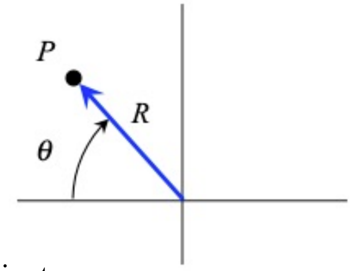
ME 274 Examination No. 1 **Name** SOLUTION
September 23, 2021

PROBLEM NO. 1 – 20 points



Particle P travels along a path described in terms of the polar components of R and θ of: $R=6+2cos\theta$, where R is in meters and θ is in radians. It is known that θ changes in time at a *constant* rate of $\dot{\theta}=3\,rad/s$. For this problem, do the following for the position of $\theta=\pi$:

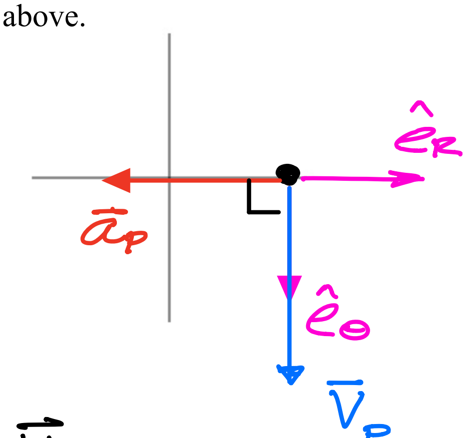
a) Calculate the *velocity vector* for P. Write your answer in polar coordinates.

$$R = 6+2\cos\theta = 4m$$
$$\dot{R} = -2\dot{\theta}\sin\theta = 0$$
$$\therefore \quad \vec{v}_P = \dot{R}\hat{e}_R + R\dot{\theta}\hat{e}_\theta = (12\hat{e}_\theta)\ m/s$$

b) Calculate the *acceleration vector* for P. Write your answer in polar coordinates.

$$\ddot{R} = -2[\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta] = -(2)(3)^2(-1) = 18\ \frac{m}{s^2}$$
$$\therefore \quad \vec{a}_P = (\ddot{R} - R\dot{\theta}^2)\hat{e}_R + (R\cancelto{0}{\ddot{\theta}} + 2\cancelto{0}{\dot{R}}\dot{\theta})\hat{e}_\theta$$
$$= [18-(4)(3)^2]\hat{e}_R = (-18\hat{e}_R)\ \frac{m}{s^2}$$

c) On the axis provided below: show the position of P, sketch the polar unit vectors $\hat{e}_R$ and $\hat{e}_\theta$, and sketch the velocity and acceleration vectors found above.



d) Determine the *rate of change of speed* of P.

$$\dot{v}_P = \vec{a}_P \cdot \frac{\vec{v}_P}{|\vec{v}_P|} = 0 \quad ; \quad \vec{a}_P \perp \vec{v}_P$$

e) Determine the *radius of curvature* of the path of P.

$$\rho = \frac{v_P^2}{\sqrt{|\vec{a}_P|^2 - \cancelto{0}{\dot{v}_P^2}}} = \frac{(12)^2}{18} = 8m$$

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PROBLEM NO. 2 – 20 points

Given the system shown in picture, we know the length of bar BD is b, the length of bar AB is d and the radius of wheel is R. At this instant, point D is right above point the center of the wheel point C, bar BD is horizontal and bar AB is vertical. We know that the wheel is currently rotating with an angular speed of ω_{CD} in counterclockwise direction and is speeding up at α_{CD}. Please answer the following questions:

a. What is the current velocity of point D $\vec{v}_D$?

b. What is the current angular velocity bar BD $\vec{\omega}_{BD}$?

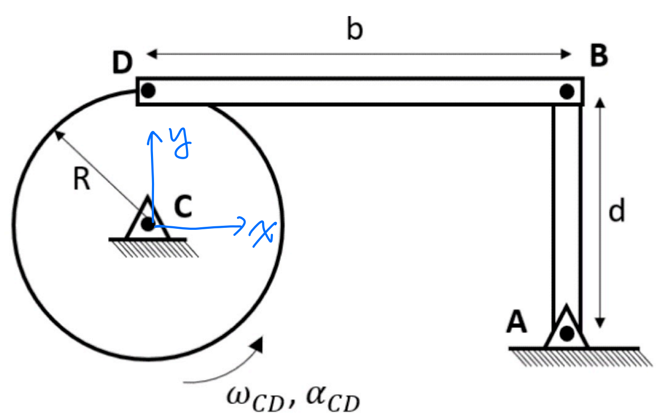
c. What is the current angular velocity bar AB $\vec{\omega}_{AB}$?

d. What is the current acceleration of point D $\vec{a}_D$?

e. What is the current angular acceleration bar BD $\vec{\alpha}_{BD}$?

f. What is the current angular acceleration bar AB $\vec{\alpha}_{AB}$?

Use $b = 20\ m$, $d = 15\ m$, $R = 5\ m$, $\omega_{CD} = 1\ rad/s$ and $\alpha_{CD} = 1\ rad/s^2$ in your work. Write your answers as vectors and include units.



$\vec{V}_D = \vec{V}_C + \vec{W}_{CD} \times \vec{r}_{D/C}$

$= 0 + 1\hat{k}\ rad/s \times 5m\hat{j}$

$= [-5\hat{i}]\ m/s$

$\vec{W}_{BD} = 0$ (check instant center)

Can also be solved as following:

BD: $\vec{V}_B = \vec{V}_D + \vec{W}_{BD} \times \vec{r}_{B/D}$

$= -5\hat{i} + W_{BD}\hat{k} \times b\hat{i}$

$= -5\hat{i} + 20W_{BD}\hat{j}$

AB: $\vec{V}_B = \vec{V}_A + \vec{W}_{AB} \times \vec{r}_{A/B}$

$= 0 + W_{AB}\hat{k} \times d\hat{j}$

$= -15W_{AB}\hat{i}$

$\because \vec{V}_B = \vec{V}_B$

$\Rightarrow \begin{cases} \hat{i}: & -5 = -15W_{AB} \\ \hat{j}: & 0 = 20W_{BD} \end{cases}$

$\Rightarrow$ $\vec{W}_{AB} = \frac{1}{3}\hat{k}\ rad/s$

$\vec{W}_{BD} = 0\hat{k}\ rad/s$

PROBLEM NO. 2 (continued)

$$\vec{a}_D = \vec{a}_C + \vec{\alpha}_{CD} \times \vec{r}_{D/C} - \omega_{CD}^2 (\vec{r}_{D/C})$$
$$= 0 + 1\hat{k} \times 5\hat{j} - 1^2 \times 5\hat{j}$$
$$= [-5\hat{i} - 5\hat{j}] \ m/s^2$$

BD: $$\vec{a}_B = \vec{a}_D + \vec{\alpha}_{BD} \times \vec{r}_{B/D} - \omega_{BD}^2 \vec{r}_{B/D}$$
$$= -5\hat{i} - 5\hat{j} + \alpha_{BD}\hat{k} \times 20\hat{i}$$
$$= -5\hat{i} + (20\alpha_{BD} - 5)\hat{j}$$

AB: $$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \vec{r}_{B/A}$$
$$= 0 + \alpha_{AB}\hat{k} \times 15\hat{j} - \frac{1}{9} \cdot 15\hat{j}$$
$$= -15\alpha_{AB}\hat{i} - \frac{5}{3}\hat{j}$$

Set $\vec{a}_B = \vec{a}_B$

$$\begin{cases} \hat{i}: & -5 = -15\alpha_{AB} \\ \hat{j}: & -\frac{5}{3} = 20\alpha_{BD} - 5 \end{cases}$$

$$\Rightarrow \begin{cases} \vec{\alpha}_{AB} = \frac{1}{3}\hat{k} \ rad/s^2 \\ \vec{\alpha}_{BD} = \frac{1}{6}\hat{k} \ rad/s^2 \end{cases}$$

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NOTE: *You are NOT asked to provide justification for your answers here in Problem 3. A correct response will receive full credit. Any work provided will not be graded, only the final answer.*

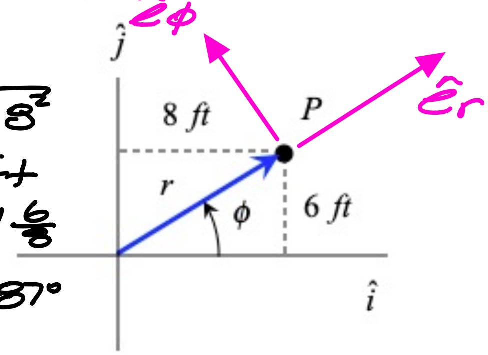
PROBLEM NO. 3 - PART A

When particle P is located at the position shown in the figure, its velocity and acceleration is known in terms of its Cartesian components as:

$$\vec{v} = \left(3\hat{i} - 4\hat{j}\right) ft/s$$

$$\vec{a} = \left(-5\hat{j}\right) ft/s^2$$

$r = \sqrt{6^2 + 8^2}$
$= 10 \text{ ft}$
$\phi = \tan^{-1}\frac{6}{8}$
$= 36.87°$



Circle the correct responses below:

2 points – At this instant:
a) the speed of P is *increasing*
b) the speed of P is *constant*
c) the speed of P is *decreasing*

$\dot{v} = \vec{a} \cdot \frac{\vec{v}}{|\vec{v}|} = (-5\hat{j}) \cdot \left(\frac{3\hat{i} - 4\hat{j}}{5}\right)$
$= 4 \text{ ft/s}^2 > 0$

2 points – If ρ is the *radius of curvature* of the path for P, then at this instant:
a) $\rho > r$
b) $\rho = r$
c) $\rho < r$

$\rho = \frac{v^2}{\sqrt{|\vec{a}|^2 - \dot{v}^2}} = \frac{5^2}{\sqrt{5^2 - 4^2}} = \frac{25}{3} < r$

2 points – At this instant:
a) r is *increasing*
b) r is *constant*
c) r is *decreasing*

$\dot{r} = \vec{v} \cdot \hat{e}_r = (3\hat{i} - 4\hat{j}) \cdot (0.8\hat{i} + 0.6\hat{j})$
$= (3)(0.8) - (4)(0.6) = 0$

2 points – At this instant:
a) ϕ is *increasing*
b) ϕ is *constant*
c) ϕ is *decreasing*

$r\dot{\phi} = \vec{v} \cdot \hat{e}_\phi = (3\hat{i} - 4\hat{j}) \cdot (-0.6\hat{i} + 0.8\hat{j})$
$= (3)(-0.6) + (-4)(0.8) = -5 \frac{ft}{s} < 0$

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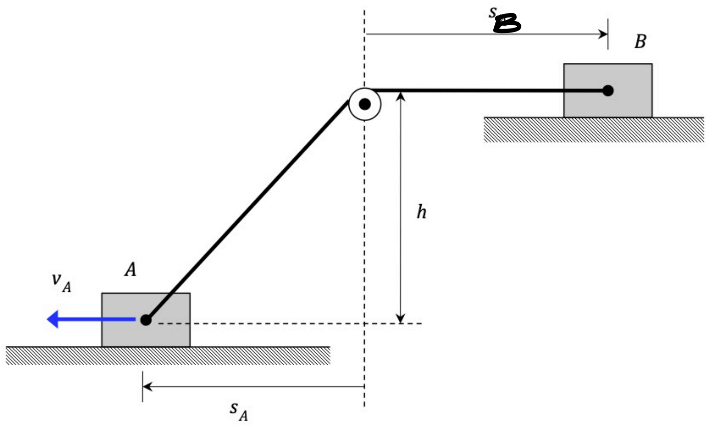
PROBLEM NO. 3 - PART B – 2 points



Blocks A and B are connected by an inextensible cable, with the cable wrapped around a small pulley. A moves to the left with a speed of v_A. Choose the correct response below:

a) The speed of B is *greater* than the speed of A.
b) The speed of B is *equal* to the speed of A.
c) The speed of B is *less* than the speed of A.

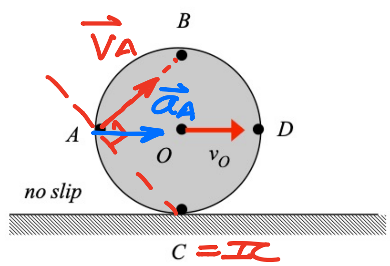
$$L = s_B + \sqrt{s_A^2 + h^2}$$

$$\frac{dL}{dt} = \dot{s}_B + \frac{1}{2}\frac{2 s_A}{\sqrt{s_A^2+h^2}}\dot{s}_A$$

$$\rightarrow \quad v_B = \frac{s_A}{\sqrt{s_A^2+h^2}} v_A$$

< 1

PROBLEM NO. 3 - PART C



The disk rolls without slipping with its center O having a *constant* speed of v_0. At the instant shown, A is located directly to the left of O. Circle the correct responses below:

2 points: At this instant:

a) The speed of A is *greater* than the speed of O.
b) The speed of A is *equal* to the speed of O.
c) The speed of A is *less* than the speed of O.

O is closer to the IC than is A.

2 points: At this instant:

a) The speed of A is *increasing*.
b) The speed of A is *constant*.
c) The speed of A is *decreasing*.

$\vec{a} \cdot \vec{v} > 0$

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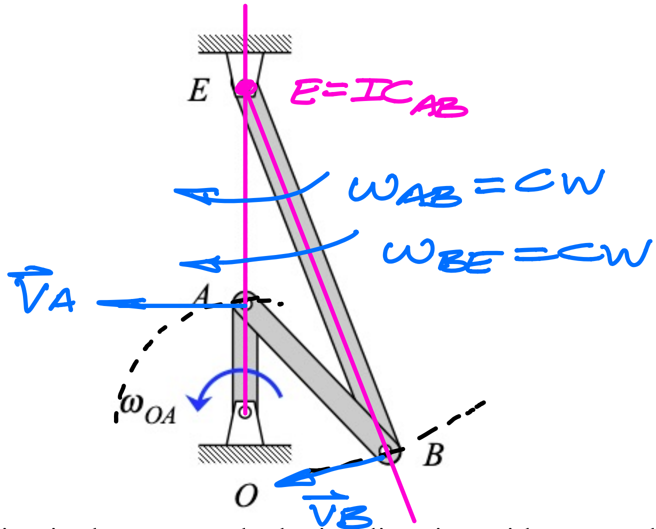
PROBLEM NO. 3 - PART D



Link OA is rotating in the counterclockwise direction with an angular speed of ω_{OA}. At the instant shown, link OA is aligned with line OE, as shown above. Choose the correct responses below (*Hint: Consider the location of the instant center for link AB.*):

2 points:

a) Link BE is rotating *counterclockwise*.
b) Link BE is *not rotating*.
c) Link BE is rotating *clockwise*.

2 points:

a) Link AB is rotating *counterclockwise*.
b) Link AB is *not rotating*.
c) Link AB is rotating *clockwise*.

2 points:

a) The speed of A is *greater* than the speed of B.
b) The speed of A is *equal* to the speed of B.
c) The speed of A is *less* than the speed of B.

A is closer to IC_{AB} than is B.