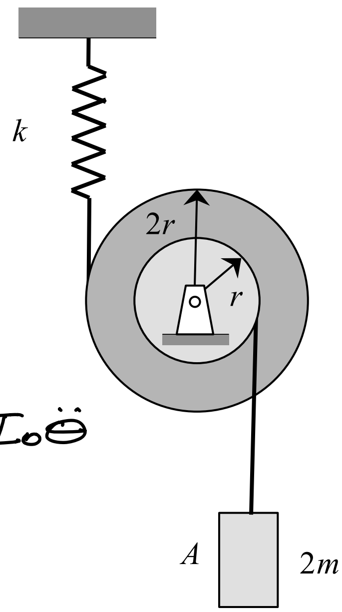


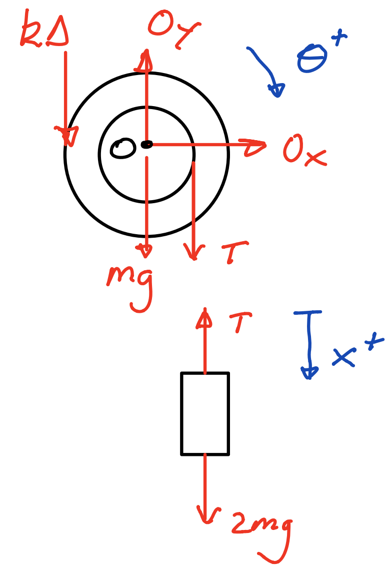


$$\sum F_x = -3kx - 2kx - kx = m\ddot{x}$$

Question C6.4

The system shown below consists of a pulley (of mass m and centroidal mass moment of inertia I_O) and block A. Determine the natural frequency for the system.





Pulley

$$\sum M_O = -(k\Delta)(2r) + Tr = I_O\ddot{\theta}$$

Block

$$\sum F_x = -T + 2mg = 2m\ddot{x}$$

Kinematics

$$\Delta = (2r)\theta$$

$$\ddot{x} = r\ddot{\theta}$$

Question C6.5

Consider the free response for a damped, single-DOF system having the following differential equation of motion:

$$m\ddot{x} + c\dot{x} + kx = 0$$

This system is known to have a damping ratio of $\zeta = 0.1$ and undamped natural frequency of $\omega_n = 10$ rad/s.

What are the new values for the damping ratio and undamped natural frequency if:

(a) The original value of m is doubled, the original value of k is doubled and the value of c is unchanged?

(b) The original value of m is doubled, the original value of k is halved and the value of c is unchanged?

(c) The original value of k is doubled, the original value of c is doubled and the value of m is unchanged?

(d) The original value of m is doubled, the original value of c is doubled and the value of k is unchanged?

$$m\ddot{x} + c\dot{x} + kx = 0$$

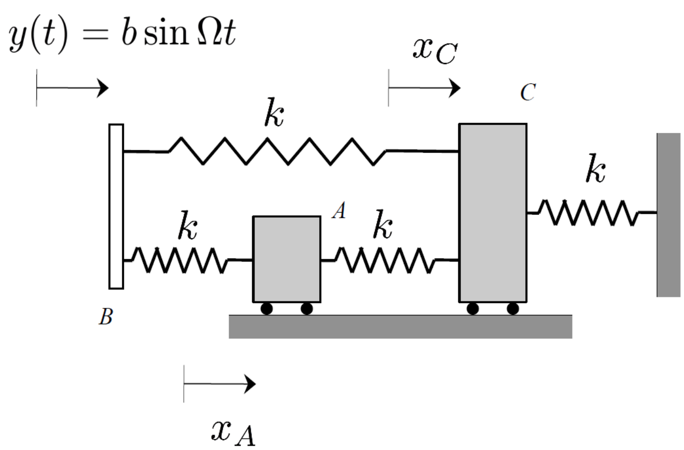
$$\div m: \quad \ddot{x} + \underbrace{\boxed{\frac{c}{m}}}_{2\zeta\omega_n}\dot{x} + \underbrace{\boxed{\frac{k}{m}}}_{\omega_n^2}x = 0$$

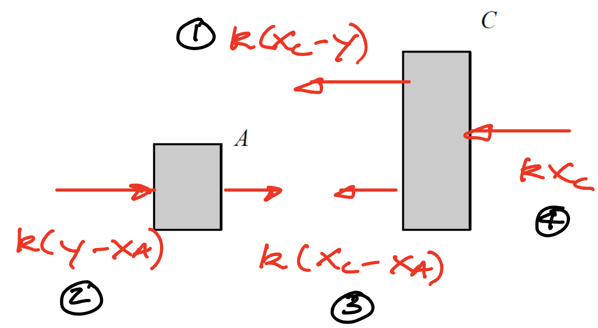
$$\omega_n = \sqrt{\frac{k}{m}}$$

$$2\zeta\omega_n = \frac{c}{m} \Rightarrow \zeta = \frac{c}{2m\omega_n} = \frac{c}{2m\sqrt{k/m}} = \frac{c}{2\sqrt{km}}$$

Question C6.6

The positions of particles A and C are described by the coordinates x_A and x_C shown below. The base B on the left side is given prescribed motion of $y(t) = b \sin \Omega t$. All springs are unstretched when $x_A = x_C = y = 0$. In the free body diagrams shown below, draw and label [in terms of k, $y(t)$, x_A and x_C] the spring force vectors on particles A and C.





① $x_c > y \Rightarrow$ Spring stretched

② $y > x_A \Rightarrow$ Spring compressed

③ $x_c > x_A \Rightarrow$ Spring stretched

④ $x_c > 0 \Rightarrow$ Spring compressed

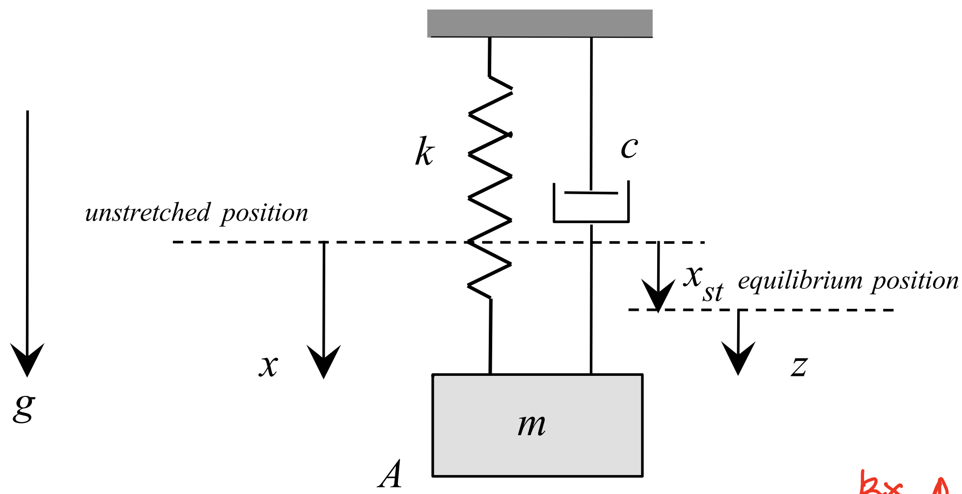
Question C6.7

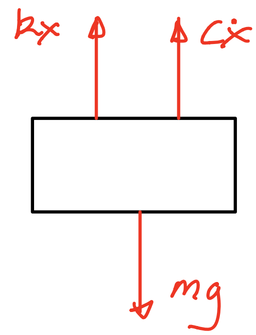
Consider the spring-mass-dashpot system shown below. Let x represent the position of block A as measured from its position when the spring is unstretched. Let z represent the position of block A as measured from its position when the system is in static equilibrium; that is, $x = x_{st} + z$.

(a) Derive the equation of motion (EOM) of the system in terms of the coordinate x.

(b) Derive the EOM of the system in terms of the coordinate z.

(c) Compare the EOMs from (a) and (b). How do they differ?





(a) $\sum F_x = -kx - c\dot{x} + mg = m\ddot{x}$

(b) For x_{st}: $\dot{x} = \ddot{x} = 0 \Rightarrow -kx_{st} + mg = 0$

(c) $x = z + x_{st}$ ← substitute into EOM in (a)

Question C6.8

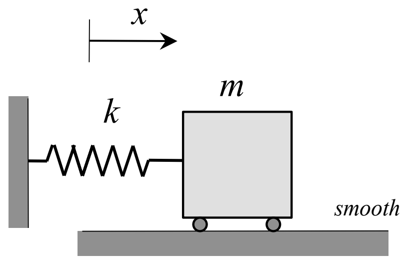
Consider Systems A and B shown below. System A is made up of a spring and block with the block moving in pure translation along a smooth horizontal surface. System B is made up of a spring and a homogeneous disk of mass m and outer radius R, with the center of the disk at O and the disk rolling without slipping on a horizontal surface. Each system has the same mass m and same spring stiffness k. Let ω_{nA} and ω_{nB} represent the natural frequencies of Systems A and B, respectively. Circle the answer below that most accurately represents the natural frequencies for the two systems:

(a) $\omega_{nA} > \omega_{nB}$

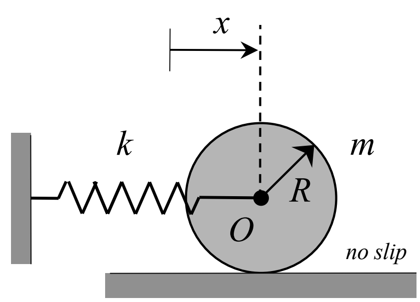
(b) $\omega_{nA} = \omega_{nB}$

(c) $\omega_{nA} < \omega_{nB}$

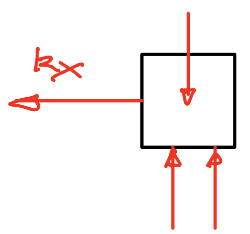
(d) More information is needed on the two systems in order to answer this question.

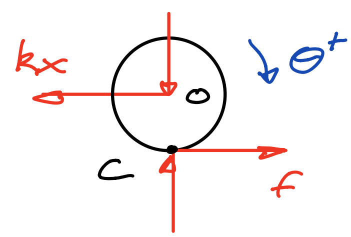


System A



System B





A: $\sum F_x = -kx = m\ddot{x}$

B: $\sum M_C = -(kx)R = I_C\ddot{\theta}$

w/ $I_C = I_O + mR^2$

$= \frac{1}{2}mR^2 + mR^2$

$\ddot{\theta} = \ddot{x}/R$

Question C6.9

Consider the standard form of the equation of motion (EOM) for the free response of a single-degree-of-freedom system:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$$

where $\zeta \geq 0$ and $\omega_n > 0$. Describe, in words, the nature of the response of the system if:

(a) $\zeta = 0$

(b) $0 < \zeta < 1$

(c) $\zeta = 1$

(d) $\zeta > 1$

See discussion in lecturebook

Question C6.10

The following equation of motion (EOM) has been derived for a single-degree-of-freedom system:

$$6\ddot{x} + 2\dot{x} - 216x = 0$$

Explain why the response governed by this EOM is not oscillatory.

$\div 6: \quad \ddot{x} + \frac{2}{6}\dot{x} \boxed{- \frac{216}{6}} x = 0$

$\hookrightarrow \omega_n = \sqrt{-\frac{216}{6}} = 6\hat{\imath} = \text{imaginary}$

Chapter 6

Question C6.11

The following equation of motion (EOM) has been derived for a single-degree-of-freedom system:

$$2\ddot{x} + 48\dot{x} + 800x = 200$$

(a) Determine the undamped natural frequency ω_n for the system.

(b) Determine the damping ratio ζ for the system.

(c) Determine the damped natural frequency ω_d for the system.

(d) Determine the static deformation x_{st} for the system.

$$\div 2: \quad \ddot{x} + \underbrace{\frac{48}{2}}_{2\zeta\omega_n}\dot{x} + \underbrace{\frac{800}{2}}_{\omega_n^2}x = \frac{200}{2}$$

$$x_{st} \Rightarrow \dot{x} = \ddot{x} = 0 \Rightarrow \omega_n^2 x_{st} = 100$$

Question C6.12

The following equation of motion (EOM) has been derived for an undamped single-degree-of-freedom system:

$$2\ddot{x} + 800x = f(t) = 40 \sin \Omega t$$

Let $x_P(t)$ represent the particular solution of this EOM. Is $x_P(t)$ in phase or 180° out of phase with the excitation $f(t)$ when $\Omega = 30$ rad/s? Provide a justification for your answer.

$\div 2: \quad \ddot{x} + \underbrace{\boxed{\frac{800}{2}}}_{\omega_n^2} x = \frac{40}{2} \sin \Omega t \quad ; \quad \omega_n = \sqrt{400} = 20 \ \frac{\text{rad}}{\text{s}}$

$x_p(t) = A \sin \Omega t + B \cos \Omega t$

$\ddot{x}_p(t) = -A\Omega^2 \sin \Omega t - B\Omega^2 \cos \Omega t$

$\therefore \ (-\Omega^2 + \omega_n^2) A \sin \Omega t + (-\Omega^2 + \omega_n^2) B \cos \Omega t = 20 \sin \Omega t$

$\cos \Omega t: \ (-\Omega^2 + \omega_n^2) B = 0 \quad \Rightarrow \quad B = 0$

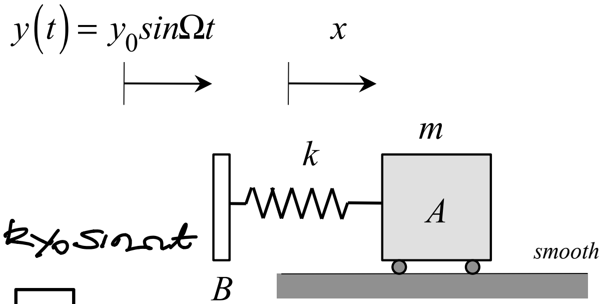
$\sin \Omega t: \ (-\Omega^2 + \omega_n^2) A = 20 \quad \Rightarrow \quad A = \dfrac{20}{-\Omega^2 + \omega_n^2}$

What is the sign of A?

Chapter 6

Question C6.13

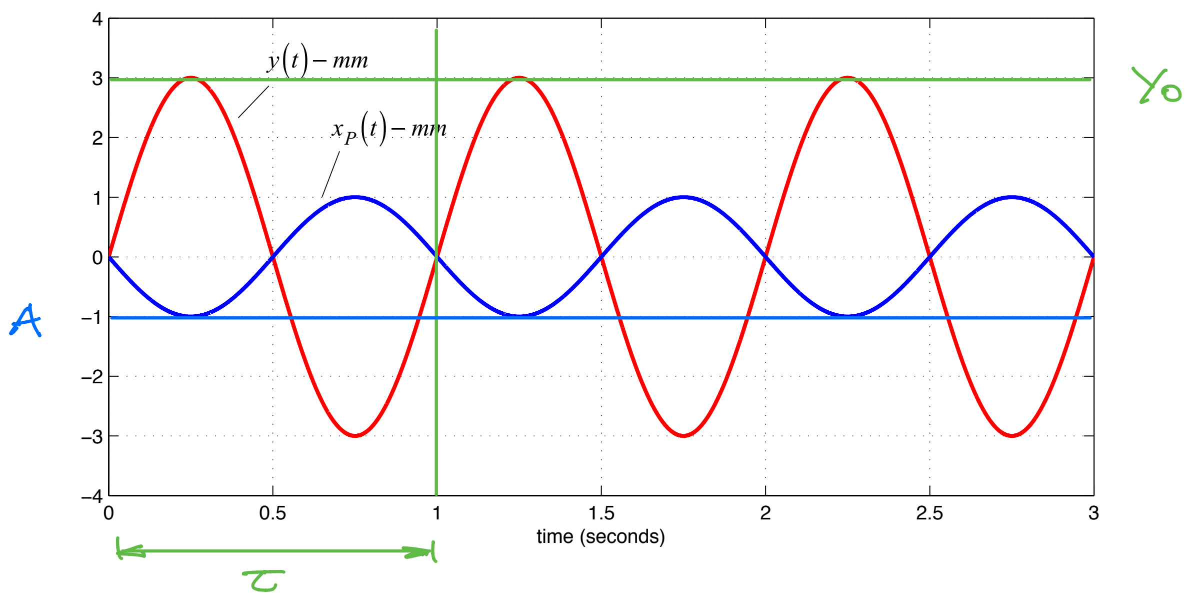
The undamped, single-degree-of-freedom system shown below is made up of block A (of mass m) and a spring of stiffness k. The spring is connected between A and base B, with B given a prescribed displacement of $y(t) = y_0 \sin \Omega t$.



EOM: $m\ddot{x} + kx = ky_0 \sin\Omega t$

$\div m$: $\ddot{x} + \underbrace{\boxed{\frac{k}{m}}}_{\omega_n^2} x = \underbrace{\boxed{\frac{k}{m}}}_{\omega_n^2} y_0 \sin\Omega t$

Let $x_P(t)$ represent the particular solution of the EOM for this system. Time histories for $x_P(t)$ and $y(t)$ are shown below.



From the plot, provide estimates for:

(a) The excitation amplitude y_0.

(b) The excitation frequency Ω.

(c) The natural frequency ω_n of the system.

(b) $\tau = \frac{2\pi}{\Omega} \Rightarrow \Omega = \frac{2\pi}{\tau}$

(c) $x_p(t) = A \sin\Omega t$; $A = \frac{\omega_n^2 y_0}{-\Omega^2 + \omega_n^2}$

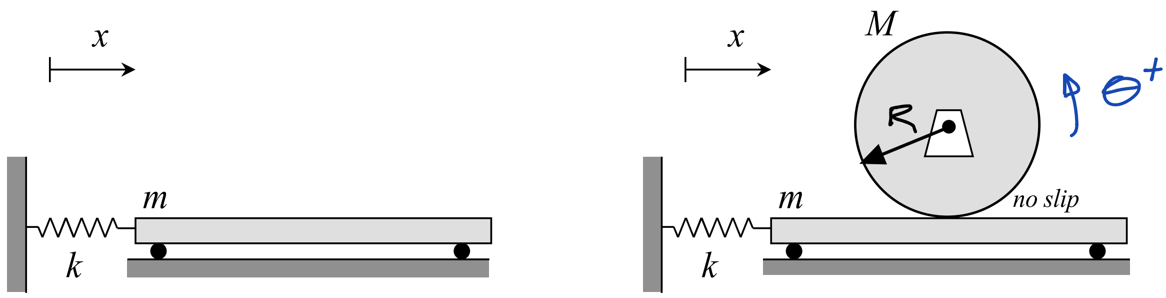
Question C6.14

Consider Systems A and B shown below. Let $(\omega_n)_A$ and $(\omega_n)_B$ represent the natural frequencies of Systems A and B, respectively. Circle the statement below that most accurately describes the natural frequencies of these two systems:

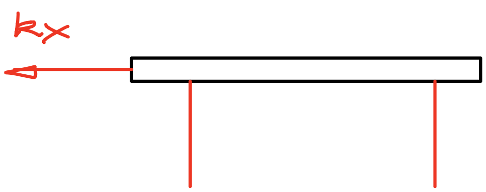
(a) $(\omega_n)_A > (\omega_n)_B$

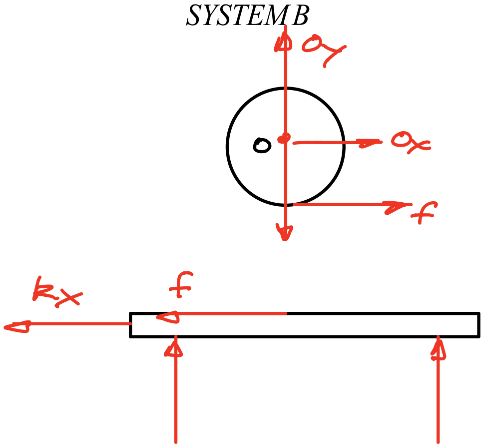
(b) $(\omega_n)_A = (\omega_n)_B$

(c) $(\omega_n)_A < (\omega_n)_B$



SYSTEM A *SYSTEM B*





$$\sum F_x = -kx = m\ddot{x}$$

↳ $m\ddot{x} + kx = 0$

Block:

$$\sum F_x = -kx - f = m\ddot{x} \quad (1)$$

Disk

$$\sum M_o = fR = I_o\ddot{\theta} \quad (2)$$

Kinematics

$$\ddot{\theta} = \ddot{x}/R \quad (3)$$

(1)–(3) ⇒

$$-kx - \frac{I_o}{R}\frac{\ddot{x}}{R} = m\ddot{x}$$

↳ $\left(m + \frac{I_o}{R^2}\right)\ddot{x} + kx = 0$

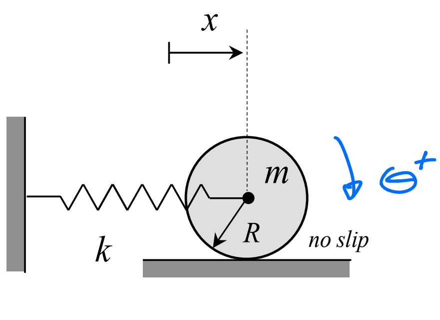
Question C6.15

Consider Systems A and B shown below. Let $(\omega_n)_A$ and $(\omega_n)_B$ represent the natural frequencies of Systems A and B, respectively. Circle the statement below that most accurately describes the natural frequencies of these two systems:

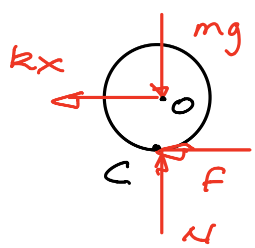
(a) $(\omega_n)_A > (\omega_n)_B$

(b) $(\omega_n)_A = (\omega_n)_B$

(c) $(\omega_n)_A < (\omega_n)_B$



System A



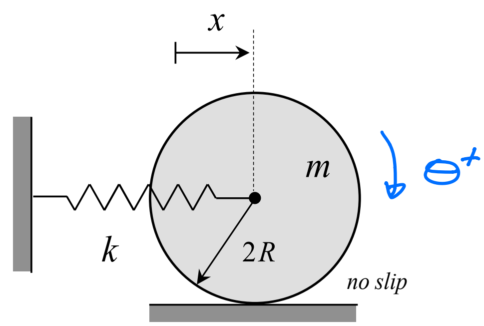
$$\sum M_c = -(kx)R = I_c\ddot{\theta} \quad (1)$$

$$\text{w/ } I_c = I_o + mR^2 = \tfrac{3}{2}mR^2 \quad (2)$$

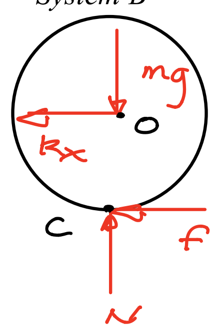
$$\ddot{\theta} = \ddot{x}/R \quad (3)$$

$(1)-(3) \Rightarrow$

$$\left(\tfrac{3}{2}mR^2\right)\frac{\ddot{x}}{R} + kRx = 0$$



System B



$$\sum M_c = -(kx)(2R) = I_c\ddot{\theta} \quad (1)$$

$$\text{w/ } I_c = I_o + m(2R)^2 = \tfrac{3}{2}m(2R)^2$$

$$= 6mR^2 \quad (2)$$

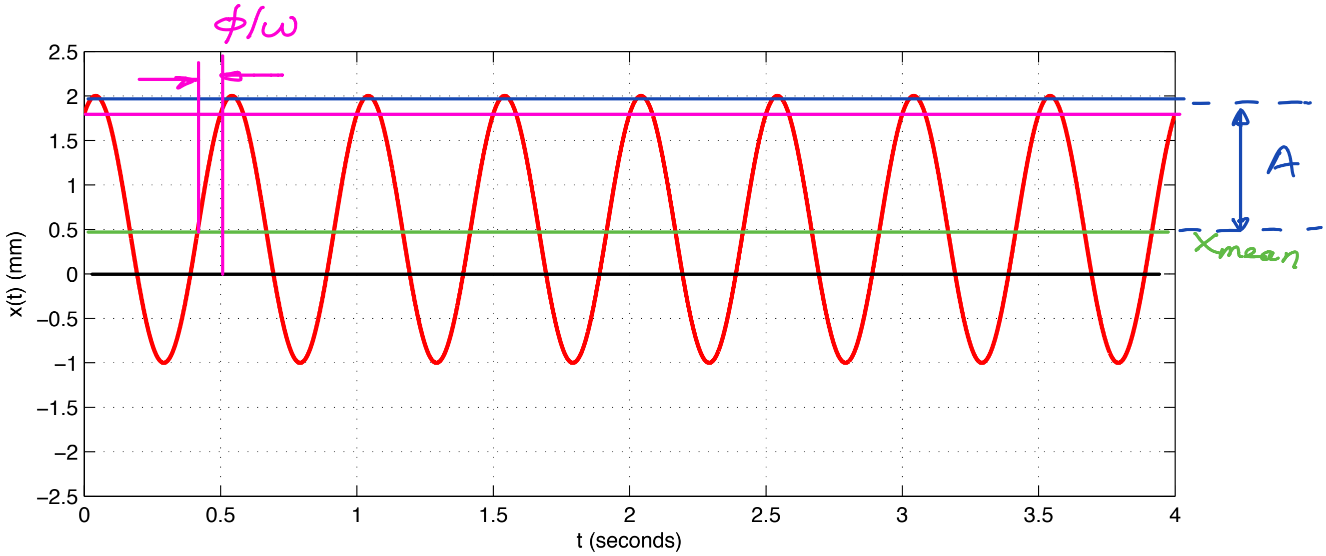
$$\ddot{\theta} = \ddot{x}/2R \quad (3)$$

$(1)-(3) \Rightarrow$

$$(6mR^2)\frac{\ddot{x}}{2R} + 2kRx = 0$$

Question C6.16

Consider the time history of the function $x(t) = x_{mean} + A\sin(\omega t + \phi)$. From this figure, provide estimates for x_{mean}, A and ϕ.



Chapter 6