Question C1.1

Particle P moves in the xy plane with an acceleration of $\vec{a}_P = a_P \hat{i}$, where $a_P = 10\ \mathrm{m/s^2} = constant$. At the instant shown, P has a speed of $v_P = 5$ m/s, with the velocity oriented as shown below. Circle the angle θ that gives the SMALLEST radius of curvature ρ for the path of P:

(a) $\theta = 0$

(b) $\theta = 30°$

(c) $\theta = 60°$

(d) $\theta = 90°$

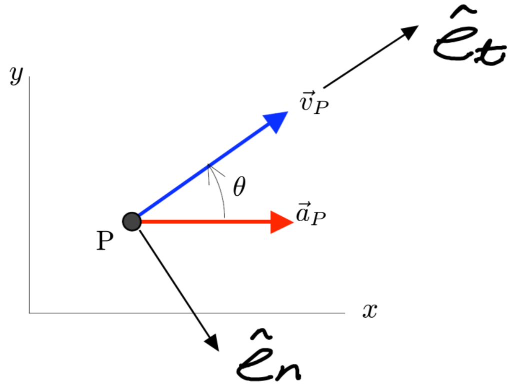
(e) $\theta = 120°$

(f) $\theta = 150°$

(g) $\theta = 180°$

(h) The radius of curvature is independent of the angle θ.

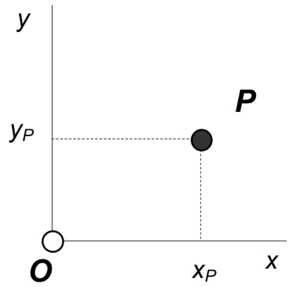
(i) More information is needed on the motion of P in order to answer this question.



Project $\vec{a}_p$ onto the $\hat{e}_n$ unit vector direction:

$$a_n = |\vec{a}_p| \sin\theta = \frac{v_p^2}{\rho} \Rightarrow \rho = \frac{v_p^2}{|\vec{a}_p| \sin\theta}$$

Question C1.2
Point P moves on a path described by $y_P = x_P^2/2$ with $x_P(t) = 3\sin\pi t$, where x_P and y_P have units of meters and t has units of seconds. Determine the acceleration vector of P at $t = 0$.



$$\dot{y}_P = \frac{dy_P}{dt} = \frac{dy_P}{dx}\frac{dx}{dt} = \dot{x}\frac{dy_P}{dx}$$

$$\ddot{y}_P = \frac{d\dot{y}_P}{dt} = \ddot{x}\frac{dy_P}{dx} + \dot{x}\underbrace{\frac{d}{dt}\left(\frac{dy_P}{dx}\right)}_{\frac{d^2y_P}{dx^2}\frac{dx}{dt}}$$

$$= \ddot{x}\frac{dy_P}{dx} + \dot{x}^2\frac{d^2y_P}{dx^2}$$

Question C1.3

Consider the path description for the motion of a point P. Circle the item below that most accurately describes the acceleration of P, $\vec{a}_P$:

(a) The rate of change of speed for P, $\dot{v}_P$, is ALWAYS the same as the magnitude of its acceleration, $|\vec{a}_P|$.

(b) The rate of change of speed for P, $\dot{v}_P$, is the same as the magnitude of its acceleration, $|\vec{a}_P|$ only if the path of P is STRAIGHT.

(c) The acceleration of point P is always PERPENDICULAR to the path of P.

(d) None of the above.

$$\vec{a}_P = \dot{v}_P \hat{e}_t + \frac{v_P^2}{\rho} \hat{e}_n$$

(a), (b) } consider $|\vec{a}_P|^2 = \dot{v}_P^2 + \left(\frac{v_P^2}{\rho_P}\right)^2$

(c) $\vec{a}_P$ has two components in general

Question C1.4

Point P represents a passenger traveling in a automobile. The velocity and acceleration of P, $\vec{v}_P$ and $\vec{a}_P$, respectively, are shown below at a given instant in time. Circle the item below that most accurately describes the motion of P:

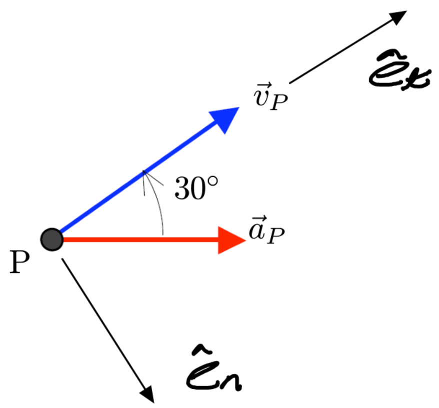
(a) The speed of P is decreasing and P is turning left.

(b) The speed of P is increasing and P is turning left.

(c) The speed of P is decreasing and P is turning right.

(d) The speed of P is increasing and P is turning right.

(e) There is insufficient information given for answering this question.



$$\vec{a}_P = \dot{v}_P \hat{e}_t + \frac{v_P^2}{\rho} \hat{e}_n = |\vec{a}_P| \cos 30^\circ \hat{e}_t + |\vec{a}_P| \sin 30^\circ \hat{e}_n$$

Question C1.5

Point P represents a passenger traveling in a automobile. The velocity and acceleration of P, $\vec{v}_P$ and $\vec{a}_P$, respectively, are shown below at a given instant in time. Circle the item below that most accurately describes the motion of P:

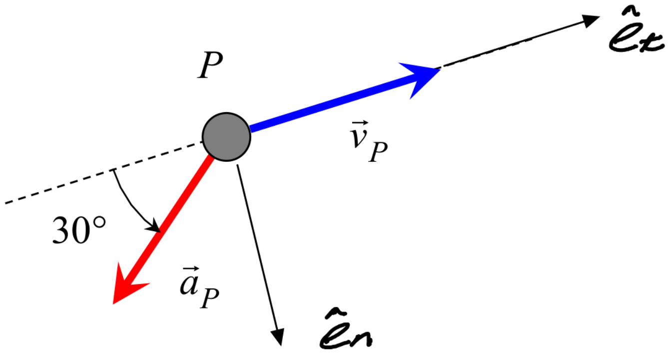
(a) The speed of P is decreasing and P is turning left.

(b) The speed of P is increasing and P is turning left.

(c) The speed of P is decreasing and P is turning right.

(d) The speed of P is increasing and P is turning right.

(e) There is insufficient information given for answering this question.



$$\vec{a}_P = \dot{v}_P \hat{e}_t + \frac{v_P^2}{\rho}\hat{e}_n = -|\vec{a}_P|\cos 30^\circ \hat{e}_t + |\vec{a}_P|\sin 30^\circ \hat{e}_n$$

Question C1.6

Particle P travels on a path described by the polar coordinates $r = 2\cos\theta$, where r is given in feet and θ is given in radians. When $\theta = \pi/3$ radians, it is known that $\dot{\theta} = -3$ rad/s and $\ddot{\theta} = 0$.

At this instant (circle the correct answer):

(a) $\dot{r} < 0$

(b) $\dot{r} = 0$

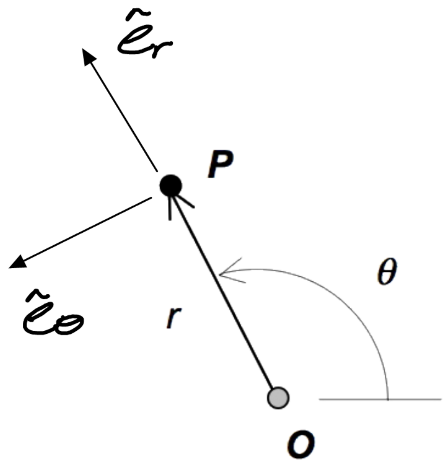
(c) $\dot{r} > 0$

At this instant (circle the correct answer):

(a) $\ddot{r} < 0$

(b) $\ddot{r} = 0$

(c) $\ddot{r} > 0$



$$\dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = (-2\sin\theta)\dot{\theta}$$

$$\ddot{r} = \frac{d}{dt}\left[-2\dot{\theta}\sin\theta\right] = -2\left[\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta\right]$$

Question C1.7

A polar description with variables r and θ is used to describe the kinematics of point P. For a position with $r = 0.5$ m and $\theta = 2$ radians, the velocity and acceleration vectors for P are known to be:

$$\vec{v}_P = (-6\hat{e}_r + 2\hat{e}_\theta) \text{ m/s}$$

$$\vec{a}_P = (10\hat{e}_r) \text{ m/s}^2$$

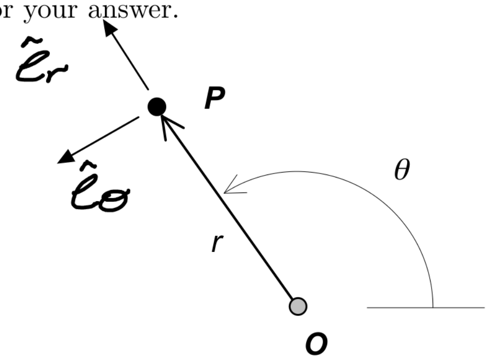
respectively. Circle the item below that most accurately describes the speed of P:

(a) The speed of P is increasing.

(b) The speed of P is not changing.

(c) The speed of P is decreasing.

Provide a justification for your answer.



Project $\vec{a}_P$ onto $\hat{e}_t$

Question C1.8

A particle P travels on a path given in terms of polar coordinates as: $r = \cos(3\theta)$, where r is given in feet, θ is given in radians and $\dot{\theta} = 2$ rad/s = *constant*. Determine the magnitude of the acceleration vector of P when $\theta = \pi/3$.

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\tilde{e}_r + (r\cancelto{0}{\ddot{\theta}} + 2\dot{r}\dot{\theta})\tilde{e}_\theta$$

$$\text{w/} \begin{cases} r = \cancelto{-1}{\cos 3\theta} \\ \dot{r} = \dfrac{dr}{dt} = \dfrac{dr}{d\theta}\dfrac{d\theta}{dt} = \dot{\theta}(-3\sin 3\theta) = -3\dot{\theta}\cancelto{0}{\sin 3\theta} \\ \ddot{r} = \dfrac{d\dot{r}}{dt} = -3\left[\cancelto{0}{\ddot{\theta}}\sin 3\theta + 3\dot{\theta}^2\cancelto{-1}{\cos 3\theta}\right] \end{cases}$$

Question C1.9

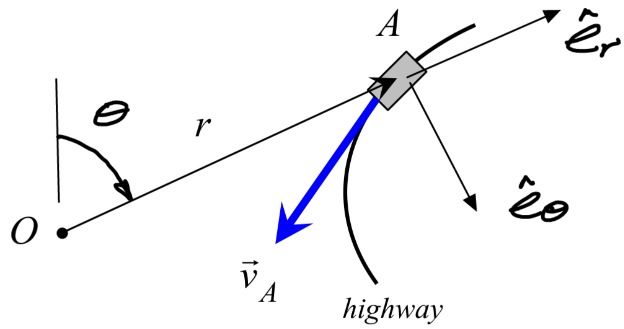
An automobile A travels along a highway with a speed of v_A. A police officer, at point O and a distance of r from A, accurately measures $\dot{r}$ (the time derivative of the distance r) with a hand-held radar device. Circle the item below that most accurately describes the size of $|\dot{r}|$ as compared to the speed v_A:

(a) $|\dot{r}| > v_A$ (the officer overestimates the speed of the automobile)

(b) $|\dot{r}| = v_A$ (the officer accurately measures the speed of the automobile)

(c) $|\dot{r}| < v_A$ (the officer underestimates the speed of the automobile)

Provide a written justification for your answer.



$$\vec{V}_A = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\hookrightarrow \quad V_A = \sqrt{\dot{r}^2 + (r\dot{\theta})^2}$$

Chapter 1

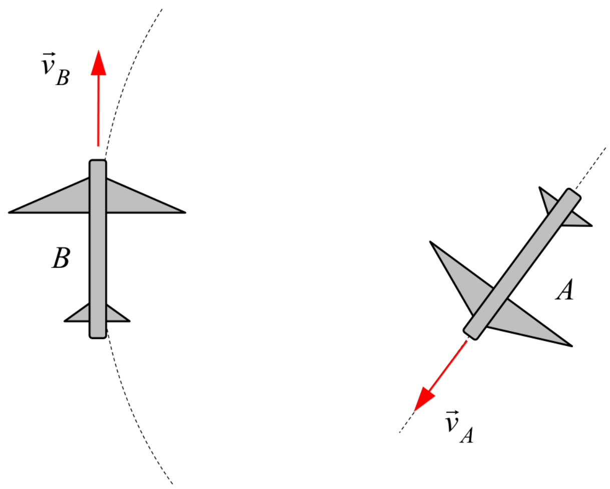
Question C1.10
Aircraft A is traveling along a straight path with a speed of v_A. Aircraft B is traveling along a circular path of radius R with a speed of v_B. Circle the answer below that most accurately describes the observed velocities of A and B:

(a) $\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$ is the velocity of A as seen by the pilot of B

(b) $\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$ is the velocity of B as seen by the pilot of A

(c) Both (a) and (b).

(d) Neither (a) nor (b).



$\vec{V}_{A/B} = \vec{V}_A - \vec{V}_B \neq$ velocity of A as seen by B
since B is rotating

Question C1.11

Blocks A and B are connected by an inextensible cable, with this cable being wrapped around pulleys at A and E. Assume that the radii of these two pulleys are small compared with the other dimensions of the problem. Block A moves downward with a speed of v_A. Let v_B be the speed of block B when $s_A > 0$. Circle the answer below that most accurately describes the speed of B as compared to the speed of A:

(a) $0 < v_B < 2v_A$

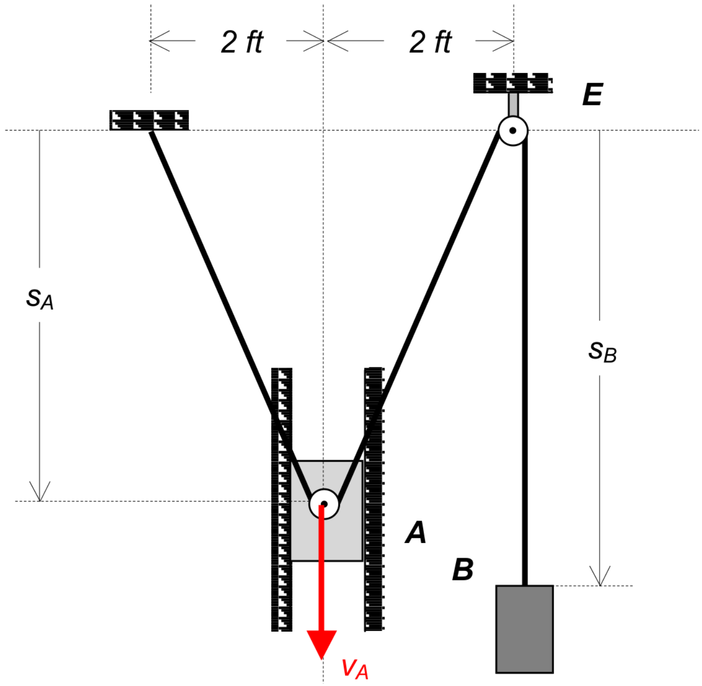
(b) $v_B = v_A$

(c) $v_B = 2v_A$

(d) $v_B > 2v_A$

(e) More information is needed about the problem in order to answer this question.

Provide a mathematical justification for your answer.



L = length of cable

$$= \left(\sqrt{s_A^2 + 2^2}\right) 2 + s_B + \text{constant}$$

$$\frac{dL}{dt} = 0 = 2\,\frac{s_A \dot{s}_A}{\sqrt{s_A^2 + 4}} + \dot{s}_B = 0$$

$$v_B = \left(\frac{s_A}{\sqrt{s_A^2 + 4}}\right)(2v_A)$$

Question C1.12

Blocks A and B are connected by an inextensible cable, as shown in the figure below. Assume that the radius of the pulley is small compared to the other dimensions of the problem. Block A moves along a horizontal path, and block B moves along a vertical path. At the instant shown, B is moving downward with a speed of v_B. Circle the answer below that most accurately describes the speed of A, v_A, as compared to the speed of B:

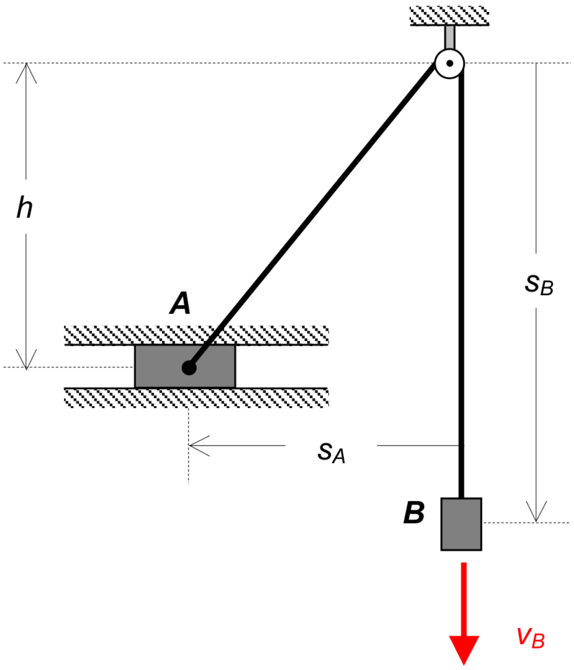
(a) $v_A > v_B$

(b) $v_A = v_B$

(c) $v_A < v_B$

(d) More information is needed about the problem in order to answer this question.

Provide an mathematical justification for your answer.



L = length of cable

$= \sqrt{s_A^2 + h^2} + s_B + \text{constant}$

$$\frac{dL}{dt} = 0 = \frac{s_A \dot{s}_A}{\sqrt{s_A^2 + h^2}} + \dot{s}_B$$

$$v_B = \frac{s_A}{\sqrt{s_A^2 + h^2}} v_A$$

Question C1.13

The instantaneous three-dimensional motion of particle P is described by its velocity and acceleration vectors written in terms of cylindrical components:

$$\vec{v}_P = \left(-9\hat{e}_R + 12\hat{e}_\theta - 36\hat{k}\right)\ \text{m/s}$$

$$\vec{a}_P = \left(-3\hat{e}_R + 4\hat{e}_\theta\right)\ \text{m/s}^2$$

For this motion:

(a) Determine the rate of change $\dot{v}_P$ of speed of P; and

(b) Determine the radius of curvature ρ_P for the path of P.

- Project $\vec{a}_P$ onto $\hat{e}_t$ to find $\dot{v}_P$
- $|\vec{a}_P|^2 = \dot{v}_P^{\,2} + \left(\frac{v_P^2}{\rho_P}\right)^2 \Rightarrow$ Solve for ρ_P