

## Chapter 1

# Particle Kinematics Homework

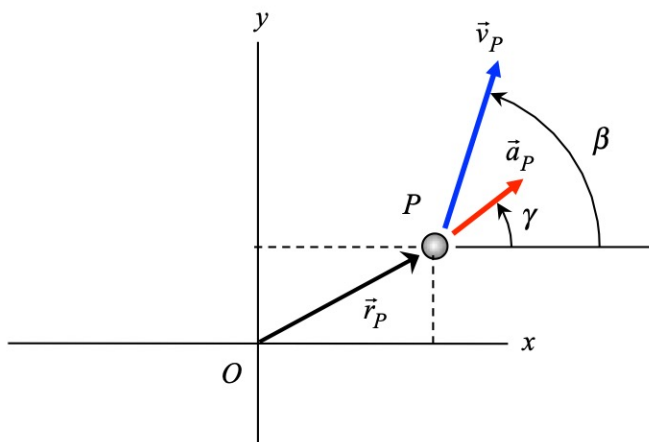


**Homework H1.A**

**Given:** Particle P moves in the  $xy$ -plane in such a way that the position vector of P as measured from the origin O is given as a function of time as:  $\vec{r}_P = \left(-\frac{1}{6}t^3 + \frac{1}{2}t^2\right)\hat{i} + \frac{1}{12}t^4\hat{j}$ , with the position vector components given in terms of meters and time  $t$  given in seconds.

**Find:** For this problem:

- At  $t = 1\text{ s}$ , determine the velocity  $\vec{v}_P$  and acceleration  $\vec{a}_P$  vectors in terms of the  $\hat{i}$  and  $\hat{j}$  Cartesian unit vectors, as well as the angles  $\beta$  and  $\gamma$  as measured from the direction of the positive  $x$ -axis.
- Repeat Part (a) above for  $t = 4\text{ s}$ .



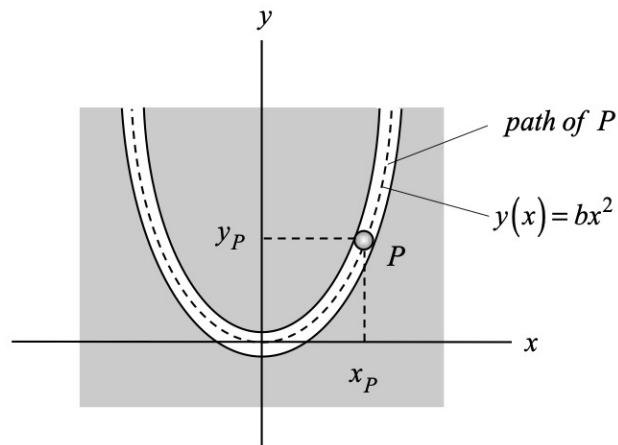
**Homework H1.B**

**Given:** Particle P moves in a slot in  $xy$ -plane with the shape of the slot being described by  $y(x) = bx^2$ , where  $x$  and  $y$  are given in terms of feet. The  $x$ -position of P is controlled in such a way that  $\dot{x} = \text{constant}$ .

**Find:** For the position where  $x = 2$  ft:

- Determine the velocity  $\vec{v}$  and acceleration  $\vec{a}$  vectors for P in terms of their  $xy$ -components and written using the Cartesian unit vectors  $\hat{i}$  and  $\hat{j}$ .
- What is the speed and the magnitude of acceleration for P?
- Make a sketch of the  $\vec{v}$  and  $\vec{a}$  vectors in the  $xy$ -plane with P in the position prescribed above.

Use the following parameters in your analysis:  $b = 0.5/\text{ft}$  and  $\dot{x} = 10$  ft/s.



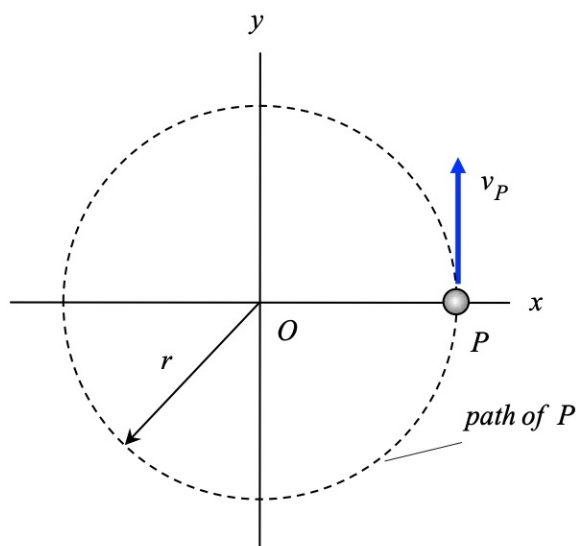
**Homework H1.C**

**Given:** Particle P moves on a circular path with a radius of  $r$ . At the position shown, P has a speed of  $v_P$ .

**Find:** Determine the acceleration vector for P,  $\vec{a}_P$ , in terms of its Cartesian components and make a sketch of  $\vec{a}_P$  for the position shown for the cases of:

- (a) the speed  $v_P$  increasing at a rate of  $2 \text{ m/s}^2$ .
- (b) the speed  $v_P = \text{constant}$ .
- (c) the speed  $v_P$  decreasing at a rate of  $1.5 \text{ m/s}^2$ .

Use the following parameters in your analysis:  $r = 0.5 \text{ ft}$  and  $v_P = 10 \text{ ft/s}$ .

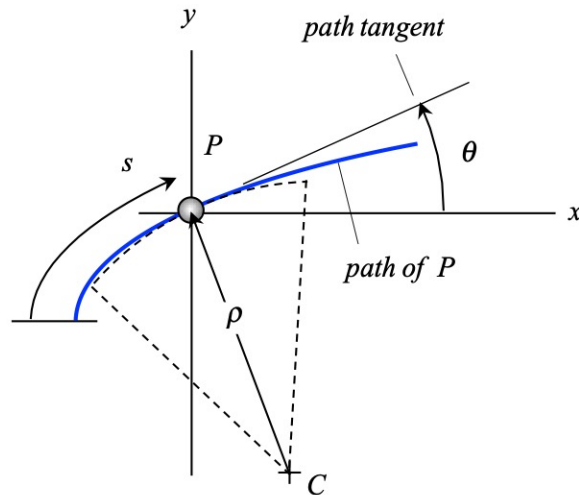


## Homework H1.D

**Given:** Particle P moves along a path with its position on the path given by the arc length of  $s$ . The speed of P is given as a function of  $s$  as:  $v_P = bs^2$ , where  $s$  is given in meters and  $v_P$  in terms of meters/second. The radius of curvature of the path is given by  $\rho$  and the path tangent is at an angle of  $\theta$  with respect to the direction of the  $x$ -axis.

**Find:** At the position of P where  $s = 3$  m:

- Make a sketch of the path unit vectors  $\hat{e}_t$  and  $\hat{e}_n$ .
- Determine the velocity and acceleration of P in terms of path unit vectors  $\hat{e}_t$  and  $\hat{e}_n$ .
- Determine the velocity and acceleration of P in terms of Cartesian unit vectors  $\hat{i}$  and  $\hat{j}$ .
- Determine the  $xy$ -components of location of the center of curvature, C, for the path.



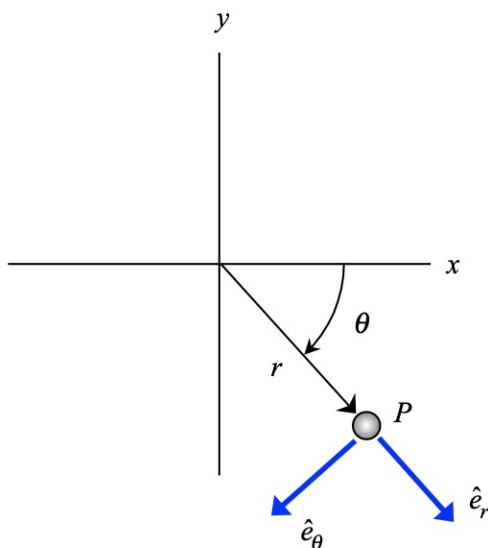
Use the following parameters in your work:  $b = 0.5/\text{m}\cdot\text{s}$ ,  $\rho = 5$  m and  $\theta = 30^\circ$ .

**Homework H1.E**

**Given:** Particle P moves with its position in the  $xy$ -plane given by  $r(t) = r_0 + bt^2$  and  $\theta(t) = ct$ , where  $r$  is in terms of m and  $\theta$  is in radians.

**Find:** For the position of P when  $t = 2$  s:

- determine the velocity and acceleration vectors of P in terms of their  $\hat{e}_r$  and  $\hat{e}_\theta$  components.
- determine the velocity and acceleration vectors of P in terms of their  $\hat{i}$  and  $\hat{j}$  components.



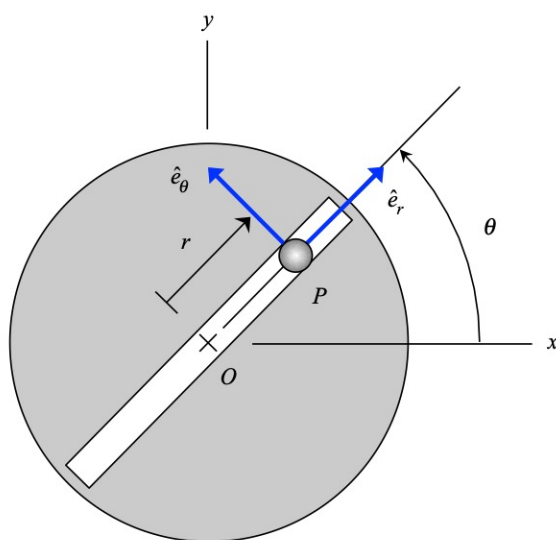
Use the following parameters in your work:  $r_0 = 2$  m,  $b = 0.5/\text{s}^2$  and  $c = (\pi/2)/\text{s}$ .

**Homework H1.F**

**Given:** A turntable rotates about a fixed, vertical axis passing through point  $O$ , with  $\theta$  representing the angle of rotation of the turntable. Particle  $P$  moves within a radial slot cut into the turntable where the radial position of  $P$  in the slot is given by  $r(\theta) = b\theta^2$ , with  $r$  being in feet and  $\theta$  in radians. The turntable rotates at a constant rate of  $\dot{\theta}$ .

**Find:** For the position of  $P$  when  $\theta = \pi/2$ :

- determine the velocity and acceleration vectors of  $P$  in terms of their  $\hat{e}_r$  and  $\hat{e}_\theta$  components.
- determine the velocity and acceleration vectors of  $P$  in terms of their  $\hat{i}$  and  $\hat{j}$  components.



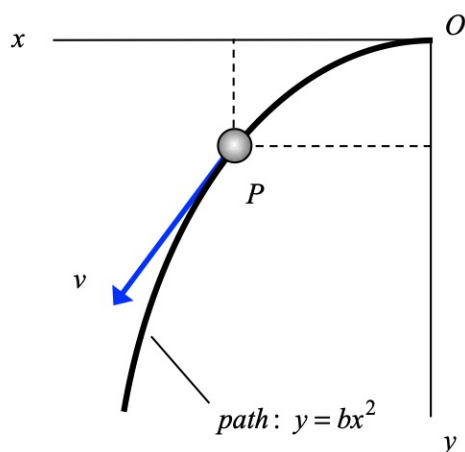
Use the following parameters in your work:  $b = 2$  ft and  $\dot{\theta} = 3$  rad/s.

**Problem H1.G**

**Given:** Particle P travels along a path defined by Cartesian components as  $y = bx^2$ , where  $x$  and  $y$  are in feet. The vertical component of the velocity of P is known to have a constant value of  $\dot{y}$ .

**Find:** For the position of  $x = 50$  ft:

- Determine the velocity and acceleration of P in terms of the Cartesian unit vectors  $\hat{i}$  and  $\hat{j}$ .
- Determine the unit tangent and unit normal vectors,  $\hat{e}_t$  and  $\hat{e}_n$ , in terms of  $\hat{i}$  and  $\hat{j}$ .
- Determine the rate of change of speed of P,  $\dot{v}$ , and the normal component of the acceleration of P.
- From the results above, determine the radius of curvature  $\rho$  of the path of P at this position.



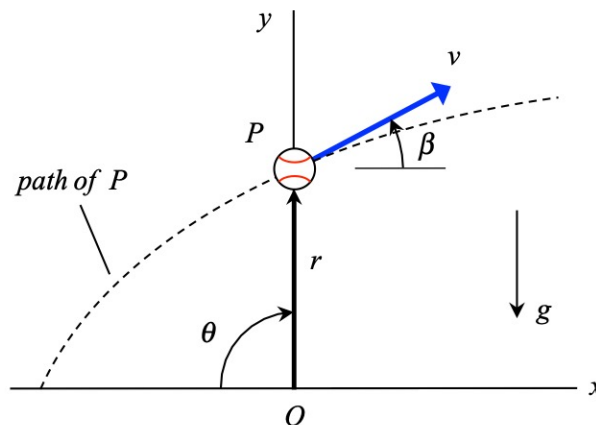
Use the following parameter in your analysis:  $\dot{y} = 10$  ft/s and  $b = 0.01$ /ft.

**Problem H1.H**

**Given:** A baseball P moves along the path in the vertical  $xy$ -plane shown below in the figure in the absence of air resistance. The velocity of P makes an angle of  $\beta$  with respect to the horizontal. The motion of P is monitored by an observer at O who is able to measure the radial distance  $r$  from O to P and the angle  $\theta$  that the line OP makes with the horizontal, as shown in the figure. Note that since the air resistance is to be considered negligible, the acceleration of P is  $g$  vertically downward. At the position shown, line OP is vertical ( $\theta = 90^\circ$ ) and  $r = 6$  ft.

**Find:** For the position shown:

- Make a sketch of P showing the polar unit vectors  $\hat{e}_r$  and  $\hat{e}_\theta$ , and the path unit vectors  $\hat{e}_t$  and  $\hat{e}_n$ .
- Determine numerical values for  $\dot{r}$ ,  $\dot{\theta}$ ,  $\ddot{r}$  and  $\ddot{\theta}$ .
- Write the path unit vectors  $\hat{e}_t$  and  $\hat{e}_n$  in terms of  $\hat{e}_r$  and  $\hat{e}_\theta$  polar unit vectors.
- Determine the rate of change of speed of P,  $\dot{v}$ , and the radius of curvature,  $\rho$ , of the path of P at this position.



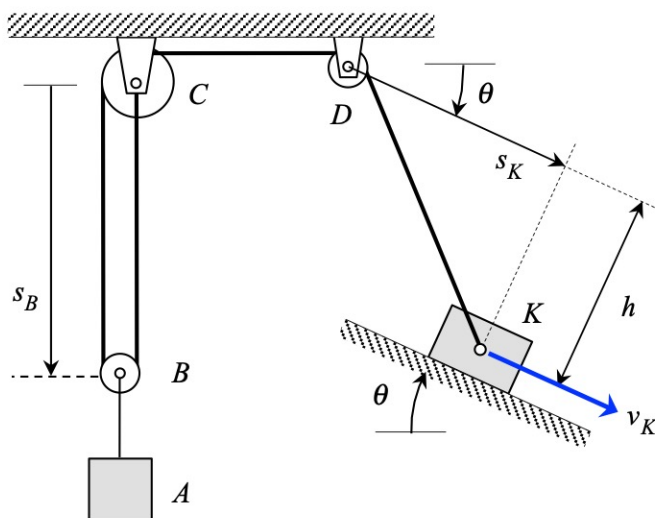
Use the following parameters in your analysis:  $v = 100$  ft/s and  $\beta = 36.87^\circ$ .

**Problem H1.I**

**Given:** Blocks A and K are connected by the cable-pulley system shown, where the radius of pulley D can be assumed to be small. Block K is moving along an incline with a speed of  $v_K$ .

**Find:** For this problem:

- Determine the speed of block A in terms of  $v_K$ ,  $s_K$  and  $h$ .
- Is the speed of A larger than, smaller than or equal to the speed of K? Explain based on your answer in Part (a).



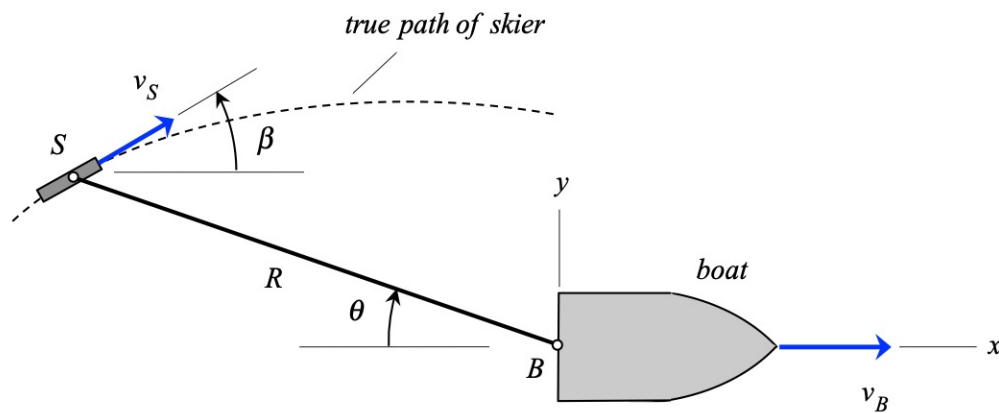
Note that the motion of A is independent of the incline angle  $\theta$ .

**Problem H1.J**

**Given:** A motor boat B, moving to the right with a speed of  $v_B$ , is towing a water skier S. The skier is connected to an inextensible towing cable BS of length  $R$  and with the cable making an angle of  $\theta$  with respect to the travel path of the boat. The skier is moving with a true speed of  $v_S$  along a true path whose tangent makes an angle of  $\beta$  measured from the travel direction of the boat.

**Find:** For this problem:

- Make a sketch of S and B, and show the direction of the polar unit vectors  $\hat{e}_R$  and  $\hat{e}_\theta$ .
- Determine the true speed of the skier,  $v_S$ , and the value of  $\dot{\theta}$ .
- Is the true speed of the skier greater than, less than or equal to the speed of the boat?



Use the following parameters in your analysis:  $v_B = 10$  m/s,  $R = 10$  m,  $\theta = 30^\circ$  and  $\beta = 20^\circ$ .

## Chapter 2

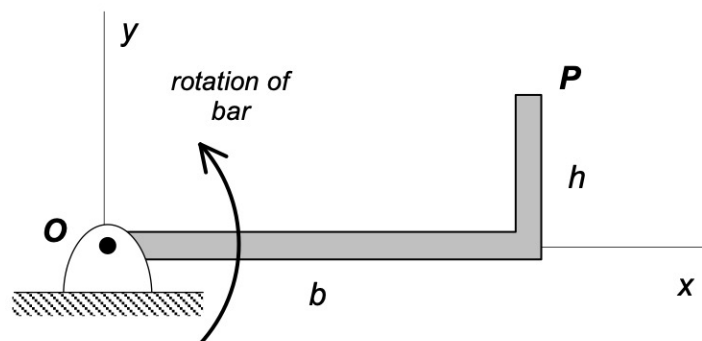
# Planar Rigid Body Kinematics Homework



**Homework H2.A**

**Given:** Bar OP rotates about a shaft passing through end O of the bar. At the instant shown, the angular velocity and angular acceleration of OP are given by  $\vec{\omega}$  and  $\vec{\alpha}$ .

**Find:** Determine the velocity and acceleration of end P. Write your answers as vectors.

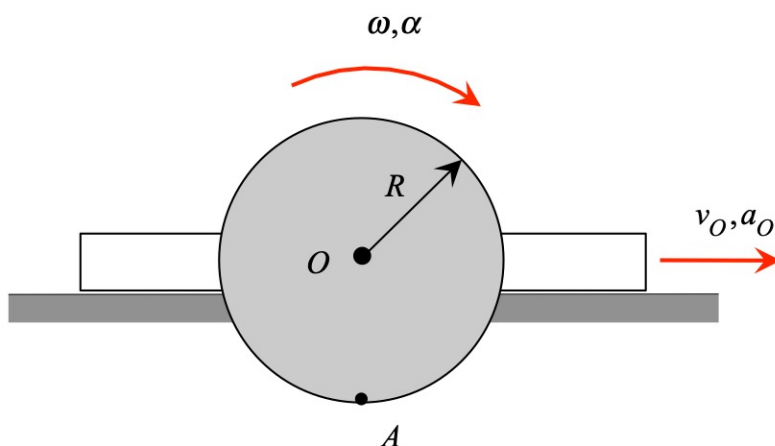


Use the following parameters in your analysis:  $\omega = 2 \text{ rad/s}$  (CCW),  $\alpha = 3 \text{ rad/s}^2$  (CW),  $b = 500 \text{ mm}$  and  $h = 200 \text{ mm}$ .

**Homework H2.B**

**Given:** A circular disk is pinned to a block at its center  $O$ , with the block being constrained to move along a horizontal surface. The angular velocity  $\vec{\omega}$  and angular acceleration  $\vec{\alpha}$  of the disk are in the directions shown in the figure. The block is moving the right with a speed of  $v_O$  and an acceleration of  $a_O$ . At the position shown, point  $A$  on the perimeter of the disk is directly below  $O$ .

**Find:** For this position, determine the velocity and acceleration of point  $A$ . Express your answers as vectors.



Use the following parameters in your analysis:  $R = 0.75$  m,  $\omega = 4$  rad/s,  $\alpha = 2$  rad/s<sup>2</sup>,  $v_O = 3$  m/s and  $a_O = 4$  m/s<sup>2</sup>.

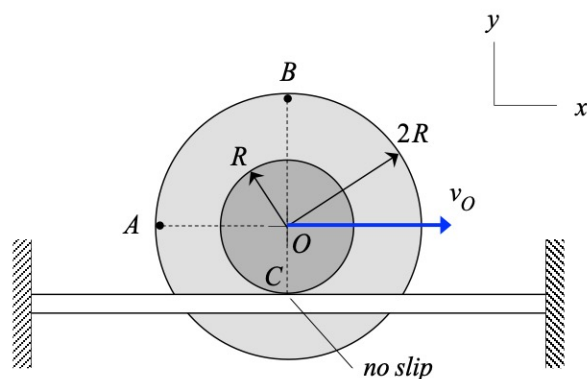
**Homework H2.C**

**Given:** A stepped disk rolls without slipping at the contact point C. The center of the disk O is moving to the right with a speed of  $v_O$ .

**Find:** For this problem:

- (a) Here the speed of O is constant. Determine the angular velocity and angular acceleration of the disk. Also, determine velocity and acceleration of points A and B on the disk. Make sketches of  $\vec{v}_A$ ,  $\vec{v}_B$ ,  $\vec{a}_A$  and  $\vec{a}_B$ .
- (b) Repeat Part (a) here with the speed of O is increasing at a rate of  $\dot{v}_O$ .

Write your answers as vectors.



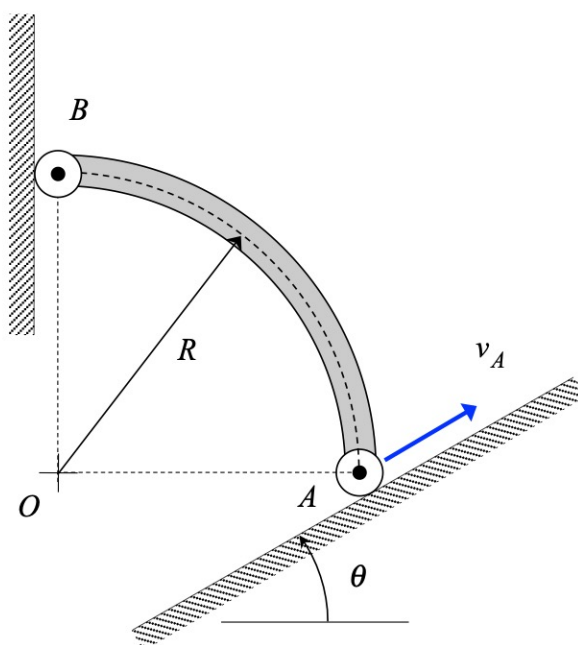
Use the following parameters in your analysis:  $R = 2$  ft,  $v_O = 10$  ft/s and  $\dot{v}_O = 2$  ft/s<sup>2</sup>.

**Homework H2.D**

**Given:** Rigid body AB is shaped as quarter-circle arc with a radius of  $R$ . End B of the bar is constrained to move along a vertical wall, whereas end A moves along an incline at an angle of  $\theta = 53.13^\circ$  with respect to the horizontal. At the instant shown, the center O of the AB arc is directly below end B, and end A moves with a constant speed of  $v_A$ .

**Find:** For this problem:

- Determine the velocity and acceleration of end B of the bar. Express your answers as vectors and in terms of the parameters of  $v_A$  and  $R$ .
- Is the speed of B increasing, decreasing or constant?

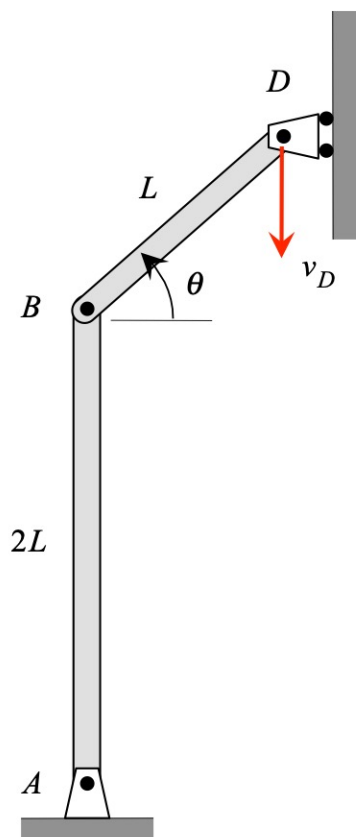


**Homework H2.E**

**Given:** Roller D of the mechanism shown is moving downward along a straight vertical surface with a constant speed of  $v_D$ . At the instant shown, link AB is vertical.

**Find:** For this position:

- Determine the angular velocities of links AB and BD. Write your answers as vectors.
- Determine the angular accelerations of links AB and BD. Write your answers as vectors.



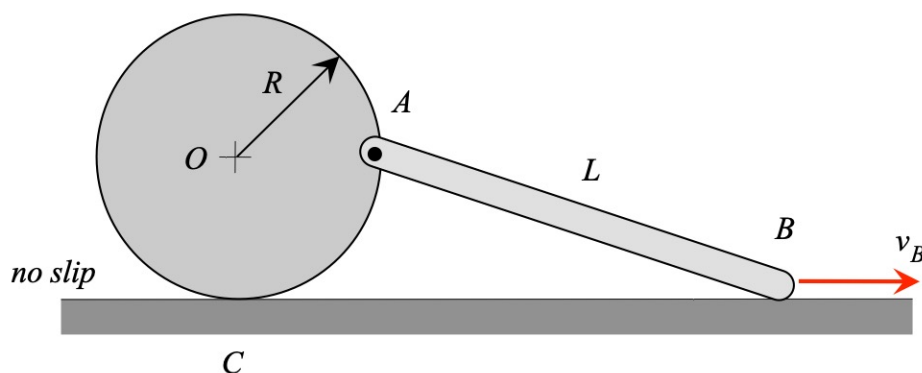
Use the following parameters in your analysis:  $\theta = 53.13^\circ$ ,  $L = 2$  m and  $v_D = 15$  m/s.

**Homework H2.F**

**Given:** The circular disk shown rolls without slipping on a straight horizontal surface. Bar AB is pinned to point A on the disk, with end B constrained to move along a smooth horizontal surface with a constant speed  $v_B$ . At the position shown, A is directly to the right of the center O of the disk.

**Find:** For this position:

- Determine the angular velocities of link AB and of the disk. Write your answers as vectors.
- Determine the angular accelerations of link AB and of the disk. Write your answers as vectors.

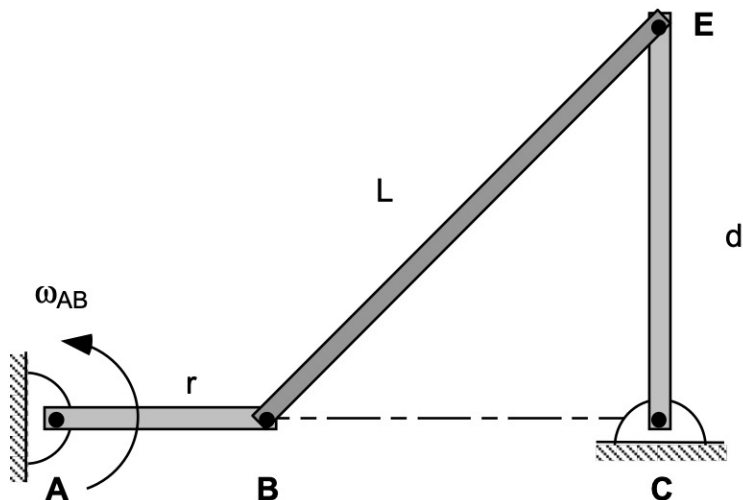


Use the following parameters in your analysis:  $R = 6$  in,  $L = 10$  in and  $v_B = 100$  in/s.

**Homework H2.G**

**Given:** The mechanism shown below is made up of links AB, BE and CE. Links AB and CE are pinned to ground at pins A and C, respectively. Link BE is pinned to links AB and CE at pins B and E, respectively. Link AB is rotating CCW at a constant rate of  $\omega_{AB}$ . In the position shown link AB is horizontal, and link CE is vertical.

**Find:** For this position, determine the angular velocity for links BE and CE.



Use the following parameters in your analysis:  $r = 0.2$  ft,  $L = 0.5$  ft,  $d = 0.4$  ft and  $\omega_{AB} = 4$  rad/s.

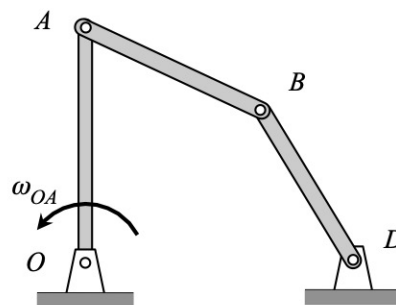
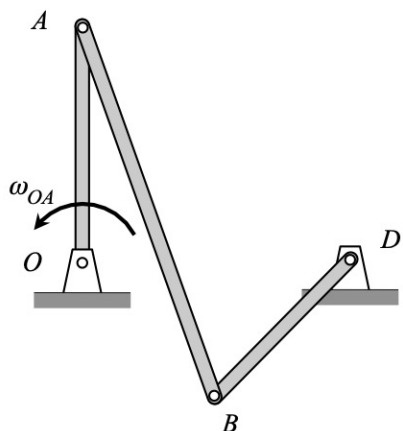
**Homework H2.H**

This problem has three parts. In each part, you are asked to use the instant center approach in answering the questions related to the problems. In all cases, the figures are drawn to scale. Please use a straight edge when making your drawings.

**PART A**

In the mechanisms shown below, link OA is rotating in the counterclockwise sense. For the position shown of EACH mechanism:

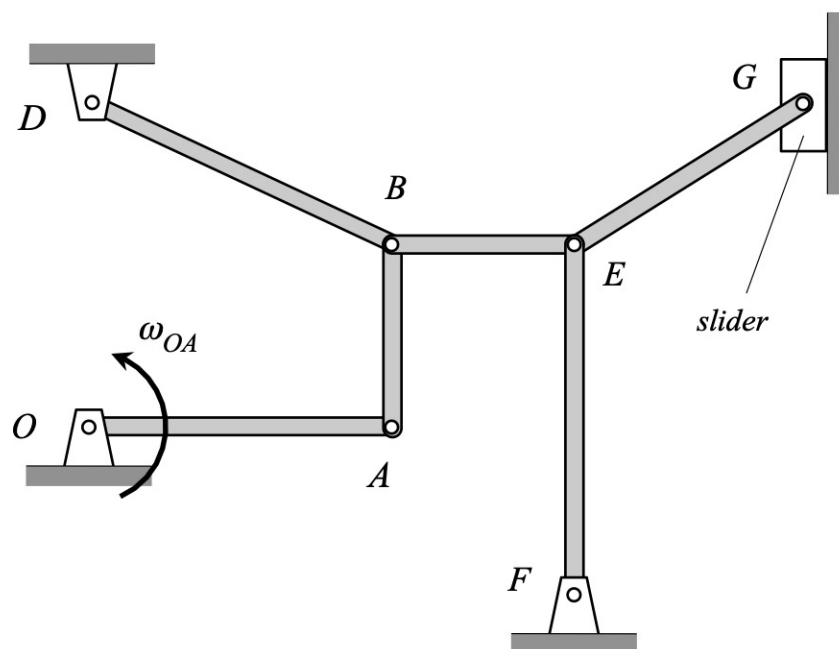
- Determine the location of the instant center for link AB.
- Determine the directions of rotation for links AB and BD. Justify your answers in words.
- Which is larger:  $|\omega_{OA}|$  or  $|\omega_{AB}|$ ? Justify your answers in words.



PART B

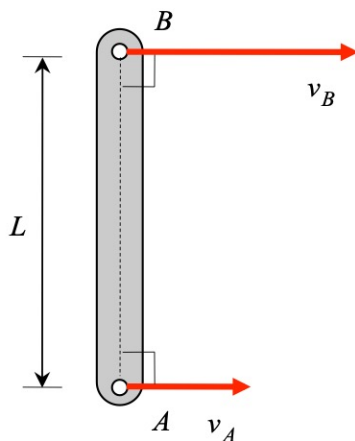
In the mechanism shown below, link OA is rotating in the counterclockwise sense.

- (a) Determine the locations of the instant centers for links AB, BE and EG.
- (b) Determine the directions of rotation for links AB, BE and EG. Justify your answers in words.



## PART C

Link AB, having a length of  $L = 5$  in, is part of a planar mechanism. At the instant shown, the velocities of points A and B are known to be both perpendicular to a line connecting A and B, with  $v_B = 3v_A = 30$  in/s. Determine the location of the instant center for link AB.

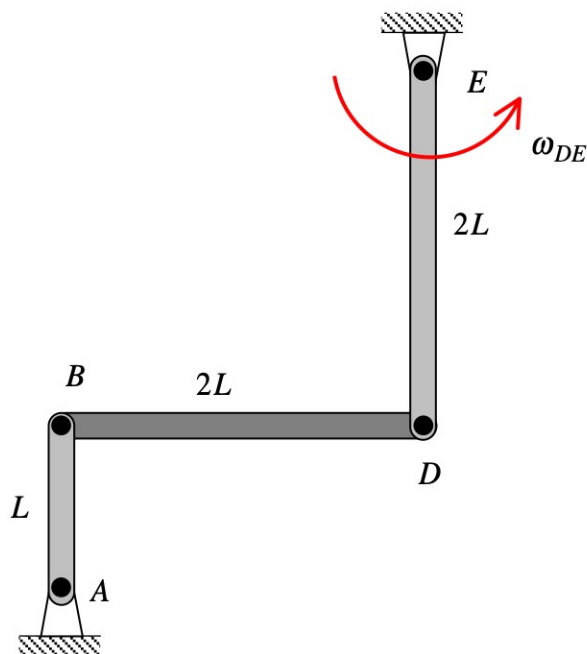


**Homework H2.1**

**Given:** A mechanism is made up of links AB, BD and DE. At the instant shown, links AB and DE are vertical, and link BD is horizontal.

**Find:** For this position:

- Determine the angular velocities of links AB and BD. Write your answers as vectors.
- Determine the angular accelerations of links AB and BD. Write your answers as vectors.



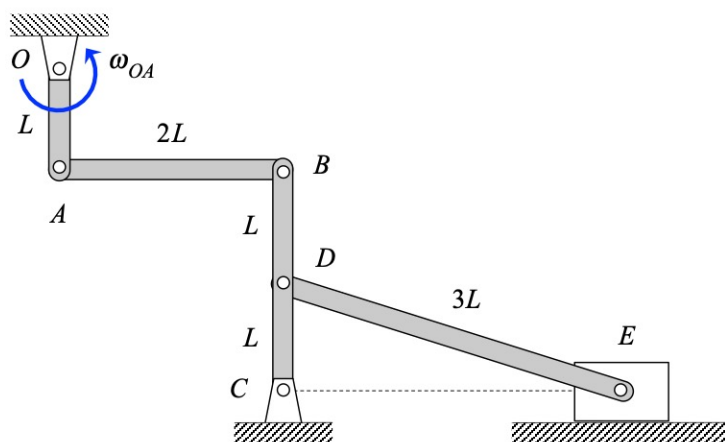
Use the following parameters in your analysis:  $L = 3$  ft and  $\omega_{DE} = 3$  rad/s = constant. You may solve the problem using the Method of Instant Centers and/or by adopting a vector approach.

**Homework H2.J**

**Given:** A mechanism is made up of rigid links OA, AB, BC and DE. A slider is pinned to end E of link DE and is constrained to move along a horizontal guide. For the position shown, link OA is rotating in the counterclockwise sense about O with a constant rotation rate of  $\omega_{OA}$ , with links OA and BC being vertically oriented and link AB being horizontally oriented.

**Find:** For the position shown:

- Use the instant center approach to determine the angular velocities of links AB, BC and DE, along with the speed of slider E.
- Use vector analysis to determine the angular accelerations of links AB, BC and DE, along with the acceleration of slider E. Is the speed of E increasing, decreasing or constant at this instant?



## Chapter 3

# Moving Reference Frame Kinematics Homework

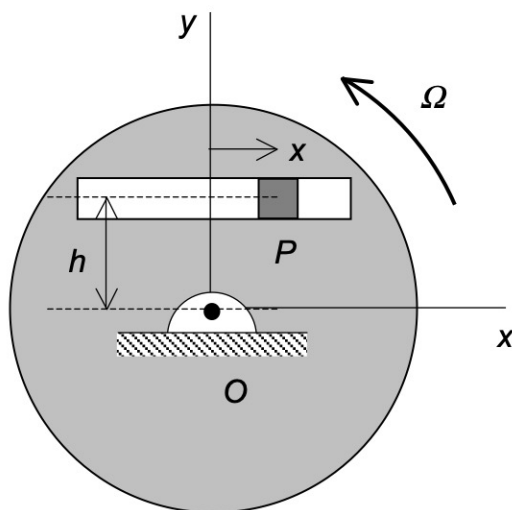


**Homework H3.A**

**Given:** Particle P moves within a straight slot cut into a rotating disk with the  $x$ -position of P increasing at a constant rate of  $\dot{x}$ . The disk is rotating with counter-clockwise sense at a speed of  $\Omega$ , changing at a rate of  $\dot{\Omega}$ . The  $xyz$  coordinate system is attached to the disk with its origin at the center O of the disk.

**Find:** For this problem:

- Determine the velocity and acceleration of particle P. Express your answers as vectors in terms of their  $x$ - $y$  components.
- Make a sketch of the velocity and acceleration vectors found above.

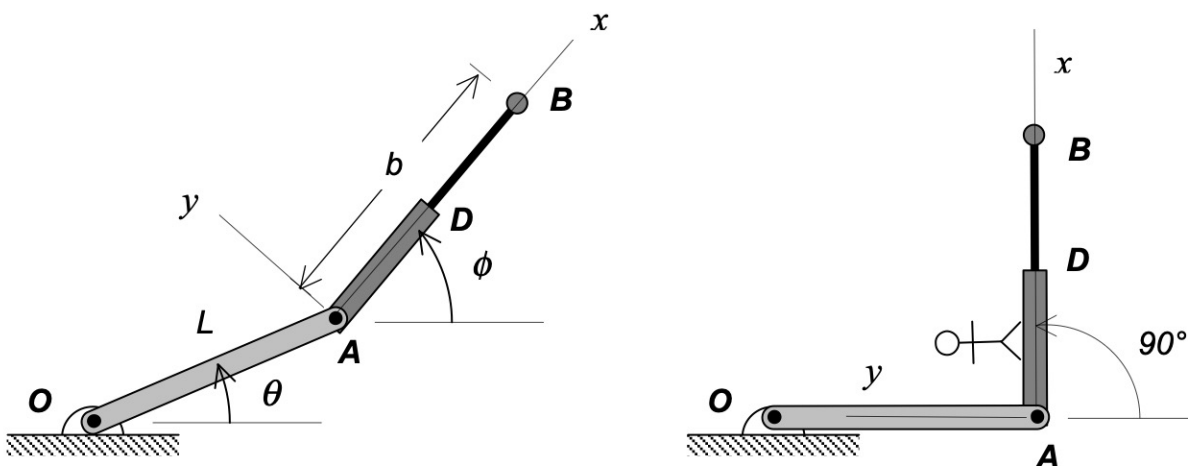


Use the following parameters in your analysis:  $x = 1.5$  ft,  $h = 0.6$  ft,  $\dot{x} = -10$  ft/s,  $\Omega = 6$  rad/s and  $\dot{\Omega} = -4$  rad/s<sup>2</sup>.

## Homework H3.B

**Given:** A robotic manipulator is made up of two links OA and ADB as shown in the figure below left. Link OA has a fixed length of  $L$ , and the length link ADB is changing at a constant rate of  $\dot{b}$ .

**Find:** For the position shown below right with  $\theta = 0^\circ$  and  $\phi = 90^\circ$ , determine the acceleration of point B on the manipulator.



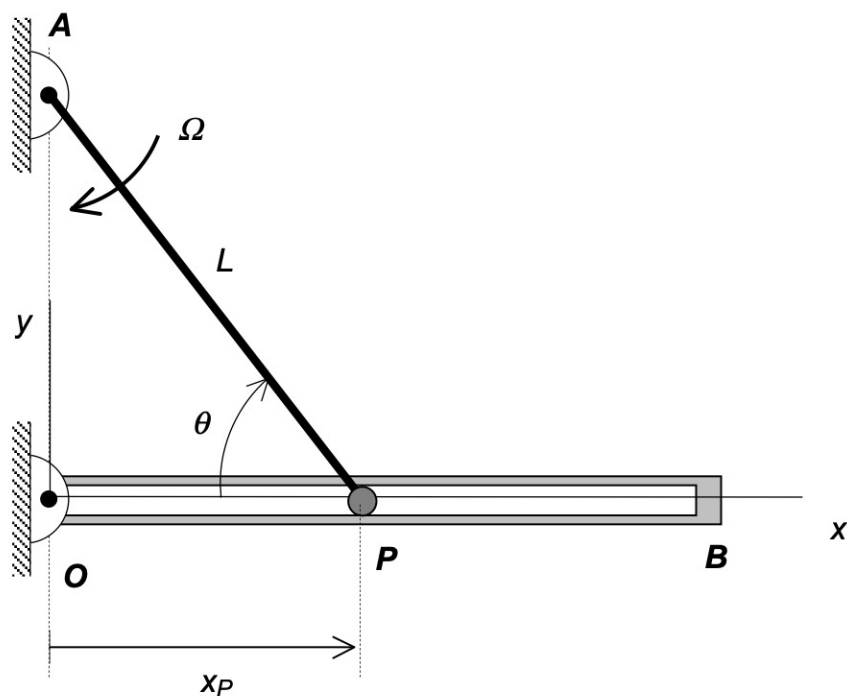
Use the following parameters in your analysis:  $b = 3$  ft,  $\dot{b} = 6$  ft/s = constant,  $\dot{\theta} = 2$  rad/s = constant,  $\dot{\phi} = 3$  rad/s = constant and  $L = 4$  ft.

**Homework H3.C**

**Given:** Arm OB is pinned to ground at end O. Rigid link AP is pinned to ground at A, with end P able to slide within a straight slot that is cut into arm OB. Arm AP is rotating in the clockwise sense with a constant rate of  $\Omega$ . The  $xyz$  coordinate system is attached to arm OB with its origin at pin O. For the position shown, link OB is perpendicular to the direction of OA, with arm AP at an angle of  $\theta$  measured from OB.

**Find:** For this position:

- Determine the angular velocity of arm OB. Write your answer as a vector.
- Determine the angular acceleration of arm OB. Write your answer as a vector.
- Determine the values of  $\dot{x}_P$  and  $\ddot{x}_P$ .



Use the following parameters in your analysis:  $\Omega = 3 \text{ rad/s}$ ,  $L = 2 \text{ m}$  and  $\theta = 53.13^\circ$ .

**Homework H3.D**

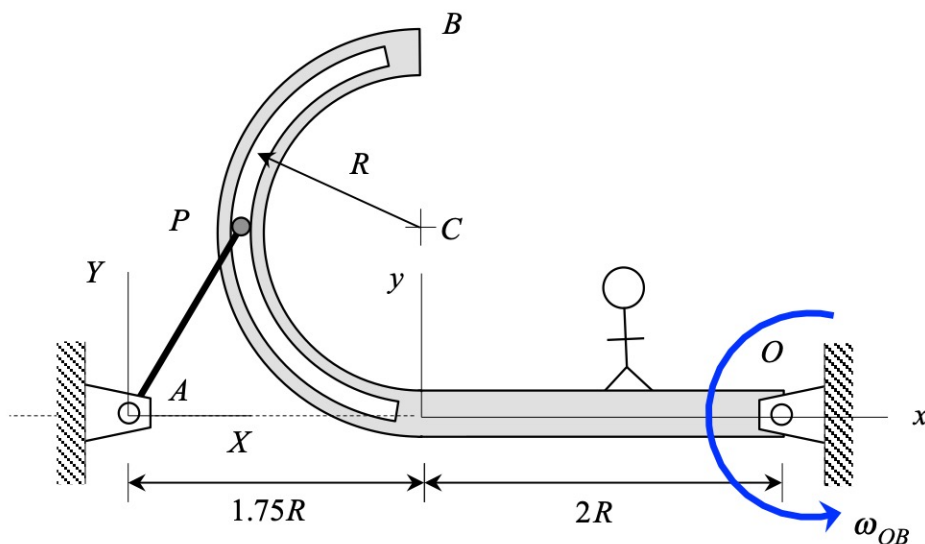
**Given:** A semi-circular slot is cut into arm OB. Arm OB is pinned to ground at end O. Pin P is constrained to move within the slot in arm OB, with P connected to ground through rigid link AP. Arm OB is rotating in the counter-clockwise sense with a constant rotation rate of  $\omega_{OB}$ . At the position shown, P is directly to the left of the center of the semi-circular slot C.

**Find:** For this position,

- Determine the angular velocity of link AP and the speed of P relative to arm OB.
- Determine the angular acceleration of link AP and the rate of change of speed of P relative to arm OB.

Use the following parameters in your analysis:  $\omega_{OB} = 3 \text{ rad/s}$  and  $R = 3 \text{ ft}$ .

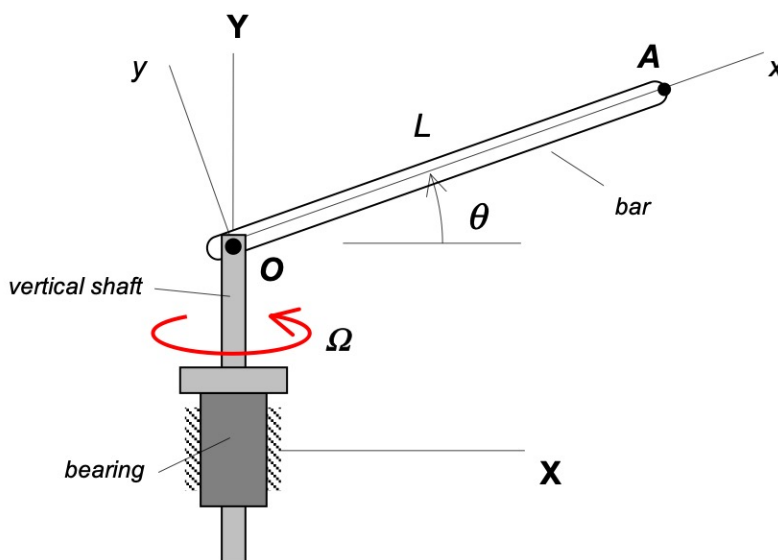
**HINT:** Use an observer attached to the slotted arm OB, and relate the kinematics of points O and P through the moving reference frame kinematics equations.



**Homework H3.E**

**Given:** A shaft rotates about a fixed vertical axis at a constant rate of  $\Omega$ , as shown below. A straight bar OA, having a length of  $L$ , is pinned to point O on the shaft, with O being on the rotation axis of the shaft. At the instant when  $\theta = 0^\circ$ , bar OA is being raised at a rate of  $\dot{\theta}$  from the horizontal plane, with this rate changing at a rate of  $\ddot{\theta}$ . A set of  $xyz$  coordinate axes is attached to bar OA with its origin at O. A second set of coordinate axes,  $XYZ$ , are fixed to ground. At the instant when  $\theta = 0^\circ$ , the  $xyz$  and  $XYZ$  axes are aligned with each other.

**Find:** For the instant when  $\theta = 0^\circ$ , determine the angular velocity and angular acceleration of bar OA.



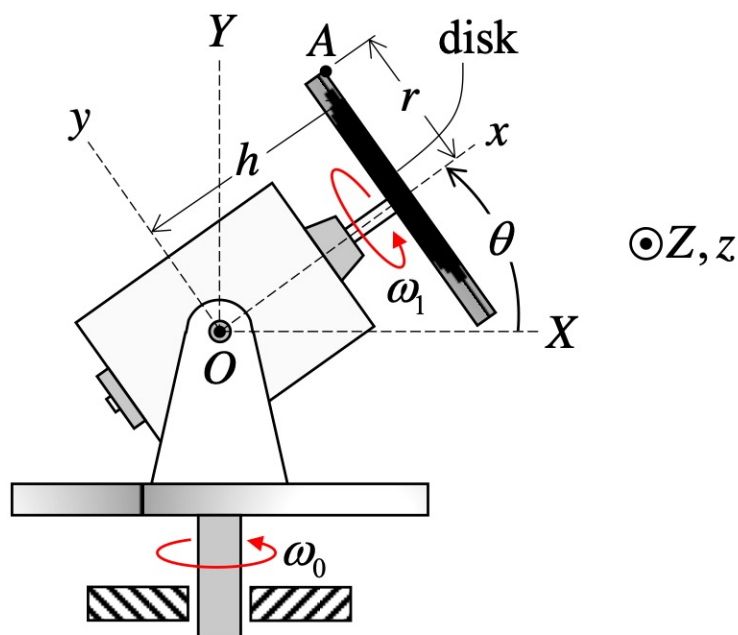
Use the following parameters in your analysis:  $\Omega = 4 \text{ rad/s}$ ,  $\dot{\theta} = 3 \text{ rad/s}$ ,  $\ddot{\theta} = -2 \text{ rad/s}^2$  and  $L = 3 \text{ ft}$ .

**Homework H3.F**

**Given:** A motor is attached to a platform that is rotating with a constant rate of  $\omega_0$  about a fixed vertical axis. The body of the motor is maintained at a constant angle of  $\theta$  with respect to the platform and with the shaft of the motor rotating at a constant rate of  $\omega_1$ .

**Find:** Determine:

- The angular velocity of the disk attached to the shaft of the motor.
- The angular acceleration of the disk attached to the shaft of the motor.

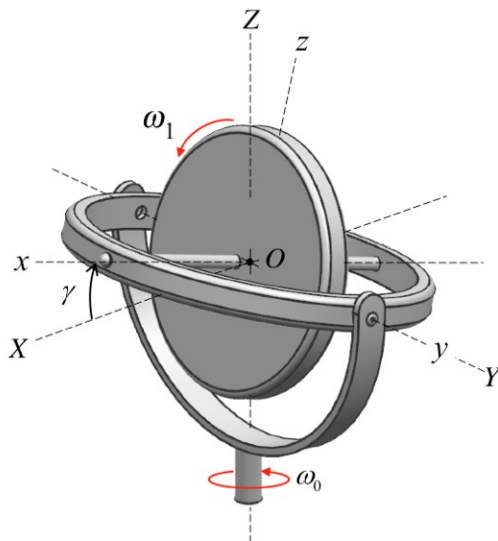


Use the following parameters in your analysis:  $\omega_0 = 2 \text{ rad/s}$ ,  $\theta = 40^\circ$ ,  $\omega_1 = 10 \text{ rad/s}$ ,  $h = 0.25 \text{ m}$ , and  $r = 0.15 \text{ m}$ .

**Homework H3.G**

**Given:** Consider the system shown below.

**Find:** Determine the angular acceleration of the rotor of the gyroscope. Assume all rotation rates are constant.

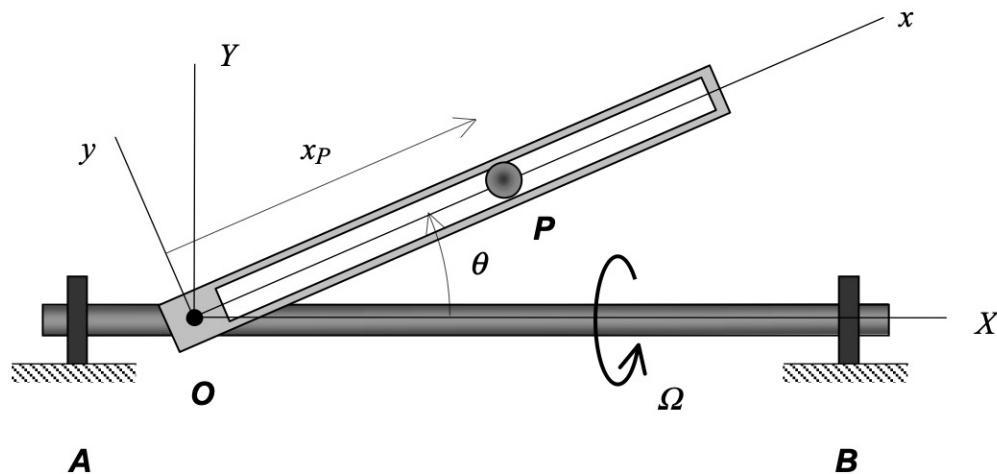


Use the following parameters in your analysis:  $\omega_0 = 10 \text{ rad/s}$ ,  $\omega_1 = 30 \text{ rad/s}$ ,  $\gamma = 40^\circ$  and  $\dot{\gamma} = 0 \text{ rad/s}$ .

## Homework H3.H

**Given:** Shaft AB rotates about a fixed axis with a constant rotational speed of  $\Omega$ . A tube is hinged on shaft AB with the angle  $\theta$  between the tube and shaft increasing at a constant rate of  $\dot{\theta}$ . Particle P moves within the tube at a constant rate of  $\dot{x}_P$  relative to the tube. The  $XYZ$  coordinate system is fixed with the  $X$ -axis aligned with the fixed rotation axis of the shaft AB. The  $xyz$  coordinate system is attached to the tube with the  $x$ -axis aligned with the tube for all time. For the position shown below, the  $z$ - and  $Z$ -axes are aligned.

**Find:** For the position shown, determine the angular velocity and angular acceleration of the tube. Write your answers as vectors in terms of their  $xyz$  components.



Use the following parameters in your analysis:  $\Omega = 4 \text{ rad/s}$ ,  $\theta = 50^\circ$ ,  $\dot{\theta} = -3 \text{ rad/s}$ ,  $x_P = 3 \text{ ft}$  and  $\dot{x}_P = 0$ .

## Chapter 4

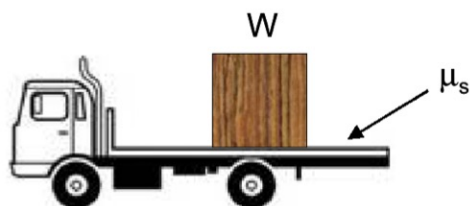
# Particle Kinetics Homework



### Homework H4.A

**Given:** The truck shown below is traveling along I-65 when a deer runs out onto the highway. The truck is initially traveling at a speed of  $v_0$  and decelerates at a constant rate.

**Find:** Find the minimum distance  $s$  over which the truck can stop to ensure that its load does not shift (and the deer is safely avoided).



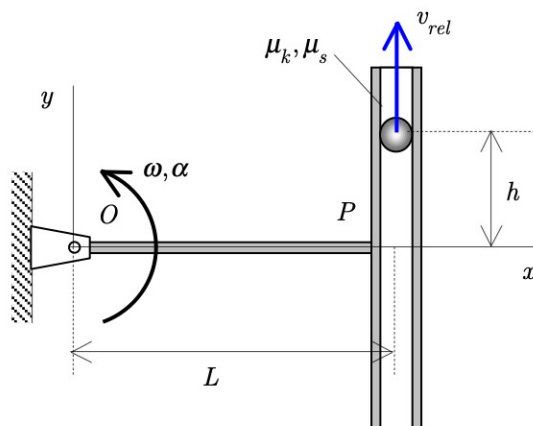
Use the following parameters in your analysis:  $v_0 = 40$  m/s,  $W = 1000$  N and  $\mu_s = 0.2$ .

**Homework H4.B**

**Given:** Particle P of mass  $m$  slides in the direction shown within a tube with a speed of  $v_{rel}$  relative to the tube as the tube rotates in the CCW sense with an angular speed of  $\omega$  and angular acceleration  $\alpha$ .

**Find:** For this problem:

- Determine the acceleration of P;
- Determine the friction force acting on P.

***HORIZONTAL PLANE***

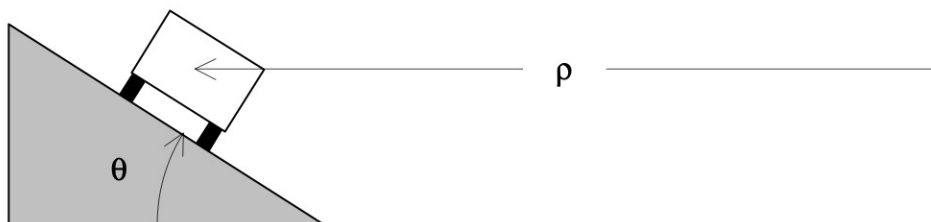
Use the following parameters in your analysis:  $m = 10$  kg,  $\omega = 5$  rad/s,  $\alpha = 2$  rad/s<sup>2</sup>,  $v_{rel} = 4$  m/s,  $\mu_s = 0.6$ ,  $\mu_k = 0.3$ ,  $L = 0.4$  m and  $h = 0.2$  m.

### Homework H4.C

**Given:** The race car, shown below, travels around a banked curve at Daytona International Speedway.

**Find:** Determine:

- (a) The speed at which the race car can circumvent the curve without the assistance of a lateral friction force;
- (b) The maximum speed the race car can circumvent the curve with the assistance of a lateral friction force.

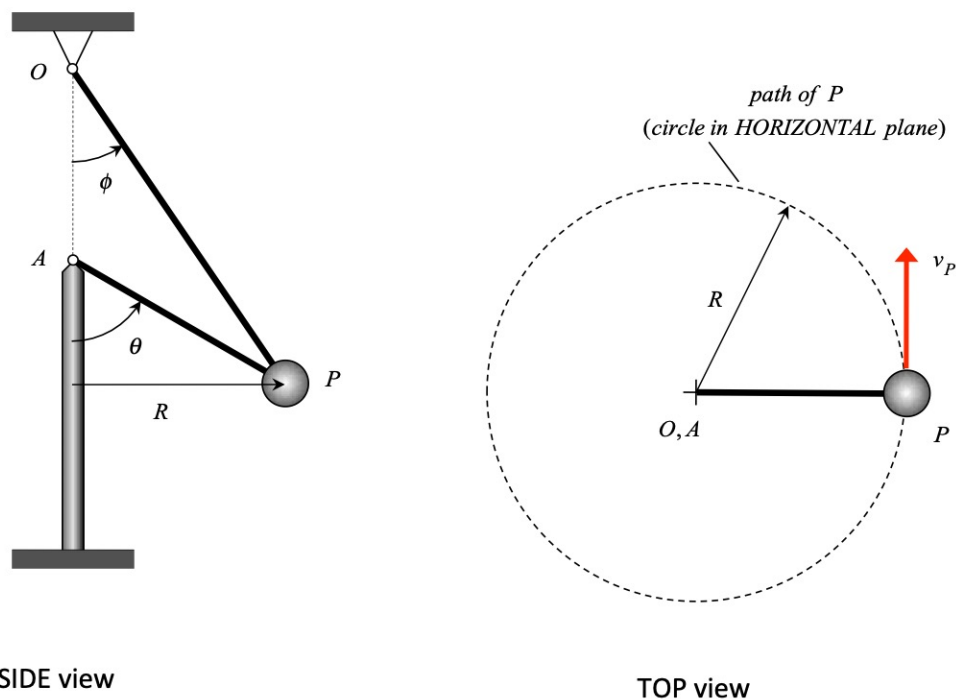


Use the following parameters in your analysis:  $\theta = 25^\circ$ ,  $\mu_s = 0.85$  and  $\rho = 450$  m.

## Homework H4.D

**Given:** Two wires, OP and AP, connect particle P (having a mass of  $m$ ) to fixed points O and A, respectively, where OA is a vertical line. The particle rotates about axis OA such that P has a constant speed of  $v_P$  and with the two wires remaining taut as particle moves on a circular path of radius  $R$ . Let  $\phi$  and  $\theta$  be the angles that wires OP and OA, respectively, make with the vertical.

**Find:** Determine the range of values for  $v_P$  for which wires OP and OA remain taut.

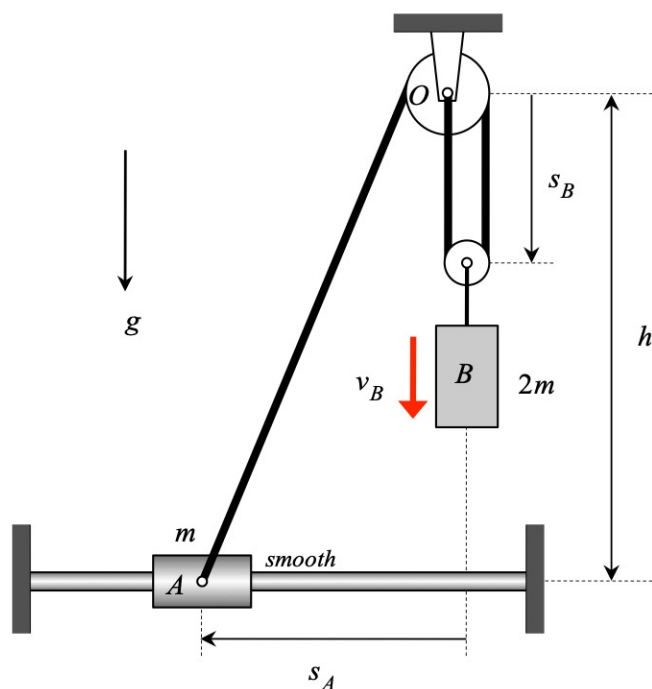


Use the following parameters in your analysis:  $R = 2$  m,  $\theta = 60^\circ$ ,  $\phi = 30^\circ$  and  $m = 4$  kg.

**Homework H4.E**

**Given:** Block A (having a mass of  $m$ ) is connected to block B (with a mass of  $2m$ ) through the cable-pulley system shown. The system is released with block B moving downward with a speed of  $v_B$  and with block A displaced to the left of the path of B by an amount of  $s_A$  and moving to the right along its horizontal guide. Consider all surfaces to be smooth. Assume that the cable remains taut during this motion and that the radii of the pulleys are small.

**Find:** Determine the acceleration of blocks A and B for this position.

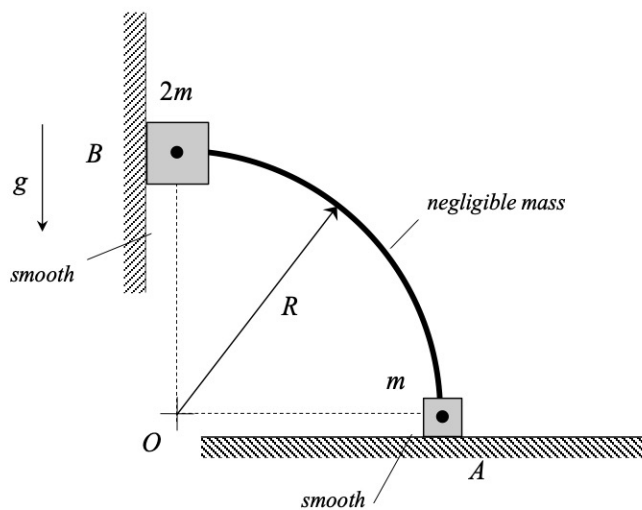


Use the following parameters in your analysis:  $m = 10$  kg,  $s_A = 0.5$  m,  $h = 2$  m and  $v_B = 10$  m/s.

**Homework H4.F**

**Given:** Blocks A and B (having masses of  $m$  and  $2m$ , respectively) are constrained to move along the smooth surfaces shown in the figure below. Member AB, in the shape of a quarter-circle arc, connects blocks A and B, with AB having a mass that is negligible compared to the masses of A and B. At the position shown, when the center O of the circular arc AB is directly below block B, the system is released from rest.

**Find:** For this position, determine the acceleration of blocks A and B on release.

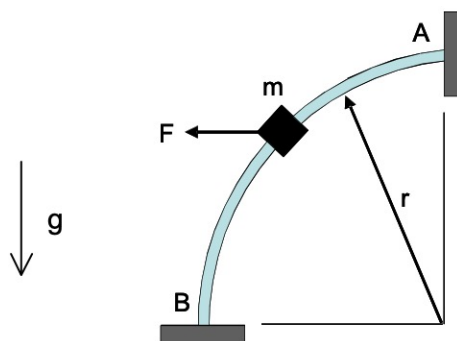


**HINT:** Note that AB is a two-force member. What does this say about the direction of the reaction forces on blocks A and B due to member AB?

**Homework H4.G**

**Given:** The collar, shown below, of mass  $m$ , starts from rest at point A. A constant force  $F$  is applied to the collar in the direction shown. Note that the mechanism lies in the vertical plane. Assume all surfaces to be smooth.

**Find:** Determine the speed of the collar when it reaches point B.

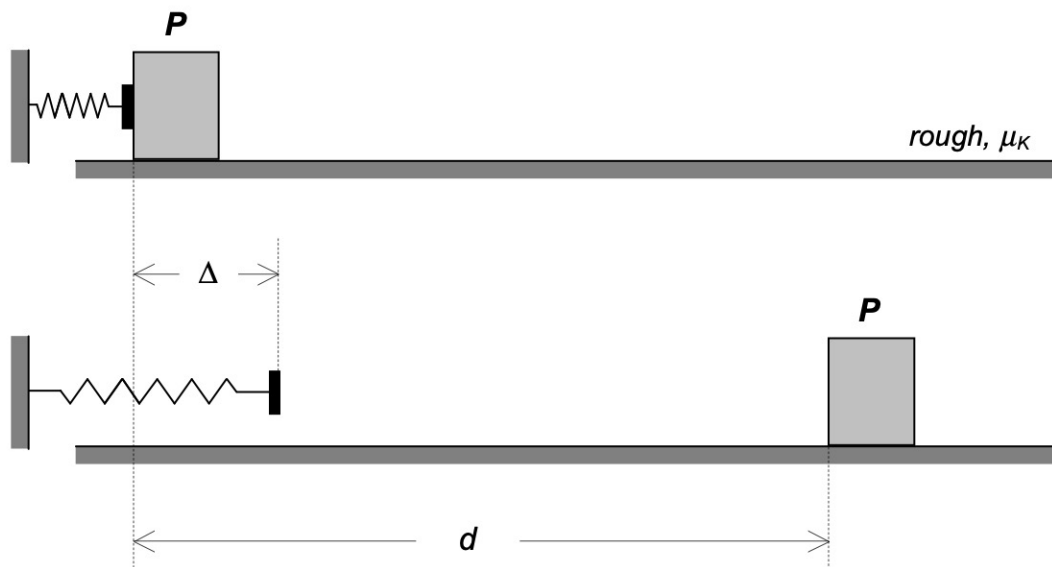


Use the following parameters in your analysis:  $mg = 10$  lb,  $F = 6$  lb, and  $r = 2$  ft.

## Homework H4.H

**Given:** Particle P, having a mass of  $m$ , is pressed against a spring (having a stiffness of  $k$ ) with the spring being compressed by an amount of  $\Delta$ . Upon release from rest, the particle travels along a rough horizontal surface for which the kinetic coefficient of friction is known to be  $\mu_k$ .

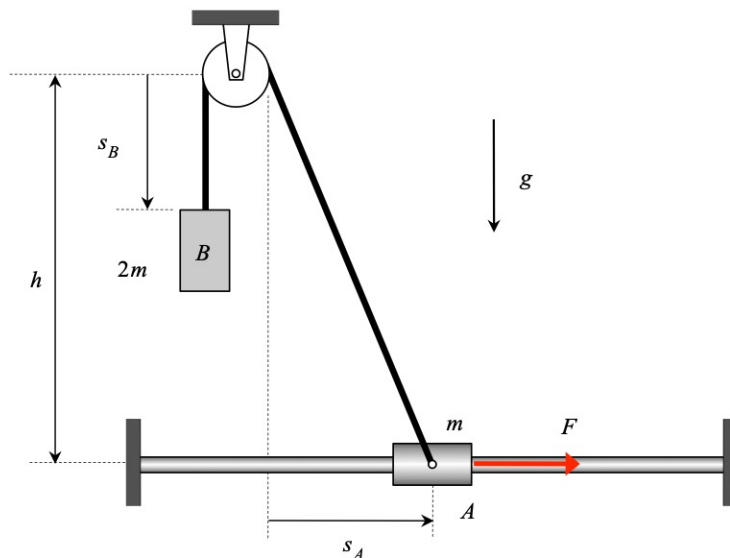
**Find:** Determine the total distance  $d$  traveled by P before coming to rest. Assume that  $d > \Delta$ . Leave your answer in symbolic form.



**Homework H4.1**

**Given:** Particles A and B (having masses of  $m$  and  $2m$ , respectively) are connected by the cable-pulley system shown. Particle A is constrained to move along a horizontal guide. A constant force  $F$  acts to the right on particle A. When A is at a position of  $s_{A1} = 0$  m, it is given an initial speed of  $v_{A1}$  to the right. Assume that the cable remains taut at all times and that all surfaces are smooth.

**Find:** Determine the speeds of A and B when A is at the position  $s_{A2}$ .

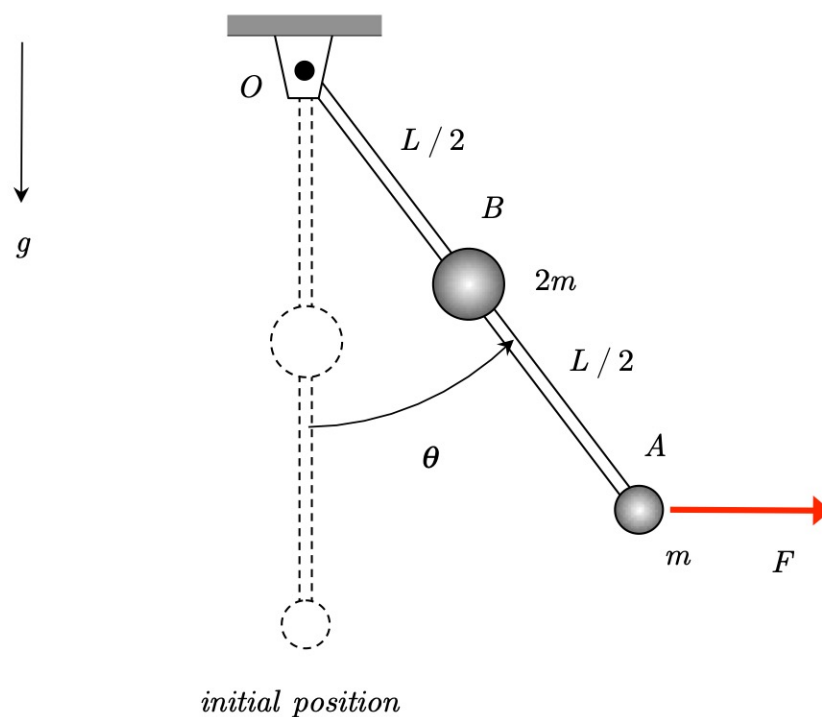


Use the following parameters in your analysis:  $m = 5$  kg,  $F = 50$  N,  $h = 2$  m,  $s_{A2} = 1.5$  m and  $v_{A1} = 10$  m/s.

## Homework H4.J

**Given:** Particles A and B (having masses of  $m$  and  $2m$ , respectively) are attached to a lightweight rigid bar as shown in the figure. A constant horizontal force  $F$  acts on particle A. At the initial state of  $\theta = 0^\circ$ , the system is at rest.

**Find:** Determine the angular speed of the bar as a function of  $\theta$  and in terms of the parameters of the problem.

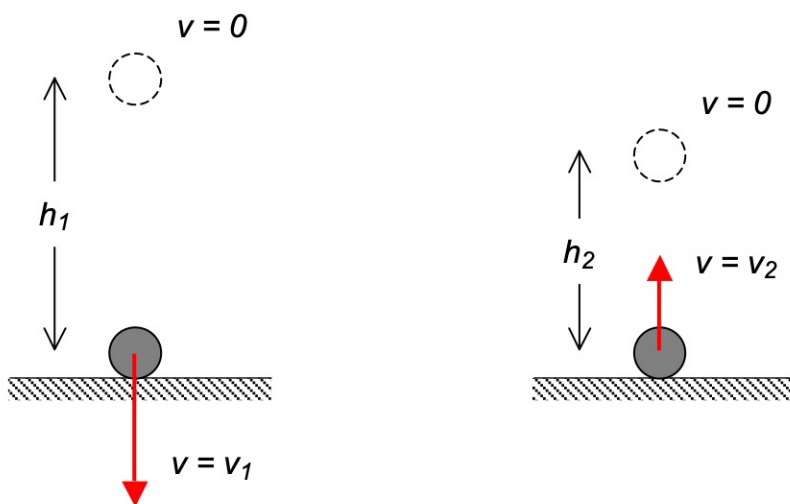


**Homework H4.K**

**Given:** A particle of mass  $m$  is dropped from rest when at a height  $h_1$  above a rigid floor. The particle impacts the floor with a speed of  $v_1$ . This impact of the particle with the floor lasts for a short duration of time  $\Delta t$ , and after the impact is complete, the particle rebounds upward with a speed of  $v_2$ . The particle continues upward reaching a maximum height of  $h_2$ .

**Find:** For this problem:

- Determine the average force acting on the particle by the floor during impact in the presence of gravity;
- Determine the average force acting on the particle by the floor during impact in the absence of gravity;
- Compare your answers from (a) and (b);
- Determine the value of  $h_2/h_1$ .

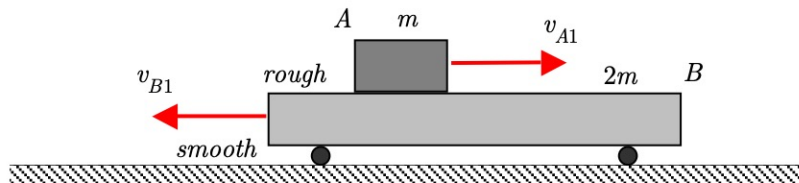


Use the following parameters in your analysis:  $\Delta t = 0.002$  s,  $m = 15$  kg,  $v_1 = 80$  m/s and  $v_2 = 50$  m/s.

**Homework H4.L**

**Given:** Block B (of mass  $2m$ ) is able to slide along a smooth horizontal surface. Block A (of mass  $m$ ) is able to slide along the rough top surface of block B, as shown in the figure. Initially, A is traveling to the right with a speed of  $v_{A1}$ , and block B is traveling to the left with a speed of  $v_{B1}$ .

**Find:** Determine the velocity of block B when block A has to come rest relative to block B.

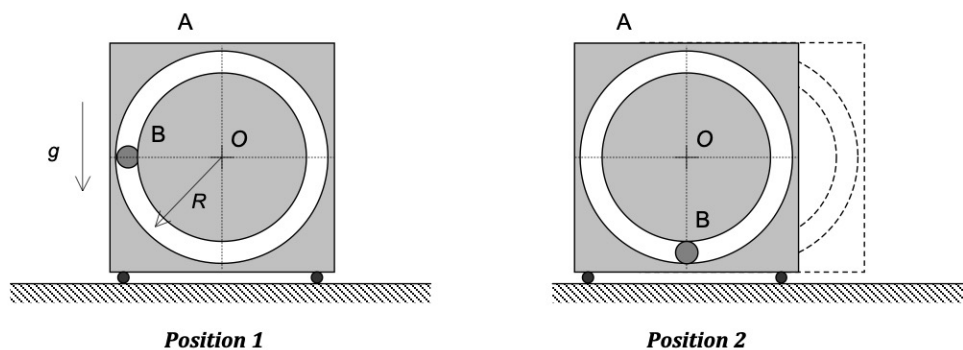


**Homework H4.M**

**Given:** Particle B (having a mass of  $m$ ) is constrained to move within a circular slot (of radius  $R$ ) that is cut into block A (having a mass of  $M$ ). The system is released from rest with particle B on a horizontal line passing through the circle's center  $O$ . Consider all surfaces to be smooth.

**Find:** For this problem:

- Determine the velocities of A and B when B has moved position 2 where B is directly below  $O$  (write your answers as vectors);
- Determine the work done on block A in moving from position 1 to position 2.



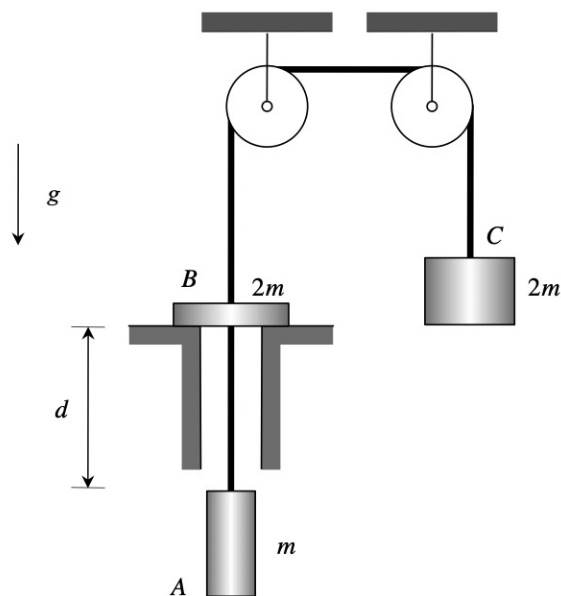
Use the following parameters in your analysis:  $m = 20$  kg,  $M = 40$  kg and  $R = 0.5$  m.

## Homework H4.N

**Given:** Blocks A and C (having masses  $m$  and  $2m$ , respectively) are connected by an inextensible cable. This cable has been fed through a third block B (having a mass of  $2m$ ), as shown in the figure below. The system is released from rest with A being located at a distance of  $D$  below B. In the subsequent motion, block A contacts block B and sticks after a short time  $\Delta t$  following their initial contact. Assume that the cable does not break and that the mass of the two pulleys is negligible.

**Find:** For this problem:

- Determine the maximum height attained by block B after its contact with and sticking to block A.
- Determine the average tension force in the cable during the impact time  $\Delta t$  between A and B.



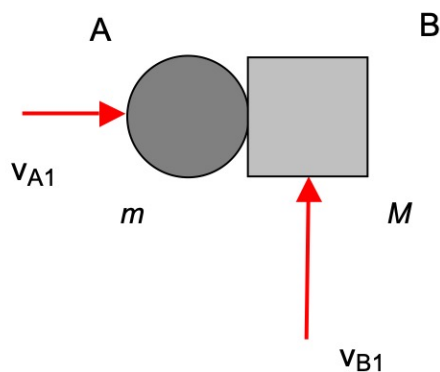
Use the following parameters in your analysis:  $m = 20$  kg,  $d = 2$  m and  $\Delta t = 0.003$  s.

**Homework H4.O**

**Given:** Blocks A and B (having masses of  $m$  and  $M$ , respectively) are initially traveling in directions perpendicular to each other with speeds of  $v_{A1}$  and  $v_{B1}$ , respectively, as shown below in the figure. After impacting each other, A is traveling to the RIGHT with a speed of  $v_{A2}$ , and B travels with a speed of  $v_{B2}$  (the direction of motion for B after impact is not known). Consider all surfaces to be smooth.

**Find:** For this problem:

- Determine the mass  $M$  of block B;
- Determine the coefficient of restitution  $e$  for the impact of A and B.



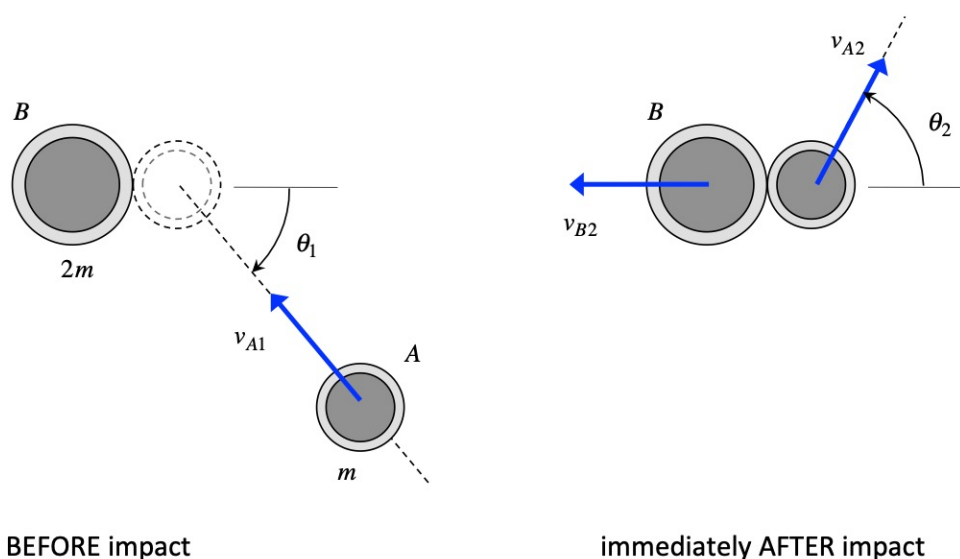
Use the following parameters in your analysis:  $m = 3$  kg,  $v_{A1} = 4$  m/s,  $v_{B1} = 4$  m/s,  $v_{A2} = 2$  m/s and  $v_{B2} = 5$  m/s.

## Homework H4.P

**Given:** Disks A and B (of masses  $m$  and  $2m$ , respectively) are able to slide freely on a smooth horizontal surface. Disk A moves toward the stationary disk B with a speed of  $v_{A1}$  in the direction shown in the figure. After impact, disk B moves immediately to the left, whereas disk A moves in a direction given by the angle  $\theta_2$ . The coefficient of restitution for the impact between the two disk is  $e$  and the contacting surfaces between A and B during impact is smooth.

**Find:** For this problem:

- Derive an expression for the angle  $\theta_2$  in terms of  $e$ .
- Knowing that  $0 \leq e \leq 1$ , use your answer from (a) to determine the range of possible angles  $\theta_2$ .

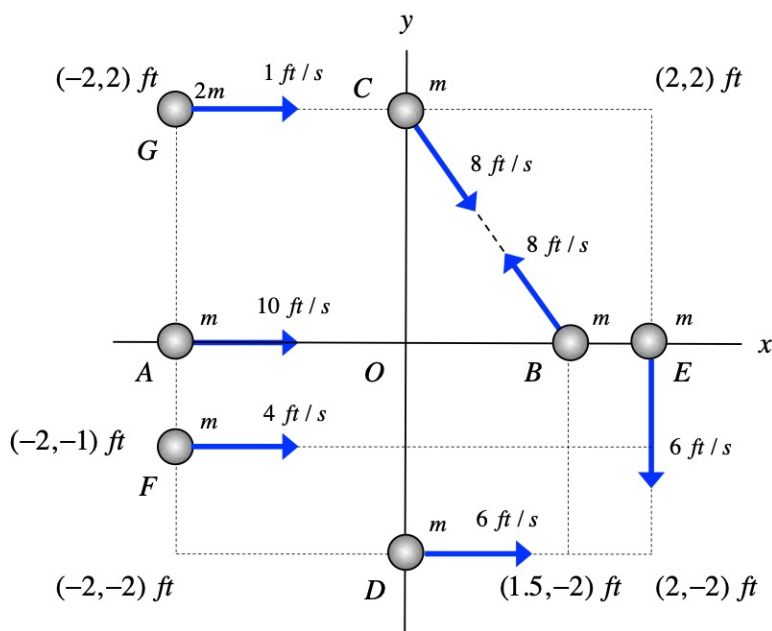


Use the following parameters in your analysis:  $\theta_1 = 30^\circ$ .

## Homework H4.Q

**Given:** Seven particles, A through G, move within a single plane. The mass of each particle is shown in the figure below, along with the velocity and position of each particle.

**Find:** Determine the total angular momentum about the fixed point O for the entire system of seven particles.

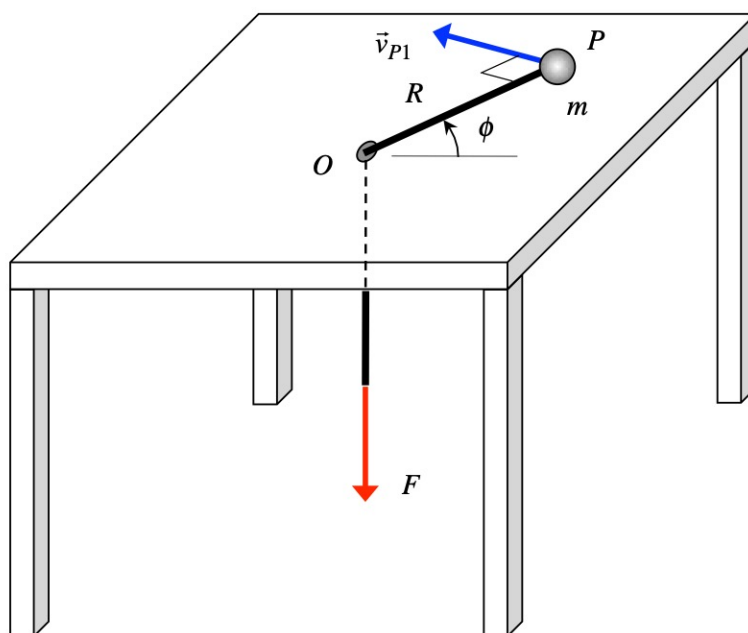


Use the following parameters in your analysis:  $m = 10$  slugs.

**Homework H4.R**

**Given:** Particle P, having a mass of  $m$ , is able to slide on the smooth, horizontal top of a table. A flexible cable is attached to P, with the cable being fed through a hole in the table at O. A constant force  $F$  acts on the other end of the cable. The system is released with P being at a radial distance  $R = R_1$  from O, and with P having a velocity perpendicular to OP with a speed of  $v_{P1}$ .

**Find:** Determine the numerical values for  $\dot{R}$  and  $\dot{\phi}$  when P has moved to a position for which  $R = R_2$ .

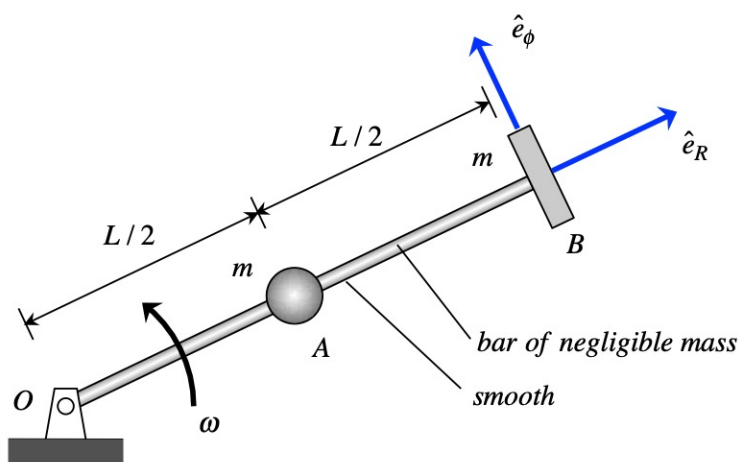


Use the following parameters in your analysis:  $m = 2$  kg,  $R_1 = 1.5$  m,  $R_2 = 0.5$  m,  $v_{P1} = 5$  m/s and  $F = 236$  N.

**Homework H4.S**

**Given:** Particle B having a mass of  $m$  is rigidly attached to arm OB, where OB has negligible mass and a length of  $L$ . OB is pinned to ground at end O and is able to rotate about O without any frictional resistance. A second particle A (also having a mass of  $m$ ) is able to slide along arm OB. When OB is rotating with an rotational speed of  $\omega = \omega_1$ , particle A is released from rest with respect to OB at the midpoint of the arm. Particle A slides outward on arm OB, eventually impacting particle B. The coefficient of restitution for the impact of A with B is  $e$ .

**Find:** Determine the velocity of A immediately after its impact with B. Write your answer as a vector in terms of its  $\hat{e}_R$  and  $\hat{e}_\phi$  components.



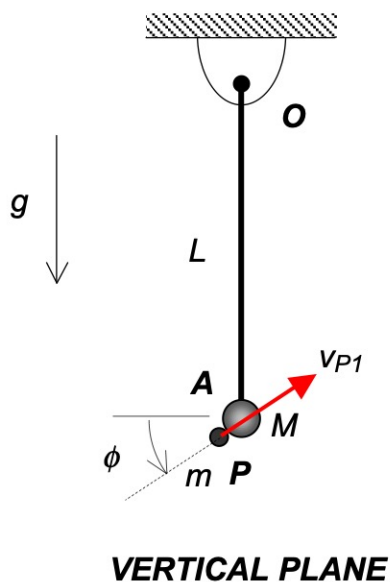
HORIZONTAL plane

Use the following parameters in your analysis:  $m = 5 \text{ kg}$ ,  $L = 2 \text{ m}$ ,  $\omega_1 = 4 \text{ rad/s}$  and  $e = 0.5$ .

**Homework H4.T**

**Given:** Rigid arm OA (having length  $L$  and having negligible mass) is pinned to ground at end O. A particle of mass  $M$  is attached to end A of OA. At instant "1", a pellet P (having a mass of  $m$ ) strikes the stationary particle A with a speed of  $v_{P1}$  in the direction shown below in the figure. At the end of a short time interval impact, P sticks to A.

**Find:** Determine the angular speed of arm OA immediately after P sticks to A.



Use the following parameters in your analysis:  $\phi = 30^\circ$ ,  $L = 4$  ft,  $mg = 4$  lb,  $Mg = 8$  lb and  $v_{P1} = 150$  ft/s.

## Chapter 5

# Rigid Body Kinetics Homework

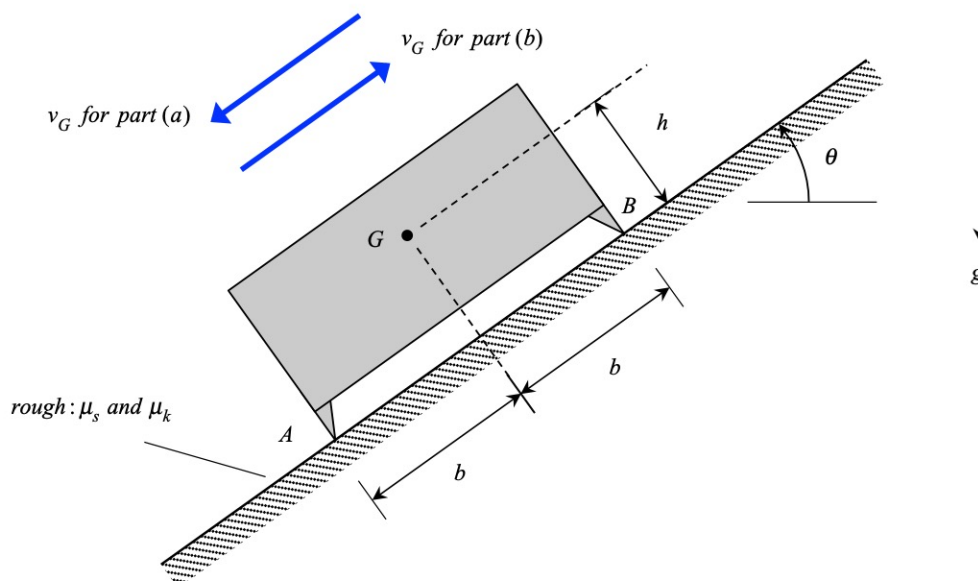


**Homework H5.A**

**Given:** A crate of mass  $m$  slides with a speed of  $v_G$  on a rough inclined surface (with coefficients of static and kinetic friction between the crate and the incline of  $\mu_s$  and  $\mu_k$ , respectively).. The center of mass of the crate is located at point G.

**Find:** For this problem:

- If the crate is moving DOWN in the incline, determine the reactions at supports A and B on the crate. Express each answer in terms of a fraction of the crate's weight  $mg$ .
- If the crate is moving UP in the incline, determine the reactions at supports A and B on the crate. Express each answer in terms of a fraction of the crate's weight  $mg$ .
- Compare your answers in (a) and (b). Explain in words why they are different.

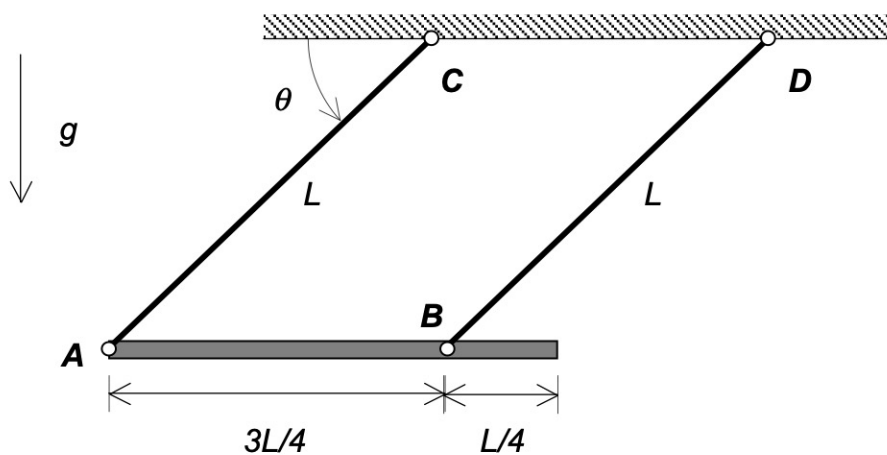


Use the following parameters in your analysis:  $b = h$ ,  $\mu_s = 0.4$ ,  $\mu_k = 0.2$  and  $\theta = 36.87^\circ$ .

**Homework H5.B**

**Given:** A thin, homogeneous bar of mass  $M$  is supported by parallel cables AC and BD, as shown below in the figure. The bar is released from rest with the cables at an angle of  $\theta$  measured from the horizontal.

**Find:** Determine the tension in cables AC and BD on release.

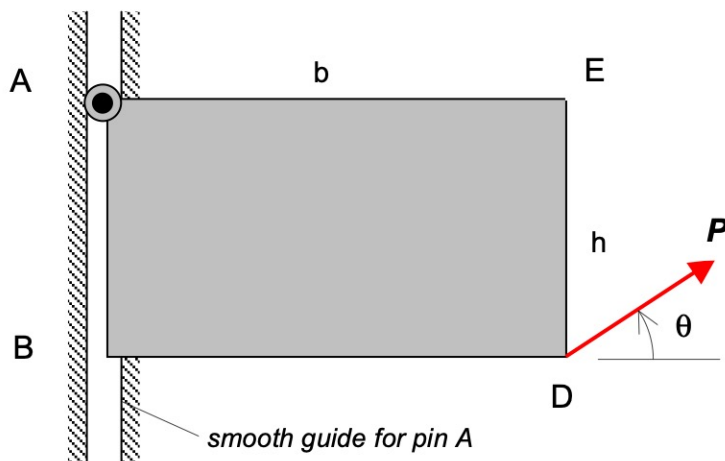


Use the following parameters in your analysis:  $M = 500$  kg,  $L = 2$  m and  $\theta = 40^\circ$ .

**Homework H5.C**

**Given:** A uniform rectangular plate of mass  $m$  is able to move on a smooth HORIZONTAL plane with corner A of the plate being constrained to move within a smooth guide in the plane of motion. The plate is initially at rest with edge AB aligned with the guide for A. A force  $P$  is applied at corner D.

**Find:** Determine the initial angular acceleration of the plate.

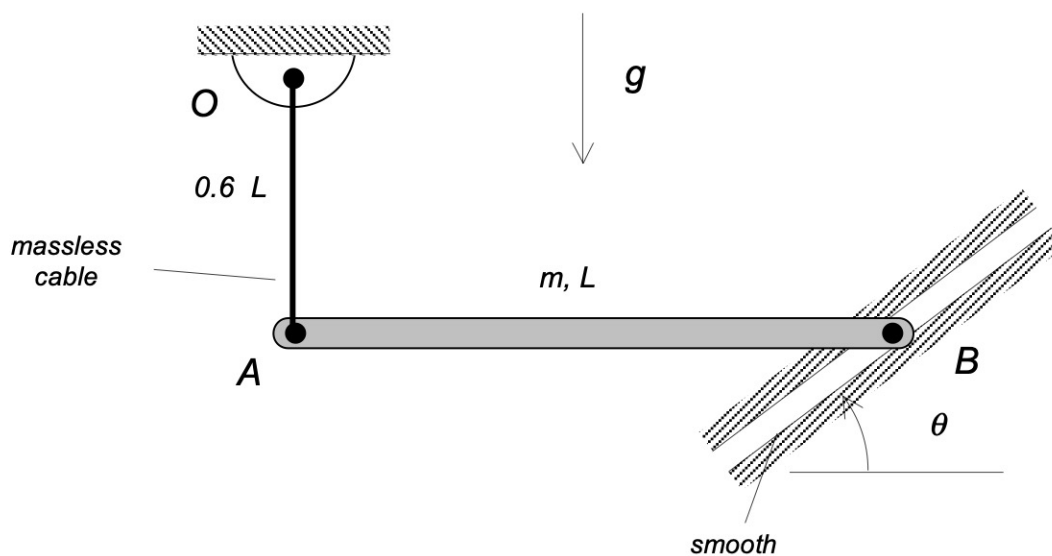


Use the following parameters in your analysis:  $m = 300$  kg,  $b = 5$  m,  $h = 2$  m,  $\theta = 36.87^\circ$  and  $P = 500$  N.

## Homework H5.D

**Given:** Link AB (having a mass of  $m$  with uniform mass distribution and length  $L$ ) is supported by a vertical cable OA (of length  $0.6L$  and negligible mass) at A, and end B is constrained to move within a smooth slot, as shown below. The bar is released from rest when it is horizontal.

**Find:** Determine the angular acceleration of AB on release.



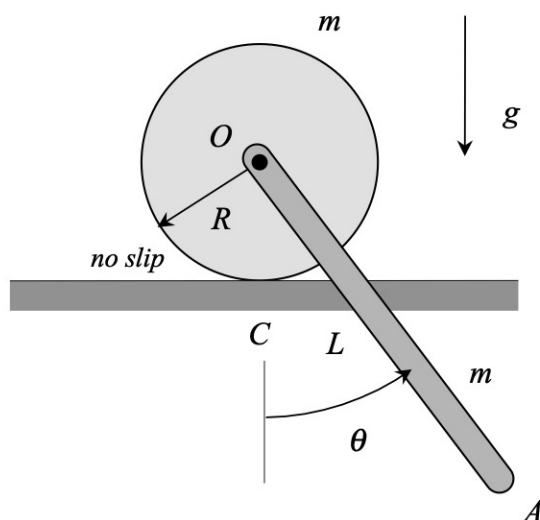
Use the following parameters in your analysis:  $m = 40$  kg,  $L = 2$  m, and  $\theta = 30^\circ$ .

**Homework H5.E**

**Given:** A thin homogeneous bar OA (of mass  $m$  and length  $L$ ) is pinned to a homogeneous disk (of mass  $m$  and radius  $R$ ) at the disk's center  $O$ . The disk is able to roll without slipping on a rough horizontal surface. The system is released from rest with the bar at an angle of  $\theta$  measured counterclockwise from vertical.

**Find:** For this problem:

- Determine the angular acceleration of the disk on release.
- Determine the angular acceleration of bar OA on release.



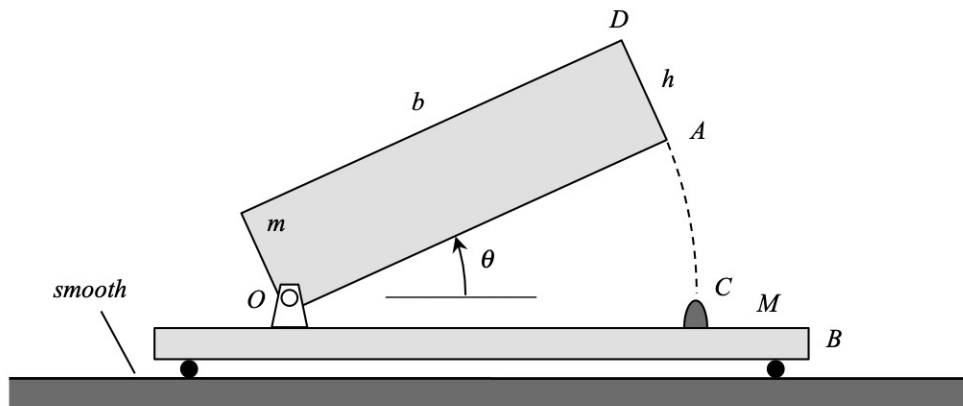
Use the following parameters in your analysis:  $m = 40$  kg,  $R = 0.5$  m,  $L = 2$  m and  $\theta = 50^\circ$ .

**Homework H5.F**

**Given:** A homogeneous plate of mass  $m$  is attached to cart B by a pin joint at corner O of the plate. The cart (having a mass of  $M$ ) is free to move along a smooth horizontal surface. The plate and cart are released from rest with the plate at angle of  $\theta$ .

**Find:**

- Determine the angular acceleration of the plate on release.
- Determine the acceleration of the cart on release.

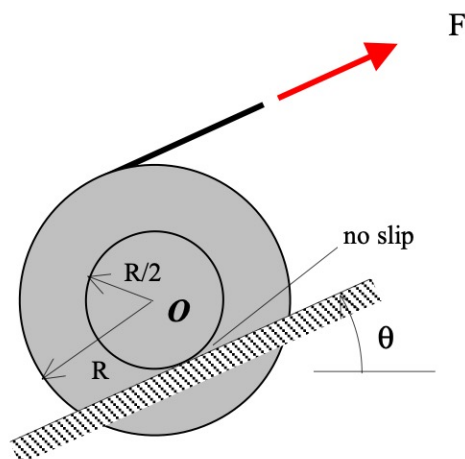


Use the following parameters in your analysis:  $m = 20$  kg,  $M = 40$  kg,  $h = 0.4$  m,  $b = 1.6$  m, and  $\theta = 40^\circ$ .

**Homework H5.G**

**Given:** The compound wheel shown below rolls without slipping up the incline on its hubs and is pulled by a constant force  $F$  applied to a cord wrapped around its outer rim. The wheel starts from rest, has a mass of  $m$ , and has a radius of gyration about its center of mass  $O$  of  $k_O$ . Assume that the cable does not slip on the wheel.

**Find:** Determine the angular velocity of the wheel after its center  $O$  has moved a distance of  $d$  up the incline.



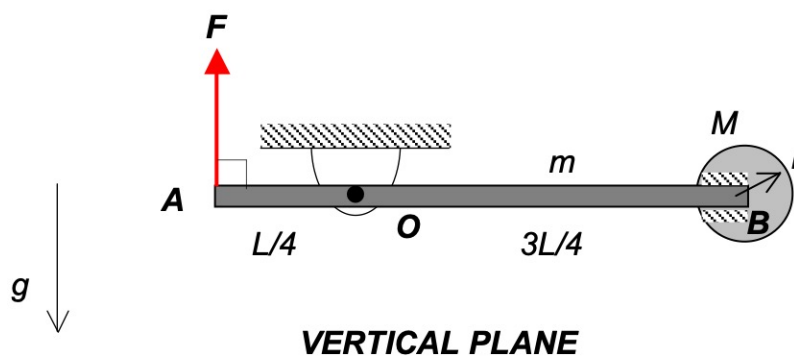
Use the following parameters in your analysis:  $m = 50$  kg,  $R = 0.4$  m,  $d = 3$  m,  $F = 300$  N,  $k_O = 0.15$  m, and  $\theta = 30^\circ$ .

**Homework H5.H**

**Given:** A thin, homogeneous bar AB (having a length of  $L$  and mass  $m$ ) is pinned to ground at point O (where O is at a distance of  $L/4$  from end A).. A homogeneous disk of mass  $M$  and radius  $r$  is welded to the bar AB with end B being at the center of the disk. A constant force  $F$  acts at end A in such a way that the line of action of  $F$  is always perpendicular to OA. The system is released from rest with AB being horizontal.

**Find:** For this problem:

- Determine the angular velocity of bar AB when AB reaches its vertical position; and
- Repeat your calculations in (a), but here treat the disk as a particle of mass  $M$ .

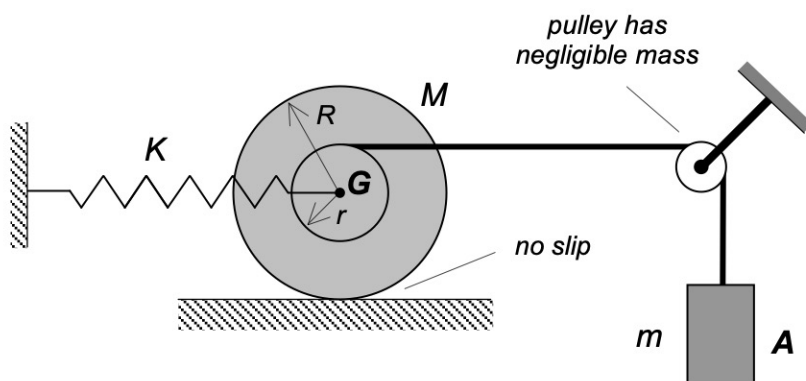


Use the following parameters in your analysis:  $L = 4$  m,  $r = 1$  m,  $m = 60$  kg,  $M = 70$  kg and  $F = 200$  N.

**Homework H5.1**

**Given:** A drum has a mass of  $M$ , outer radius of  $R$ , inner radius of  $r$ , a centroidal radius of gyration of  $k_G$  and a centroid at the geometric center  $G$ . A cable is wrapped around the inner radius of the drum and is connected to particle  $A$  having a mass of  $m$ . The cable is also wrapped over an ideal pulley. A spring of stiffness  $K$  is attached between  $G$  of the drum and ground. The system is released from rest with the spring unstretched.

**Find:** Determine the speed of  $A$  after it has dropped through a distance of  $d$ .

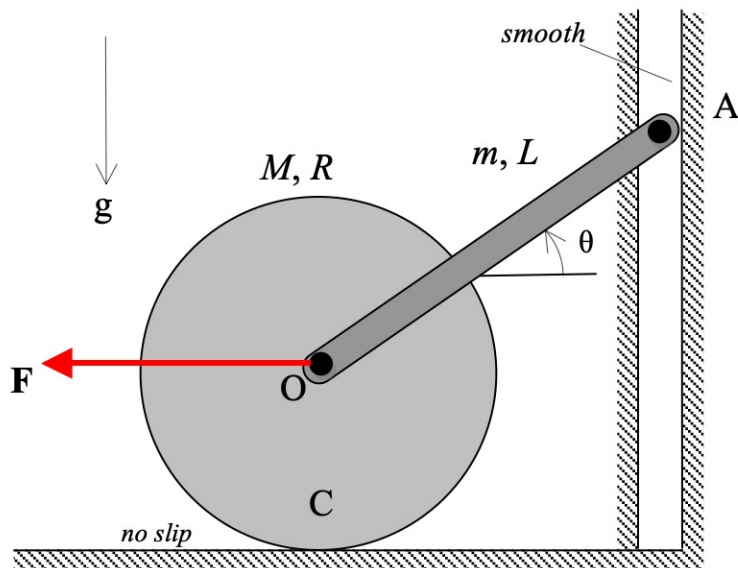


Use the following parameters in your analysis:  $M = 40$  kg,  $m = 20$  kg,  $R = 0.6$  m,  $r = 0.3$  m,  $k_G = 0.3$  m,  $K = 10$  N/m and  $d = 2$  m.

## Homework H5.J

**Given:** A homogeneous disk (mass  $M$  and radius  $R$ ) is attached to a homogeneous, thin rod OA (mass  $m$ ) at its center O. A constant force  $F$  is applied at O and point A is confined so that it moves along a smooth, vertical slot. The disk rolls without slipping. If the system is released from rest with  $\theta = \theta_1$ .

**Find:** Determine the velocity of point A when  $\theta = \theta_2$ .



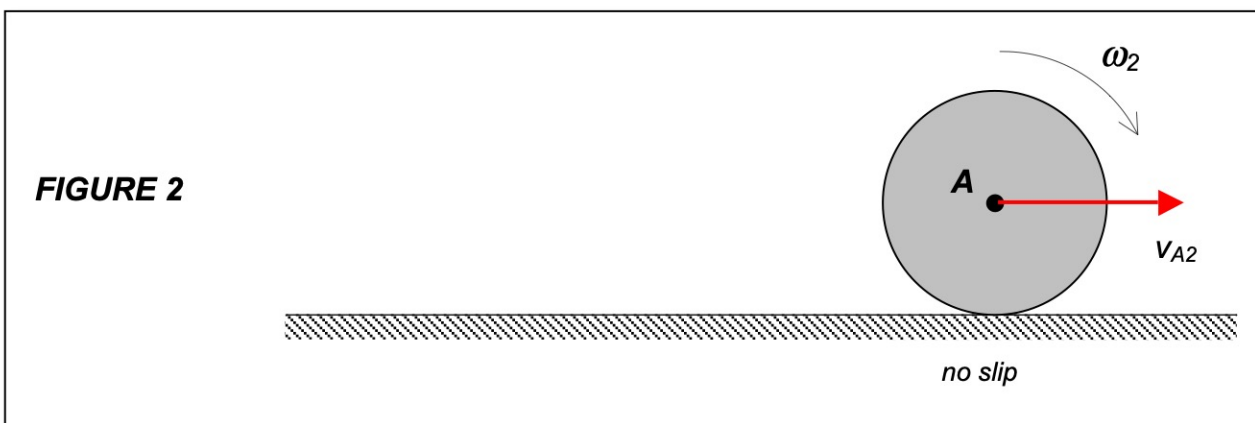
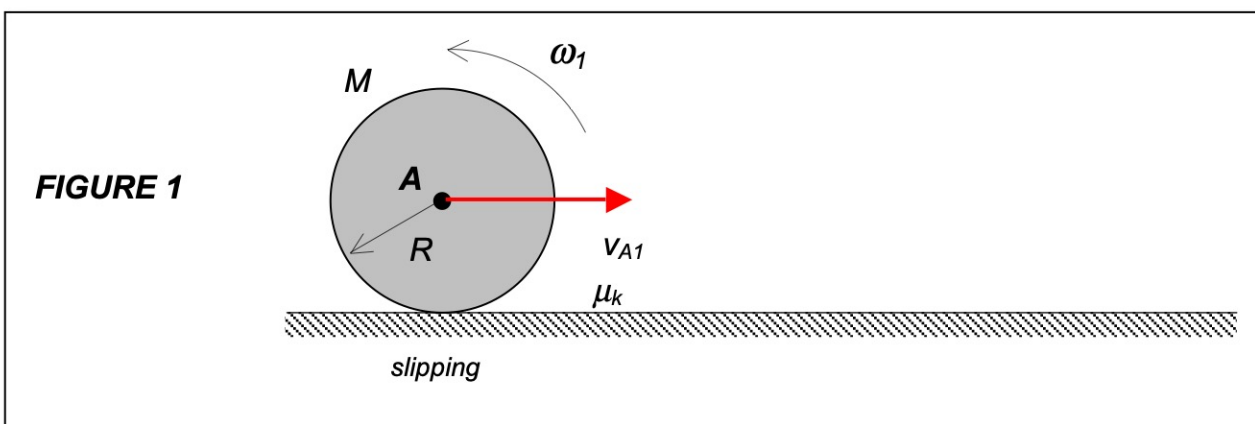
Use the following parameters in your analysis:  $M = 16$  kg,  $m = 8$  kg,  $R = 1.5$  m,  $L = 3$  m,  $F = 100$  N,  $\theta_1 = 53.13^\circ$  and  $\theta_2 = 0^\circ$ .

**HomeworkH5.K**

**Given:** A homogeneous disk (with mass  $M$  and outer radius of  $R$ ) is placed on a rough surface. When placed on this surface, the center of the disk  $A$  is moving to the right with a speed of  $v_{A1}$  and has a counterclockwise rotation rate of  $\omega_1$ , as shown in Figure 1 below. In Figure 2 below is shown the instant at which the disk ceases to slip as it continues to move on the horizontal surface.

**Find:** For this problem:

- Determine the speed of  $A$ ,  $v_{A2}$ , at the instant in Figure 2 when the disk ceases to slip on the horizontal surface; and
- Determine the elapsed time during the motion as the disk moves from the position in Figure 1 to the position in Figure 2.

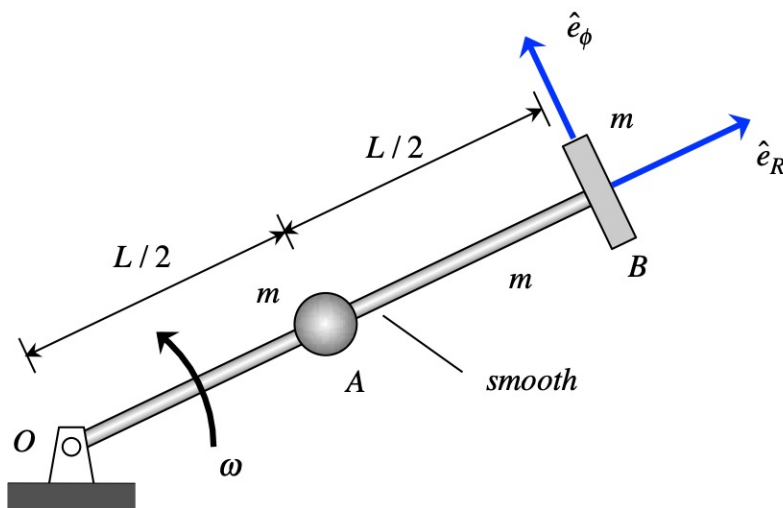


Use the following parameters in your analysis:  $M = 80$  kg,  $R = 0.75$  m,  $\mu_k = 0.4$ ,  $v_{A1} = 10$  m/s and  $\omega_1 = 9$  rad/s.

**Homework H5.L**

**Given:** Particle A, of mass  $m$ , is able to slide on a smooth homogeneous bar of length  $L$  and mass  $m$ . The bar is pinned to ground at end O and particle B (of mass  $m$ ) is rigidly attached to the other end. The bar is given an initial rotation rate of  $\omega_1$  when the A is at the midpoint of the bar, after which A slides outward on the bar. Eventually particle A impacts particle B, an impact having a coefficient of restitution of  $e$ .

**Find:** Determine the velocity of A immediately after impact. Express your answer in terms of its  $R$ - $\phi$  components.



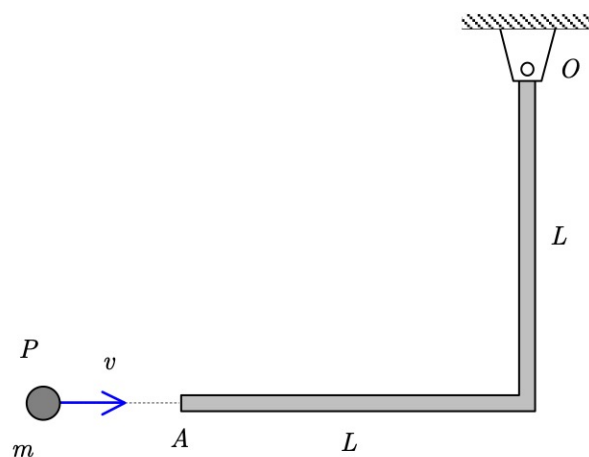
**HORIZONTAL plane**

Use the following parameters in your analysis:  $\omega_1 = 8 \text{ rad/s}$ ,  $e = 0.8$ ,  $m = 10 \text{ kg}$  and  $L = 0.5 \text{ m}$ .

**Homework H5.M**

**Given:** Particle P (of mass  $m$ ) strikes end A of a stationary homogeneous L-shaped bar (of mass  $M$ ) with a speed of  $v$ . The coefficient of restitution for the impact of P with end A of the bar is known to be  $e$ .

**Find:** Determine the angular speed of bar OA immediately after the impact.

***HORIZONTAL PLANE***

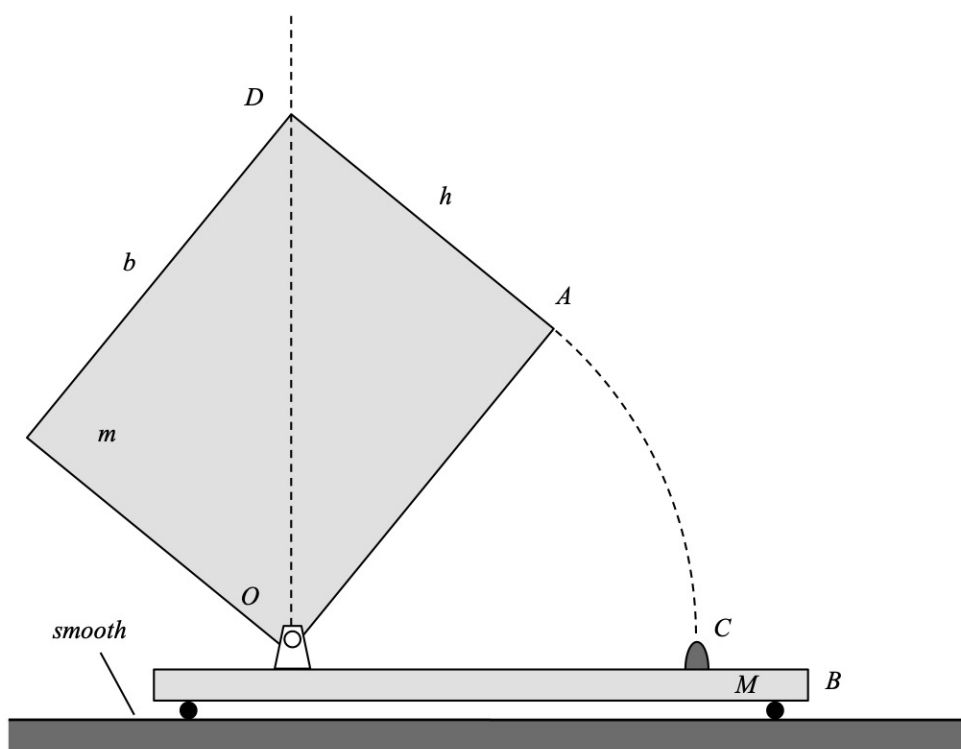
Use the following parameters in your analysis:  $M = 40$  kg,  $m = 20$  kg,  $L = 3$  m,  $v = 60$  m/s and  $e = 0.5$ .

## Homework H5.N

**Given:** A homogeneous rectangular plate of mass  $m$  is pinned to cart B at corner O, where cart B is constrained to move along a smooth horizontal surface. The system is released from rest with corner D displaced slightly to the right of a vertical line passing through the pin at O. As a result, the plate eventually impacts bumper C on the cart, with the coefficient of restitution between the plate and the bumper being  $e$ .

**Find:** For this problem:

- Determine the velocity of the center of mass of the plate immediately before the plate contacts the bumper C. Write your answer as a vector.
- Determine the velocity of the center of mass of the plate immediately after the plate contacts the bumper C. Write your answer as a vector.

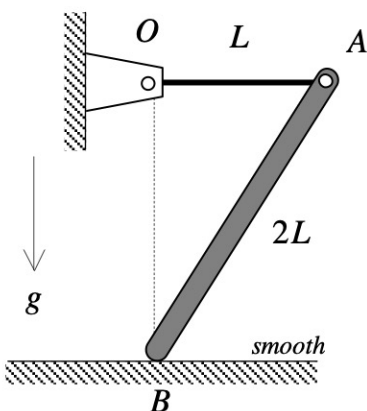


Use the following parameters in your analysis:  $m = 20$  kg,  $M = 50$  kg,  $b = 3$  m,  $h = 2$  m and  $e = 0$ .

**Homework H5.O**

**Given:** A thin homogeneous bar OA (of negligible mass and of length  $L$ ) is pinned to ground at O. The other end of the bar is pinned to a second thin homogeneous bar AB (of mass  $m$  and length  $2L$ ) at A. End B of bar AB is able to slide on a smooth, horizontal surface. At the instant of release from rest, OA is horizontal and end B of bar AB is located directly below O.

**Find:** Determine the angular acceleration of bar AB on release. Write your answer as a vector.

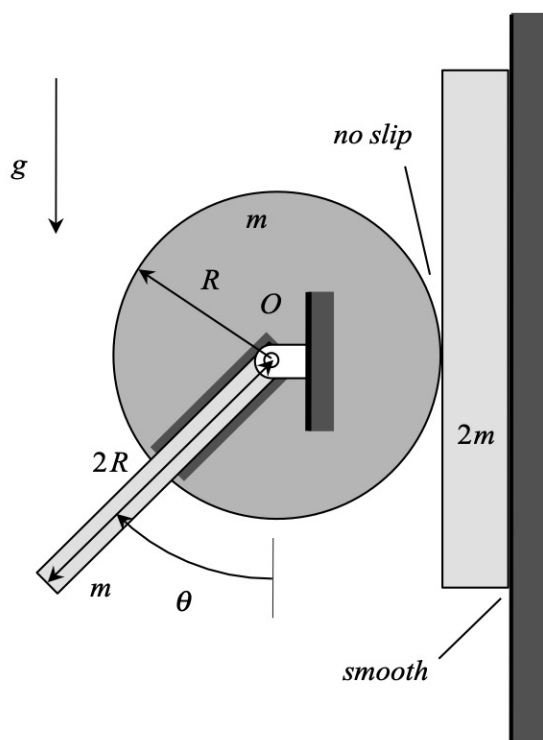


Use the following parameters in your analysis:  $m = 50$  kg and  $L = 3$  m.

**Homework H5.P**

**Given:** A homogeneous disk of mass  $m$  and outer radius  $R$  is able to rotate about a frictionless bearing at its center  $O$ . A thin, homogeneous bar of mass  $m$  and length  $2R$  is welded to the disk with the bar aligned with a radial direction on the disk and one end at  $O$ . A block of mass  $2m$  is able to slide along a smooth, vertical wall, with the block being in no-slip contact with the outer surface of the disk, as shown in the figure. The system is released from rest with  $\theta = 90^\circ$ .

**Find:** Determine the angular velocity of the disk after the disk has rotated through an additional angle of  $90^\circ$  after release.



Use the following parameters in your analysis:  $m = 5 \text{ kg}$  and  $R = 0.1 \text{ m}$ .

## Chapter 6

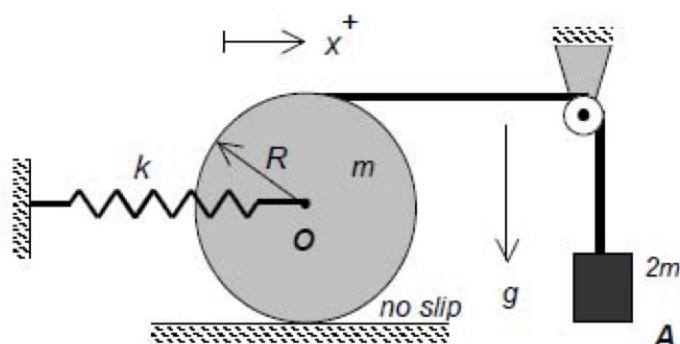
# Vibrations Homework



**Homework H6.A**

**Given:** A homogeneous drum (of mass  $m$  and outer radius  $R$ ) rolls without slip on a rough horizontal surface. A spring of stiffness  $k$  is attached between the center  $O$  of the drum and ground such that the spring remains horizontal at all times. Block A (of mass  $2m$ ) is connected to an inextensible cable that is wrapped around the outer radius of the drum, as shown in the figure. Let  $x$  represent the motion of  $O$  (measured positively to the right), and let the spring be unstretched when  $x = 0$ . Assume that the cable does not go slack at any time.

**Find:** For this problem, derive the single differential equation of motion (EOM) for the system in terms of the coordinate  $x$ , its time derivatives, and, at most, the following parameters:  $g$ ,  $m$ ,  $R$ , and  $k$ .

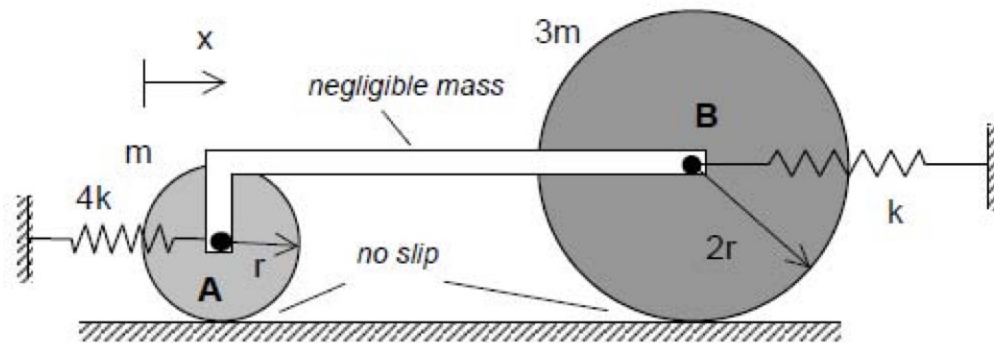


## Homework H6.B

**Given:** Two homogeneous wheels, having masses of  $m$  and  $3m$  and outer radii of  $r$  and  $2r$ , respectively, are connected by a rigid, L-shaped bar, where the mass of the bar is negligible compared to the mass of the wheels. The two wheels roll without slipping on a rough, horizontal surface. Two springs, having stiffness of  $4k$  and  $k$ , connect points A and B, respectively, to ground, where A and B are the centers of the two wheels. The coordinate  $x$  gives the position of Point A measured from the position at which the two springs are unstretched, with  $x$  being measured positive to the right (as shown below).

**Find:** For this problem:

- Derive the single differential equation of motion (EOM) for the system in terms of the coordinate  $x$ ; and
- Determine the natural frequency of free oscillation for the system.



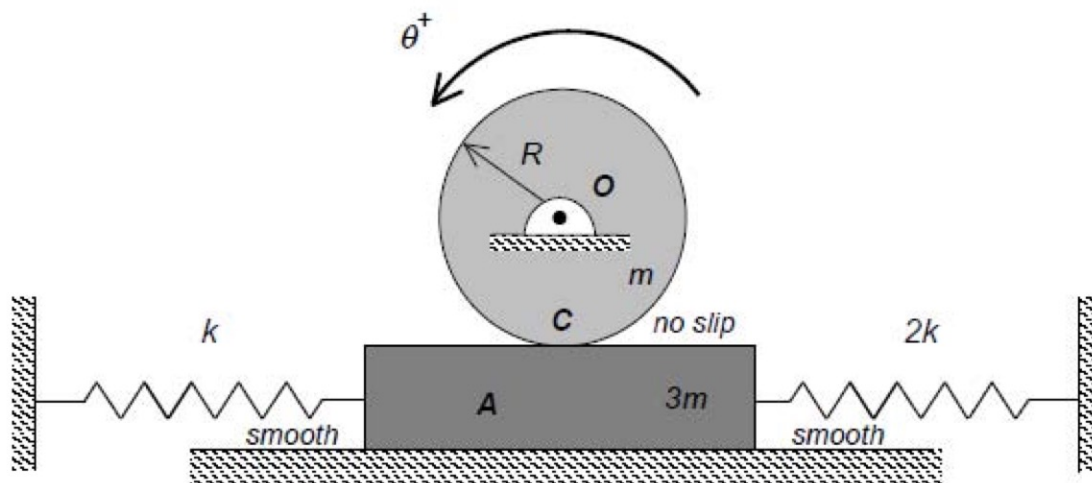
Use the following parameters in your analysis:  $m = 20$  kg,  $k = 500$  N/m, and  $r = 0.5$  m.

## Homework H6.C

**Given:** A homogeneous drum having a mass of  $m$  and outer radius  $R$  is pinned to ground at its center  $O$ . This drum is in geared contact with block A. Block A, having a mass of  $3m$ , is able to slide along a smooth horizontal surface and in such a way that the block does not slip in its contact with drum. Two springs, having stiffnesses of  $k$  and  $2k$ , are attached between block A and ground, as shown in the figure below. Let  $\theta$  represent the rotation of the drum with  $\theta$  being measured positive counterclockwise. When  $\theta = 0$  rad the springs are unstretched.

**Find:** For this problem:

- Draw individual free body diagrams of the drum and block;
- Derive the single differential equation of motion (EOM) for the system in terms of the coordinate  $\theta$ , its time derivatives, and, at most, the following parameters:  $m$ ,  $R$ , and  $k$ ;
- Based on the EOM derived above, determine the natural frequency of the system. Express the answer in both rad/s and Hz; and
- Assuming the system is released when the springs are unstretched with  $\dot{\theta}(0) = \omega_0$  (CCW), determine the response of the system  $\theta(t)$ , for  $t > 0$ .



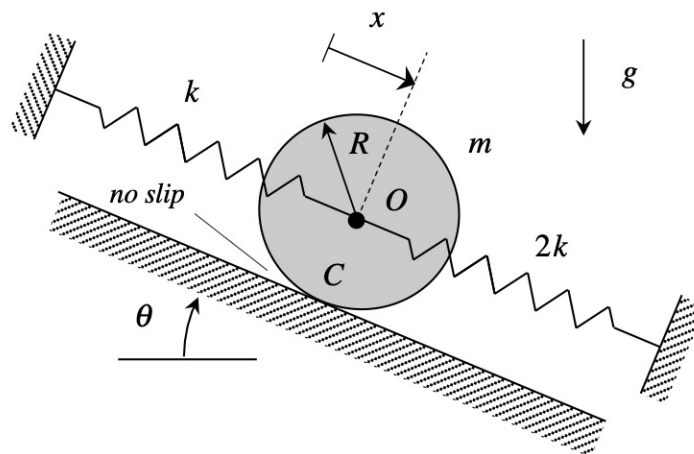
Use the following parameters in your analysis:  $m = 0.75$  kg,  $k = 5500$  N/m,  $R = 0.25$  m, and  $\omega_0 = 0.5$  rad/s.

**Homework H6.D**

**Given:** A homogeneous disk of mass  $m$  and outer radius  $R$  is able to roll without slipping on a rough, inclined surface. The center of the disk  $O$  is attached to ground with two springs of stiffnesses  $k$  and  $2k$ , as shown in the figure. Let  $x$  represent the motion of  $O$  along the incline as the disk rolls, where  $x = 0$  when the springs are unstretched.

**Find:** For this problem:

- Derive the dynamical equation of motion (EOM) of the system in terms of the coordinate  $x$ ;
- From the EOM, determine the static displacement of  $O$ ,  $x_{st}$ ;
- Rewrite the EOM of the system in terms of the variable  $z = x - x_{st}$ , where  $z$  represents the position of  $O$  relative to its static equilibrium position; and,
- Determine the natural frequency of the system in terms of, at most, the given parameters of the problem:  $m$ ,  $k$  and  $R$ .

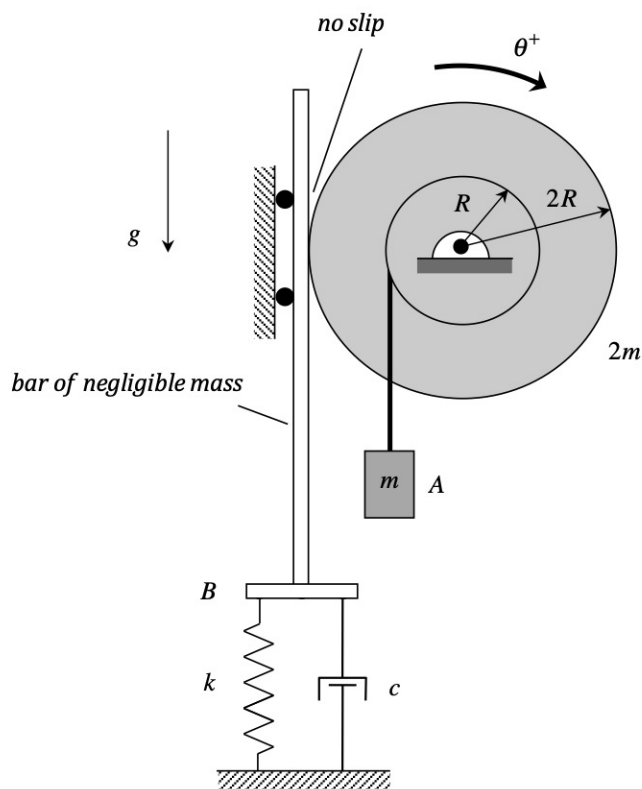


## Homework H6.E

**Given:** A stepped drum (of mass  $2m$ ) is pinned to ground at its center, with the inner and outer radii of the drum given by  $R$  and  $2R$ , respectively. The radius of gyration of the drum about its center of mass is given by  $k_O$ . A cable is wrapped around the inner radius of the drum with the other end of the cable connected to particle A that has a mass of  $m$ . A rigid bar (having negligible mass) is in no-slip contact with the outer radius of the drum, with the bar being attached to connector B. In turn, B is connected to a grounded spring of stiffness  $k$  and a grounded dashpot having a damping coefficient  $c$ , as shown in the figure. As the system moves, the cable is known to not slip on the drum nor does the cable go slack. Let  $\theta$  represent the rotation of the drum, with  $\theta$  being defined positive in the clockwise direction. The mass of B is to be considered to be negligible.

**Find:** For this problem:

- Derive the dynamical equation of motion (EOM) of the system in terms of the coordinate  $\theta$ ;
- From the EOM, determine the static rotation of the drum,  $\theta_{st}$ ;
- Rewrite the EOM of the system in terms of the variable  $z = \theta - \theta_{st}$ , where  $z$  represents the rotation of the drum relative to its static equilibrium rotation;
- Determine the natural frequency of the system in terms of, at most, the given parameters of the problem:  $m$ ,  $R$ ,  $k$  and  $k_O$ ; and,
- Determine the ratio of the parameters  $c/\sqrt{km}$  that is required for critical damping to exist in the response of the system. Use  $R/k_O = 1$ .

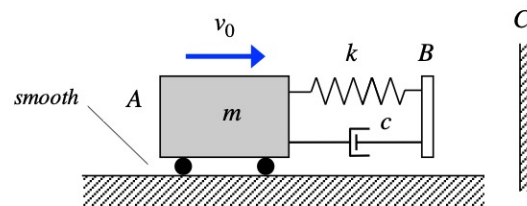


**Homework H6.F**

**Given:** A block of mass  $m$  is attached to a spring of stiffness  $k$  and a dashpot with a damping coefficient  $c$ , with the opposite ends of the spring and dashpot joined to connector B, where the mass of B can be considered to be negligible. A is initially traveling to the right with a speed of  $v_0$  when it strikes a stationary wall at C. B immediately sticks to C after impact. Let  $x$  represent the motion of A after B has stuck to the wall, with  $x$  being measured positively to the right.

**Find:** For this problem:

- Derive the dynamical equation of motion (EOM) of the system in terms of the coordinate  $x$  for motion occurring after B sticks to C;
- Determine the undamped natural frequency  $\omega_n$ , the damping ratio  $\zeta$  and the damped natural frequency  $\omega_d$  for the system; and,
- Determine the maximum compression of the spring after B impacts and sticks to C.

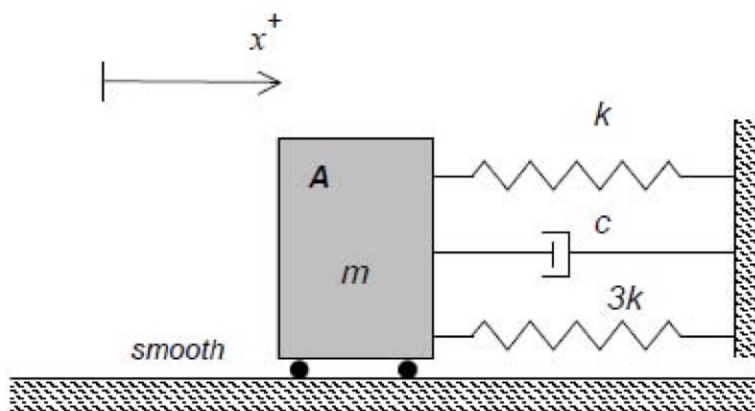


**Homework H6.G**

**Given:** Block A, having a mass of  $m$  is able to slide along a smooth horizontal surface. Two springs, having stiffness of  $k$  and  $3k$ , are connected between block A and ground. A dashpot with damping constant  $c$  is also connected between A and ground, as shown in the figure below. Let  $x$  represent the motion of block A measured positively to the right. When  $x = 0$  m, the springs are unstretched.

**Find:** For this problem:

- Draw a free body diagram of block A;
- Derive the single equation of motion for the system in terms of the coordinate  $x$ , its derivatives, and, at most, the parameters  $m$ ,  $c$ , and  $k$ ;
- Determine the numerical values for: the undamped natural frequency  $\omega_n$ , the damping ratio  $\zeta$ , and the damped natural frequency  $\omega_d$ ; and
- Determine the response of the system  $x(t)$  for  $t > 0$ , assuming the system is released with  $\dot{x}(0) = v_0$  when the springs are unstretched.



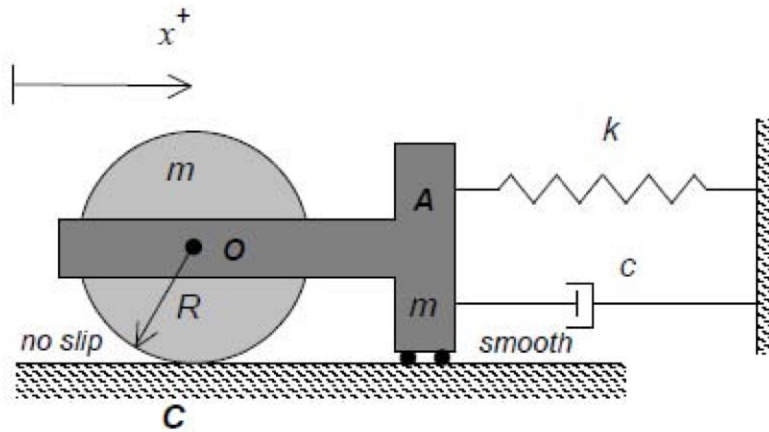
Use the following parameters in your analysis:  $m = 24$  kg,  $k = 600$  N/m,  $c = 10$  Ns/m, and  $v_0 = 4$  m/s.

## Homework H6.H

**Given:** A homogeneous wheel of mass  $m$  and outer radius  $R$  rolls without slipping on a horizontal surface. Block A (also having a mass of  $m$ ) is pinned to the center  $O$  of the wheel and is able to slide without friction on the same horizontal surface. A spring (of stiffness  $k$ ) and a dashpot (of damping constant  $c$ ) are connected between block A and ground. Let  $x$  represent the motion of block A measured positively to the right. When  $x = 0$  m, the spring is unstretched.

**Find:** For this problem:

- Draw individual free body diagrams for block A and the wheel;
- Derive the single differential equation of motion for the system in terms of the coordinate  $x$ , its time derivatives, and the following parameters:  $m$ ,  $R$ ,  $c$ , and  $k$ ;
- Determine numerical values for: the undamped natural frequency  $\omega_n$ , the damping ratio  $\zeta$ , and the damped natural frequency  $\omega_d$ ; and
- Determine the response of the system  $x(t)$  for  $t > 0$ , assuming the system is released when the springs are unstretched with  $\dot{x}(0) = v_0$ .



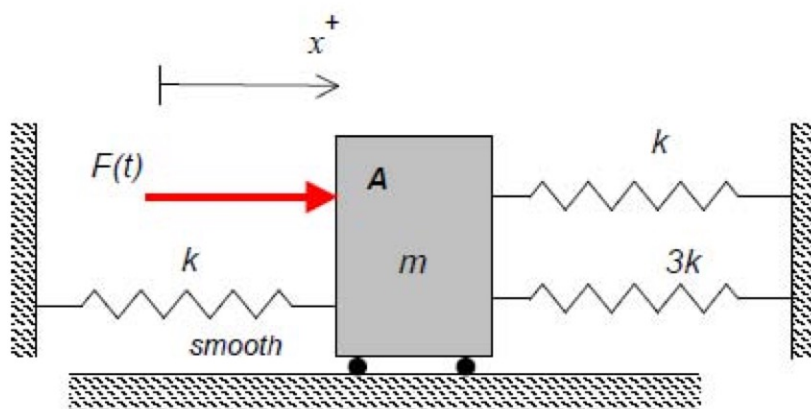
Use the following parameters in your analysis:  $m = 4$  kg,  $k = 2250$  N/m,  $R = 0.1$  m,  $c = 60$  kg/s, and  $v_0 = 8$  m/s.

**Homework H6.1**

**Given:** Block A, having a mass of  $m$ , is able to slide along a smooth horizontal surface. Three springs are connected between block A and ground, as shown in the figure below. A force  $F(t) = F_0 \sin \omega t$  acts horizontally on block A. Let  $x$  represent the motion of block A measured positively to the right, and let  $x = 0$  m designate the state at which the springs are unstretched.

**Find:** For this problem:

- Draw a free body diagram of block A;
- Derive the single differential equation of motion for the system in terms of the coordinate  $x$ ; and
- Derive the particular solution  $x_p(t)$  for the equation of motion derived above.



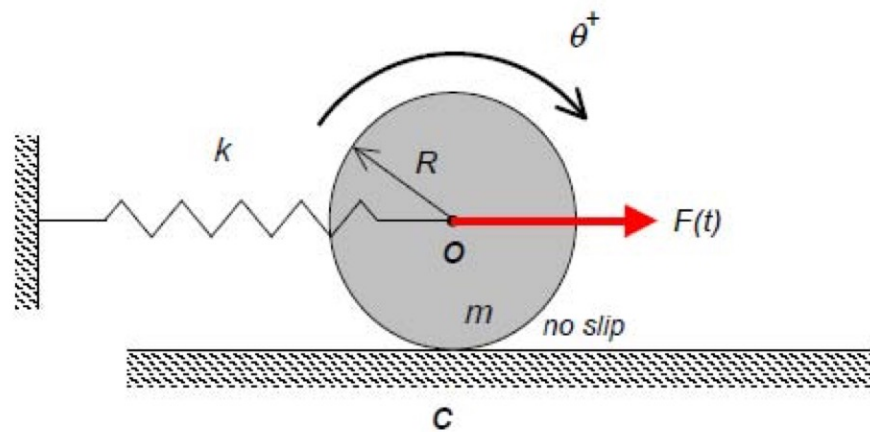
Use the following parameters in your analysis:  $m = 10$  kg,  $k = 3200$  N/m,  $F_0 = 150$  N, and  $\omega = 15$  rad/s.

**Homework H6.J**

**Given:** A homogeneous disk, having a mass of  $m$  and outer radius of  $R$ , rolls without slipping on a rough horizontal surface. A spring of stiffness  $k$  is connected between the center  $O$  of the disk and ground on the left side of the disk. A force  $F(t) = F_0 \sin \omega t$  acts horizontally at point  $O$  on the disk. Let  $\theta$  represent the rotation of the disk measured positive clockwise, and let the spring be unstretched when  $\theta = 0$  rad.

**Find:** For this problem:

- Draw a free body diagram of the disk;
- Derive the single differential equation of motion for the system in terms of the coordinate  $\theta$ ; and
- Derive the particular solution  $\theta_p(t)$  for the equation of motion obtained above.



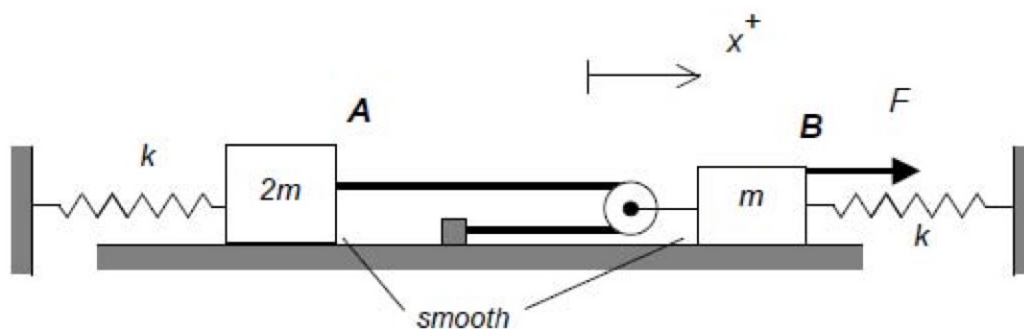
Use the following parameters in your analysis:  $m = 2$  kg,  $k = 4800$  N/m,  $R = 0.5$  m,  $F_0 = 50$  N, and  $\omega = 60$  rad/s.

**Homework H6.K**

**Given:** Blocks A and B (having masses of  $m$  and  $2m$ , respectively) are connected by a cable-pulley system as shown below. Two springs, each of stiffness  $k$ , are attached between blocks A and B and ground, as shown below. A horizontal force  $F$  is applied to B. The mass of the pulley is negligible, and the cable remains taut during all motion. Let  $x$  describe the position of B, and let  $x = 0$  correspond to the state at which the springs are unstretched.

**Find:** For this problem:

- Draw a free body diagram for each block; and
- Derive the differential equation of motion for the system in terms of the coordinate  $x$ .
- Derive the particular solution for  $x(t)$ .

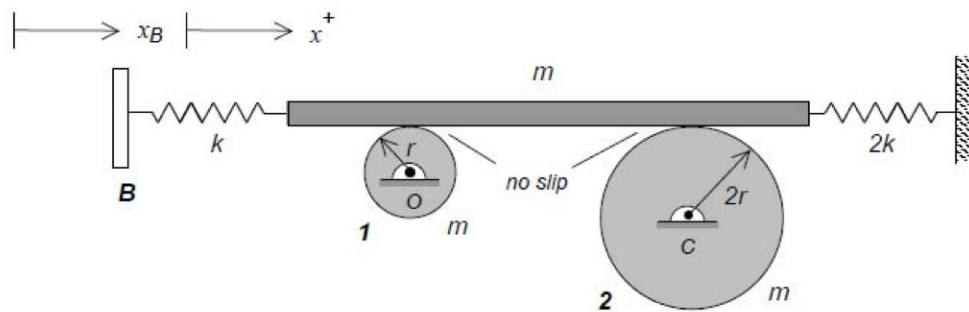


**Homework H6.L**

**Given:** A system is made up of two homogeneous disks and a bar, each of which has a mass of  $m$ . The two disks are pinned to ground at their centroids, O and C, as shown below. The bar is able to translate without slipping at its contact points with the disks. A spring of stiffness  $2k$  is attached between the right end of the bar and the fixed wall. A second spring, of stiffness  $k$ , is attached between the left end of the bar and block B. Block B is given a prescribed motion of  $x_B = b \sin \omega t$ . Let  $x$  represent the translation of the bar, with  $x = x_B = 0$  m corresponding to the state where the springs are unstretched.

**Find:** For this problem:

- Draw a free body diagram for each disk and the block;
- Derive the single differential equation of motion for the system in terms of the coordinate  $x$ ; and
- Determine the natural frequency of the system.



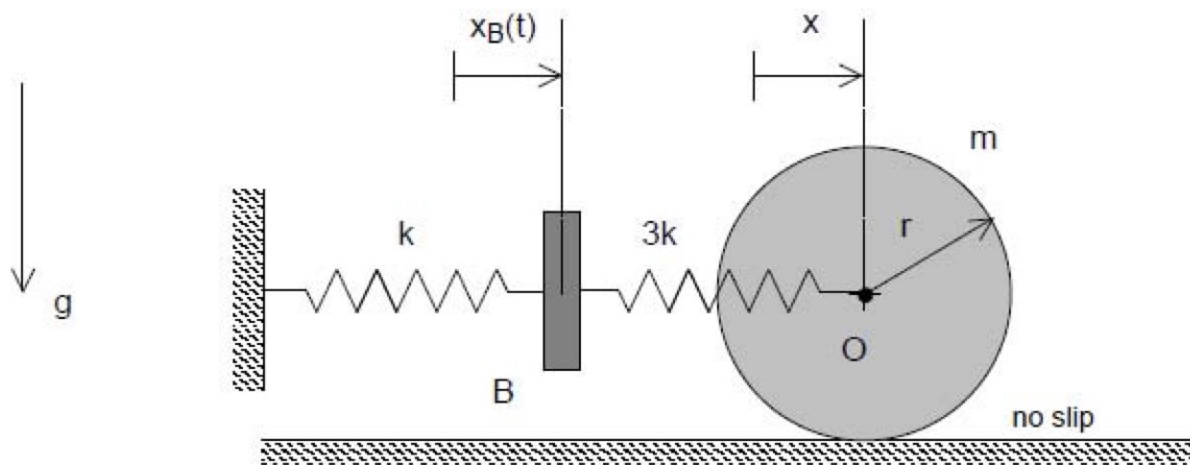
Use the following parameters in your analysis:  $m = 20$  kg and  $k = 1000$  N/m.

**Homework H6.M**

**Given:** A homogeneous disk (of mass  $m$  and outer radius  $R$ ) rolls without slipping on a rough, horizontal surface. A spring (of stiffness  $3k$ ) is attached between the center  $O$  of the disk and a moveable base  $B$ . A second spring (of stiffness  $k$ ) is attached between point  $B$  and ground. Base  $B$  is given a prescribed motion of  $x_B(t) = b \sin \Omega t$ . The coordinates  $x$  and  $x_B$  are both zero when the springs are unstretched.

**Find:** For this problem:

- Derive the differential equation of motion for the disk in terms of the coordinate  $x$ ;
- Determine the numerical value for the natural frequency of this system;
- Determine the numerical value of  $X$ , if the particular solution of the system is written as  $x_p(t) = X \sin \Omega t$ ; and
- Determine if the disk is moving in-phase or out-of-phase with the base  $B$ .



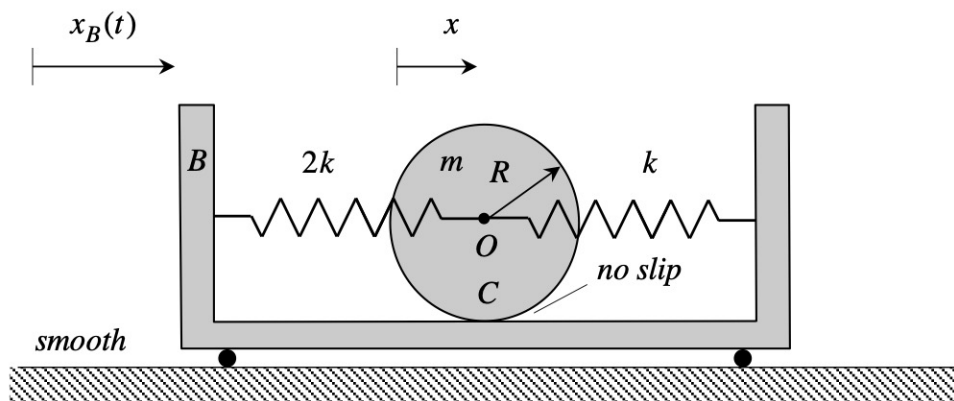
Use the following parameters in your analysis:  $m = 80$  kg,  $k = 640$  N/m,  $r = 0.25$  m,  $b = 0.16$  m, and  $\Omega = 10$  rad/s.

**Homework H6.N**

**Given:** A homogeneous disk of mass  $m$  and with an outer radius of  $R$  rolls without slipping on a rough horizontal surface on cart B. The disk is connected at its center O with two springs (of stiffnesses  $2k$  and  $k$ ) to B, as shown in the figure. The base B is given a prescribed motion of  $x_B(t) = b \sin \Omega t$ . Let  $x$  measure the position of O from its position when the springs are unstretched.

**Find:** For this problem:

- Derive the differential equation of motion (EOM) for the system in terms of the coordinate  $x$ ;
- Determine the natural frequency  $\omega_n$  of the system;
- Determine the amplitude of the motion described by particular solution of the EOM; and,
- For the motion found in (c) above, does O move in phase or out of phase with B?



Use the following parameters in your analysis:  $m = 200$  kg,  $k = 10,000$  N/m,  $b = 0.1$  m, and  $\Omega = 15$  rad/s.