ME 274 - Examination No. 2 **Name** SOLUTION

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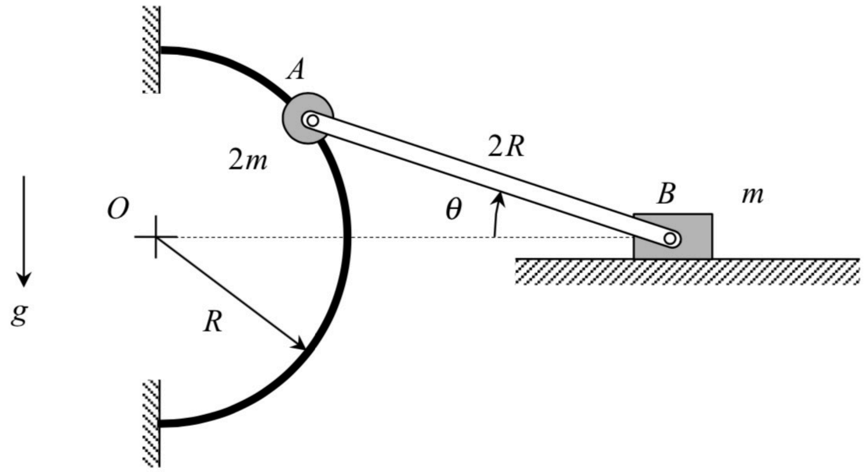
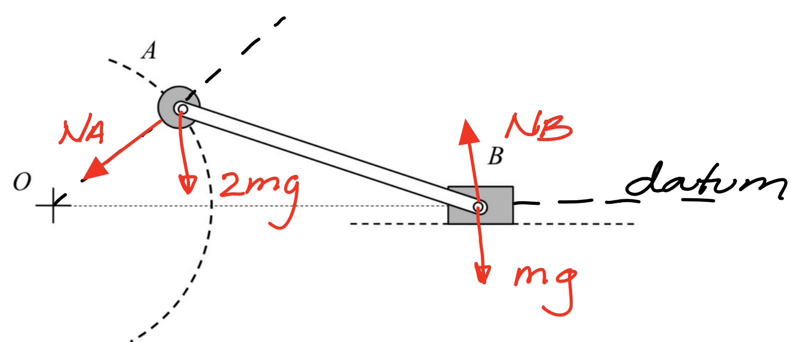
PROBLEM NO. 1 – 20 points



Given: Particles A and B (having masses of $2m$ and m, respectively) are connected by a rod having negligible mass and length of $2R$. Particle A is constrained to move on a stationary, vertically-oriented circular guide with a radius of R, whereas particle B moves along a straight, horizontal surface. Consider all surfaces to be smooth. The system is released from rest with rod AB oriented at an angle of $\theta = \theta_1$ above the horizontal (Position 1).

Find: It is desired to know the speeds of particles A and B, v_{A2} and v_{B2}, when the system has reached a position of $\theta = \theta_2 = 0$ (Position 2). To this end, do the following:

STEP 1: Draw the free body diagram of the system made up of A, B and the rod for an arbitrary angle θ. Use the sketch of the system provided below.



STEP 2:

Part a): Provide a short argument, in words, that supports the claim that energy is conserved for the system defined in Step 1 above for motion between Positions 1 and 2.

- N_A & N_B do no work since ⊥ to paths of A & B
- Weights do work, however, that work is included in potential

→ No non-conservative work done ⇒ energy conserved

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PROBLEM NO. 1 (continued)

Part b): Write down the work-energy equation for this system corresponding to motion of the system between Positions 1 and 2. Leave this equation in terms of v_{A2} and v_{B2}, and the parameters defined above.

$U_{1\to 2}^{(nc)} = 0$; see part a)

$T_1 = 0$

$T_2 = \frac{1}{2}(2m)v_{A2}^2 + \frac{1}{2}m v_B^2$

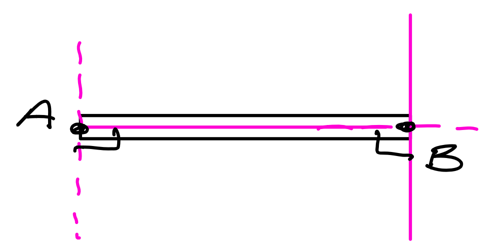
$V_1 = 2(mg)(2R\sin\theta_1) = 4mgR\sin\theta_1$

$V_2 = 0$; at datum

$\cancel{T_1}^{0} + V_1 + \cancel{U_{1\to 2}^{(nc)}}^{0} = T_2 + \cancel{V_2}^{0}$

STEP 3: Develop the kinematics equation(s) needed to solve this problem for Position 2. *HINT: Consider the location for the instant center for link AB at Position 2.*

$B = IC_{AB} \Rightarrow \boxed{v_{B_2} = 0}$



STEP 4: Solve the above equations for v_{A2} and v_{B2}. Leave your answers in terms of the variables defined in the problem statement above.

$4\cancel{m}gR\sin\theta_1 = \cancel{m}v_{A2}^2 + \frac{1}{2}m\cancel{v_{B2}^2}^{0}$

$\hookrightarrow v_{A2} = 2\sqrt{gR\sin\theta_1}$

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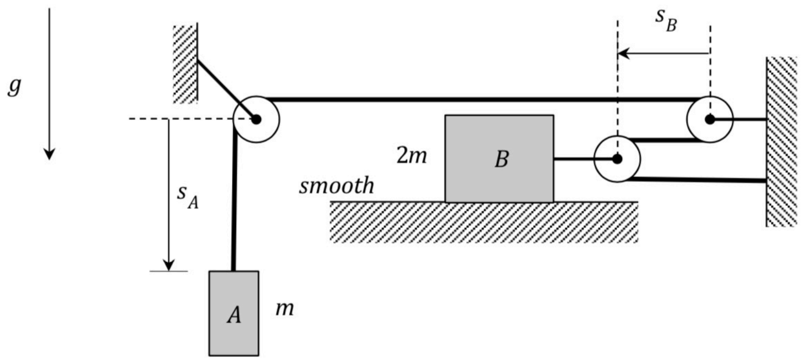
PROBLEM NO. 2 – 20 points



Given: Blocks A and B (having masses of m and $2m$, respectively) are connected by an inextensible cable through the cable-pulley system shown above. Consider the pulleys to be ideal (massless and frictionless). Block B is constrained to move along a smooth, horizontal surface, with block A being connected to the free end of the cable.

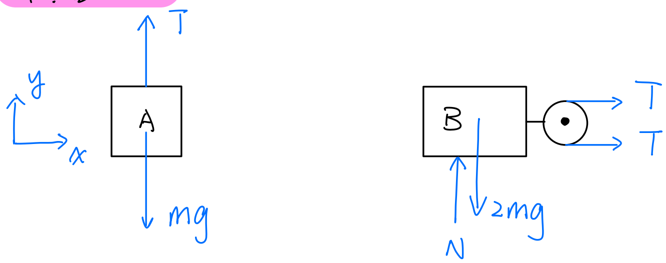
Find: For this problem:

a) Draw an *individual* free body diagram (FBD) for each block. Use the diagrams below for your FBDs.

b) Determine the acceleration of each block and the tension in the cable when the system is released from rest. Express your answers in terms of, at most, m and g. Write your acceleration answers as vectors.

Problem No. 2 solution

F.B.D.



Given m, g.

Find $\vec{a_A}$, $\vec{a_B}$, T

Kinetics:

A_y: $T - mg = ma_A$ ①

B_x: $2T = 2ma_B$ ②

② $\Rightarrow$ $T = ma_B$ ③

①, ③ $\Rightarrow$ $a_B = a_A + g$ ④

Kinematics:

Rope is constant length: $L = S_A + S_B + S_B +$ Constant

$\Rightarrow 0 = \ddot{S}_A + 2\ddot{S}_B$

$\Rightarrow \ddot{S}_A = -2\ddot{S}_B$

Since $a_A = -\ddot{S}_A$, $a_B = -\ddot{S}_B$, $a_A = -2a_B$ ⑤

④ + ⑤ $\Rightarrow$ $a_B = -2a_B + g$ $\Rightarrow$ $a_B = \frac{1}{3}g$. $\Rightarrow$ $a_A = -\frac{2}{3}g$, $T = \frac{1}{3}mg$

$$\boxed{\begin{aligned} \vec{a_A} &= -\tfrac{2}{3}g\,\hat{j} \\ \vec{a_B} &= \tfrac{1}{3}g\,\hat{i} \\ T &= \tfrac{1}{3}mg \end{aligned}}$$

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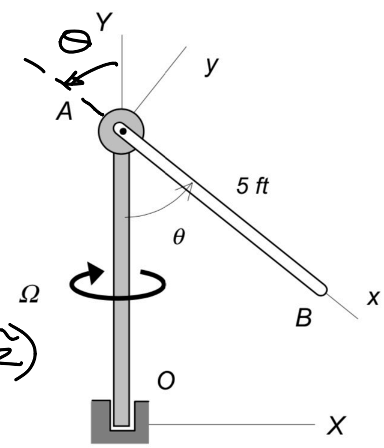
NOTE: *You are NOT asked to provide justification for your answers here in Problem 3. A correct response will receive full credit. Any work provided will not be graded, only the final answer.*

PROBLEM NO. 3 - PART A

Shaft OA rotates about a fixed vertical axis at a *constant* rate of Ω. Arm AB is pinned to OA at A, and is being raised at a *constant* rate of $\dot{\theta}$. An *observer and the xyz axes are attached to arm AB*, whereas *XYZ* are a set of *stationary* axes. The following equation is to be used to calculate the acceleration of point B using the observer defined here:

$$\vec{a}_B = \vec{a}_A + \left(\vec{a}_{B/A}\right)_{rel} + \vec{\alpha}\times\vec{r}_{B/A} + 2\vec{\omega}\times\left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}_{B/A}\right)$$

Complete the expressions for the following terms in this acceleration equation for the position of $\theta = 36.87°$:



3A.1: 2 points – Write the following in terms of its *xyz components*

$\vec{\omega} = -\Omega\hat{J} + \dot\theta\hat{k}$

$= -\Omega(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) + \dot\theta\hat{k}$

3A.2: 2 points – Write the following in terms of its *xyz components*

$\vec{\alpha} = -\cancelto{0}{\dot\Omega}\hat{J} - \Omega\cancelto{0}{\dot{\hat{J}}} + \cancelto{0}{\ddot\theta}\hat{k} + \dot\theta\dot{\hat{k}} = \dot\theta(\vec{\omega}\times\hat{k})$

$= \left[\dot\theta\left[\Omega\cos\theta\,\hat{i} - \Omega\sin\theta\,\hat{j}\right] + \dot\theta\hat{k}\right]\times\hat{k}$

$= -\dot\theta\Omega\sin\theta\,\hat{i} - \dot\theta\Omega\cos\theta\,\hat{j}$

$\hat{J} = -\cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

3A.3: 2 points – Write the following in terms of its *xyz components*

$\vec{a}_A = \vec{0}$

3A.4: 2 points – Write the following in terms of its *xyz components*

$\left(\vec{v}_{B/A}\right)_{rel} = \vec{0}$

3A.5: 2 points – Write the following in terms of its *xyz components*

$\left(\vec{a}_{B/A}\right)_{rel} = \vec{0}$

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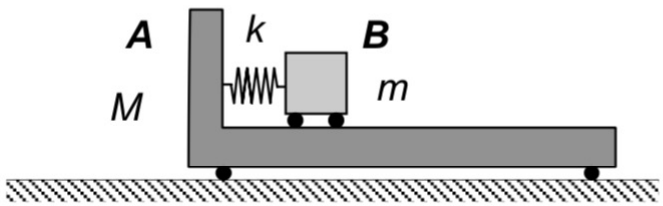
PROBLEM NO. 3 - PART B



An L-shaped block A (having a mass of M) is constrained to move along a horizontal surface. A second block B (having a mass of m) moves along the top surface of A, as shown in the figure. A spring of stiffness k and unstretched length L_0 is attached between A and B. The system is released from rest with the distance between A and B being $L_0/2$. Consider all surfaces to be smooth. During the motion of the system after release:

B.1: 1 point – The <u>energy</u> for *block A alone* is conserved – *TRUE* or *FALSE* F_{sp} does work

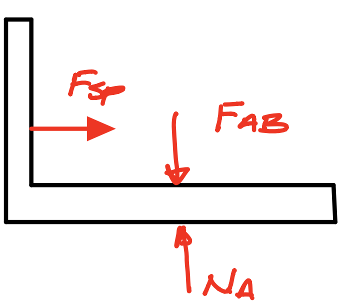
B.2: 1 point – The <u>energy</u> for *block B alone* is conserved – *TRUE* or *FALSE* F_{sp} does work

B.3: 1 point – The <u>energy</u> for *blocks A and B together* is conserved – *TRUE* or *FALSE* No work on A+B

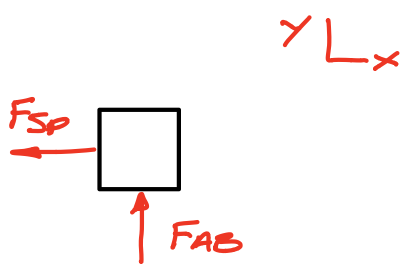
B.4: 1 point – The <u>linear momentum</u> for *block A alone* is conserved – *TRUE* or *FALSE* A: $\sum F_x \neq 0$

B.5: 1 point – The <u>linear momentum</u> for *block B alone* is conserved – *TRUE* or *FALSE* B: $\sum F_x \neq 0$

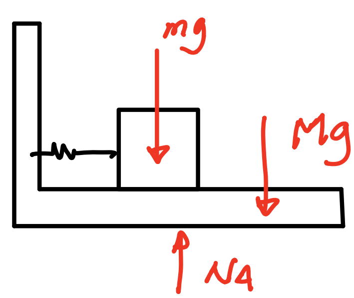
B.6: 1 point – The <u>linear momentum</u> for *blocks A and B together* is conserved – *TRUE* or *FALSE* A+B: $\sum F_x = 0$



A alone



B alone



A+B

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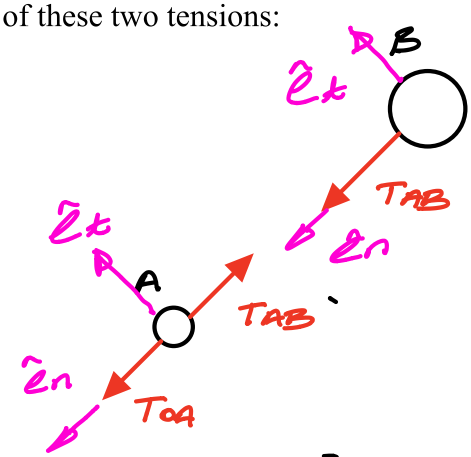
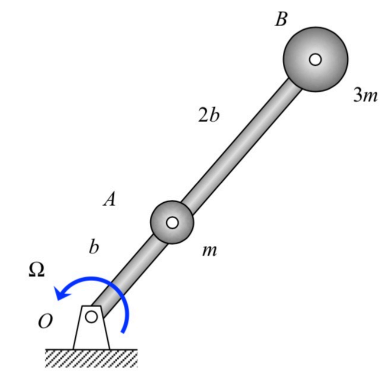
PROBLEM NO. 3 - PART C

Bar OB rotates about a vertical axis at end O with a constant rate of Ω. Particles A and B (having masses of m and 3m, respectively) are rigidly attached to bar OB, as shown. Let T_{OA} and T_{AB} represent the tensions in sections OA and AB, respectively, of the bar. Circle the correct response below that relates the sizes of these two tensions:

C.1: 2 points –

a) $T_{OA} > T_{AB}$
b) $T_{OA} = T_{AB}$
c) $T_{OA} < T_{AB}$





HORIZONTAL plane

$$B: \sum F_n = T_{AB} = (3m)\frac{(3b\Omega)^2}{3b} = 9mb\Omega^2$$

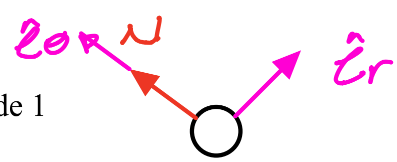
$$A: \sum F_n = T_{OA} - T_{AB} = m\frac{(b\Omega)^2}{b} \Rightarrow T_{OA} = T_{AB} + mb\Omega^2$$
$$= 10mb\Omega^2$$

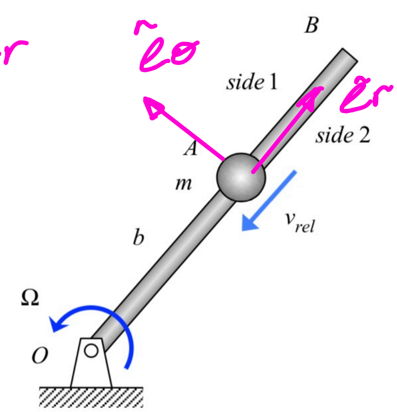
PROBLEM NO. 3 - PART D

Bar OB rotates about a vertical axis at end O with a constant rate of Ω. Particle A (having a mass of m) is allowed to slide along OB, as shown. At the instant shown, A is moving inward with a speed of v_{rel} relative of OB when A is a distance of b from O. Circle the correct responses below that describes correctly to contact between A and OB:

D.1: 2 points –

a) A is in contact with OB on side 1
b) A is NOT in contact with OB
c) A is in contact with OB on side 2





HORIZONTAL plane

$$\sum F_\theta = N = m a_\theta$$
$$= m(b\cancel{\dot{\Omega}}^{0} + 2\dot{b}\Omega)$$
$$= 2m(-v_{rel})\Omega < 0 \Rightarrow \text{contact side } 2$$