**Given:** Pellet P having a mass of m is pulled through a barrel (having negligible mass) by means of radial force $F = 60R$, where F is in Newtons and R is in meters. The barrel is constrained to move in a HORIZONTAL plane by rotating about shaft passing through point O. The system is released with $R = R_1$, $\dot{R} = \dot{R}_1$ and $\dot{\theta} = \dot{\theta}_1$.

Find: For the instant when $R = R_2$:

   a) determine the rotation rate of the barrel, $\dot{\theta}_2$.

   b) determine the value of $\dot{R}_2$.

Use the following parameters in your analysis: $m = 20kg$, $R_1 = 1.5\ meters$, $\dot{R}_1 = 4\ m/sec$, $\dot{\theta}_1 = 8\ rad/sec\ (CCW)$ and $R_2 = 3\ meters$.

a) Work – Energy

b) angular Momentum
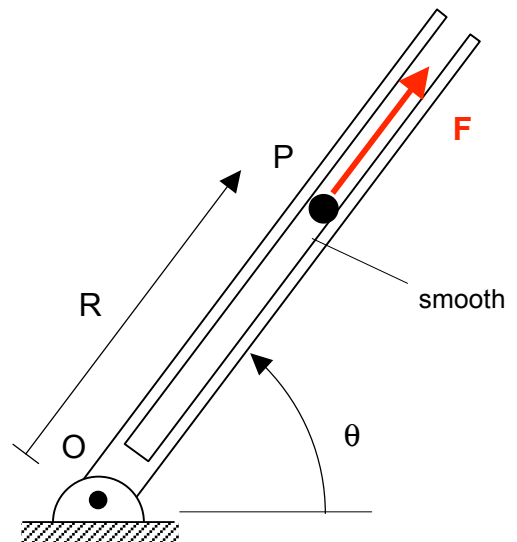
HORIZONTAL PLANE

Given: A uniform plate of mass *m* is able to move on a smooth horizontal plane with corner A of the plate being constrained to move within a smooth guide in the plane of motion. The plate is initially at rest with edge AB aligned with the guide for A. A force *P* is applied at corner D, with *P* acting perpendicular to the guide for A.

Find: Determine the initial angular acceleration of the plate. Write your answer as a vector.

Use the following parameters in your analysis: $m = 100\,kg$, $b = 8\,meters$, $h = 6\,meters$, and $P = 400\,N$.

Newton - Euler

A   b   E

h

P

B

D

smooth guide for pin A
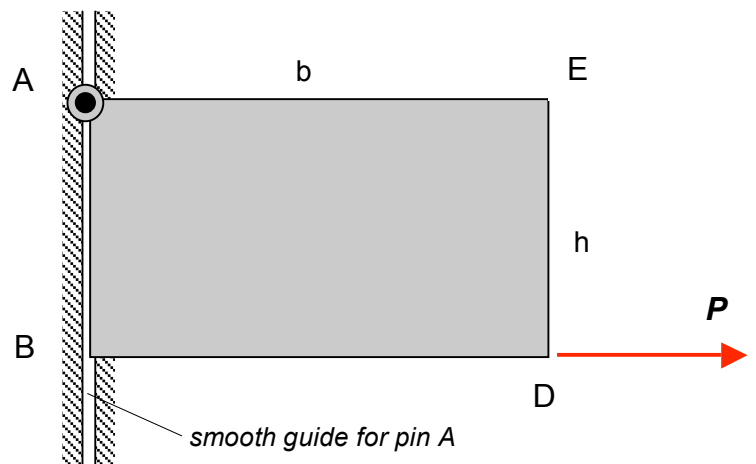
HORIZONTAL PLANE

Given: Blocks A and B are connected by the cable-pulley shown. The system is released from rest. Consider all surfaces to be smooth and that the masses of the pulleys are small compared to the masses of A and B.

Find: Upon release,

a) determine the acceleration of block B. Write your answer as a vector.

b) determine the tension in the cable.

Use the following parameters in your analysis: $m = 5kg$, $M = 20kg$ and $\theta = 36.87°$.

Newton's Method

g

A

M

θ

O

m

B
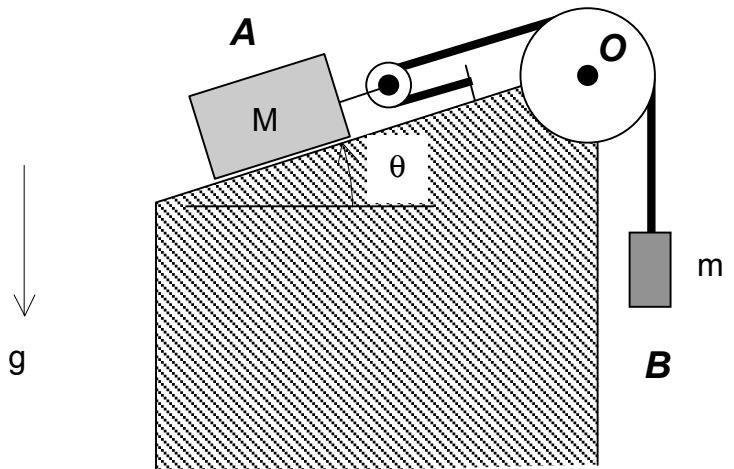
Given: Block B, having a mass of m, is pressed against a spring (of stiffness k) that is attached to cart A. Cart A (having a mass of M) rests on a horizontal surface. The system is released from rest with the spring compressed by an amount of Δ. After release, block B impacts A, with this impact having a coefficient of restitution of e. Assume all surfaces to be smooth. (Note that since B is simply pressed against the spring, the spring *can push but not pull* on B.)

Find: For this problem,

      a) determine the velocities of A and B immediately BEFORE impact.

      b) determine the velocities of A and B immediately AFTER impact

Write your answers as vectors.

Use the following parameters in your analysis: $m = 20kg$, $M = 40kg$, $k = 3000\ N/m$, $\Delta = 0.2\ meters$ and $e = 0.5$.

A

M   2Δ   B
                                          k
                                     m ⊣WWW⊣
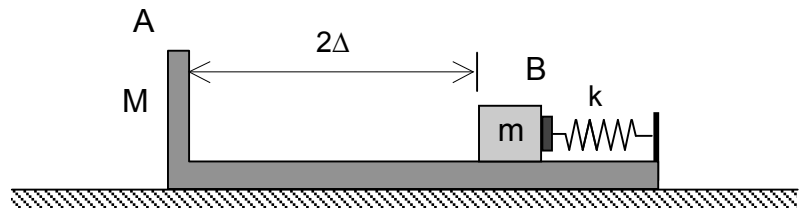
a) Linear Impulse momentum

    Work Energy

b) Linear impulse momentum

   Coefficient of Resistution

Given: A small object of mass m is placed on the inner surface of a conical dish that is rotating at a constant rate of ω. The coefficients of static and kinetic friction between the object and the dish are known to be μ_s and μ_k, respectively.

Find: Determine the *maximum* rotation rate ω for which the object does not slip on the dish.

Use the following parameters in your analysis: $m = 5kg$, $r = 0.92 \; meters$, $\mu_s = 0.4$, $\mu_k = 0.1$ and $\theta = 36.87°$.

Newton's Method

Name _____
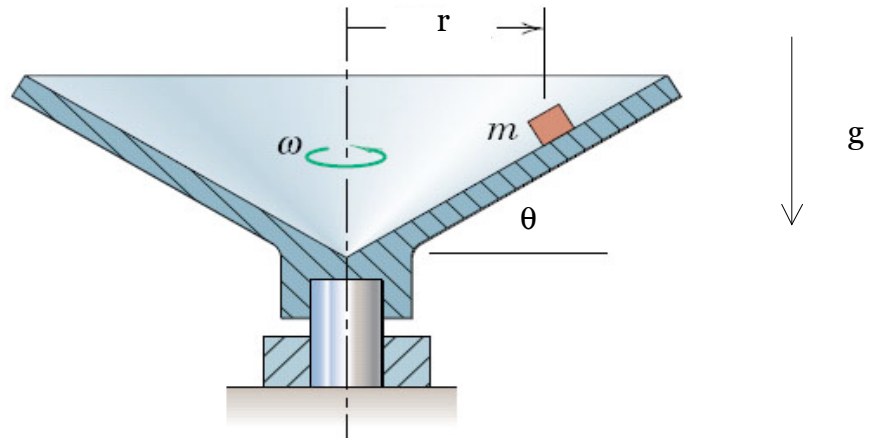
Given: Particle B, having a mass of m, is pressed against a spring (of stiffness k) that is attached to cart A. Cart A (having a mass of M) rests on a horizontal surface. The system is *released from rest* when B is at Position 1 with the spring compressed by an amount of Δ. After release, B travels within a slot cut into cart A, with the slot having straight horizontal and circular sections (the circular section has a radius of r and center at O). A position 2, B has reached the upper horizontal slot but has not yet impacted the cart at the right end of this slot.

Find: Determine the velocities of A and B when B is at Position 2. Write your answers as vectors.

Note that since B is simply pressed against the spring, the spring *can push but not pull* on B. Assume all surfaces to be smooth.

Use the following parameters in your analysis: $m = 30kg$, $M = 60kg$, $k = 3000\ N/m$, $\Delta = 0.5\ meters$ and $r = 0.2\ meters$.
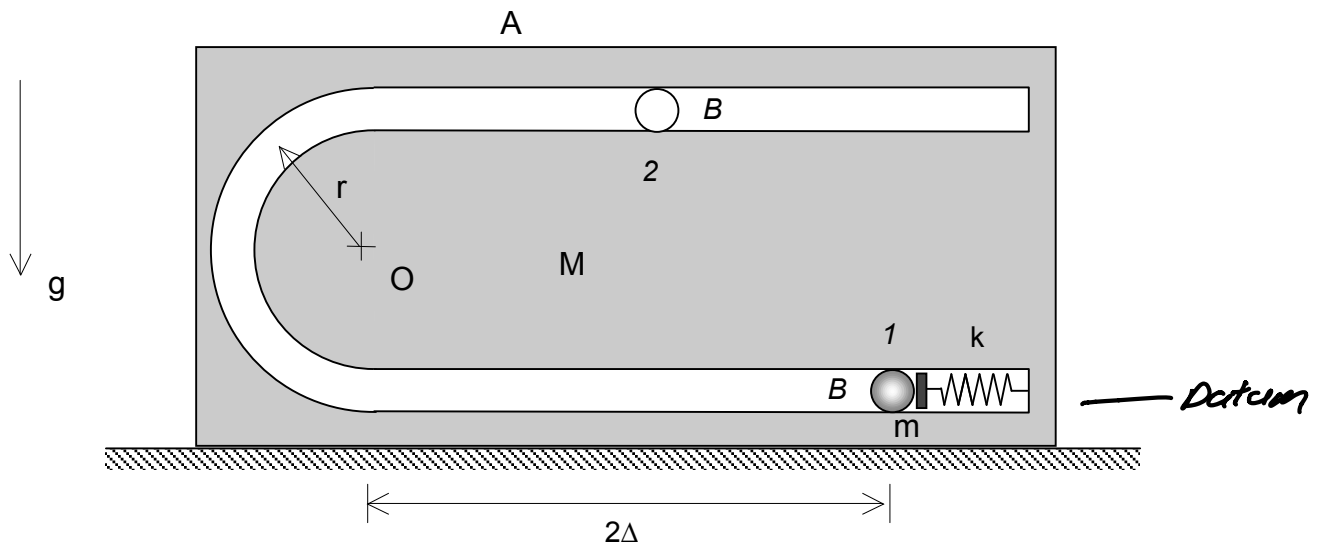
A

B

2

r

M

O

g

1   k

B

m

Datum

2Δ

Work – Energy & Linear Impulse Momentum

Given: The homogeneous disk shown below has a mass of m and outer radius of r. The disk is placed on a rough horizontal surface (coefficients of static and kinetic friction of μ_s and μ_k, respectively) with a uniform speed (i.e., zero angular velocity) of v_{O1} to the right.

Find: Determine the elapsed time, Δt, during which the disk travels to the right before slipping ceases between the disk and the horizontal surface. Note that slipping ceases when the contact point C has zero velocity.

Use the following parameters in your analysis: $m = 100\,kg$, $r = 0.5\ meters$, $\mu_s = 0.8$, $\mu_k = 0.5$ and $v_{O1} = 10\ m/sec$.
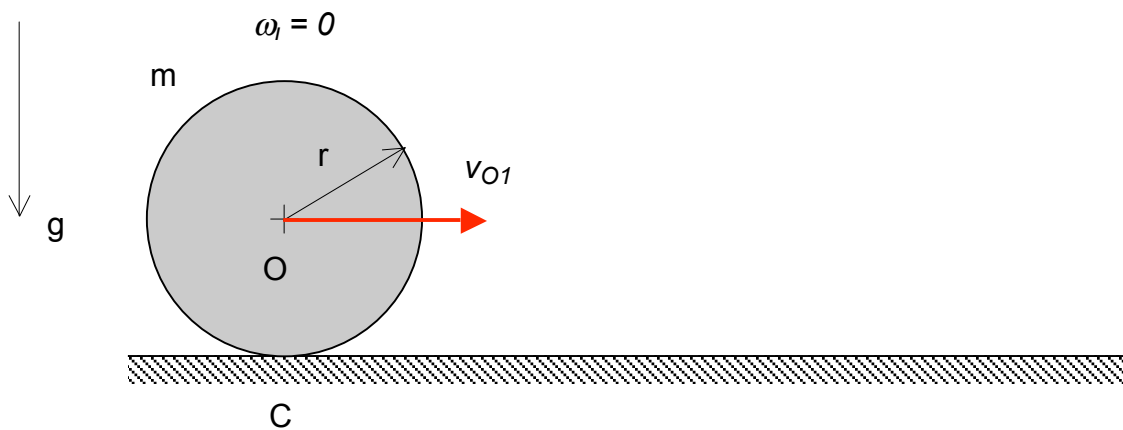
Linear Impulse Momentum
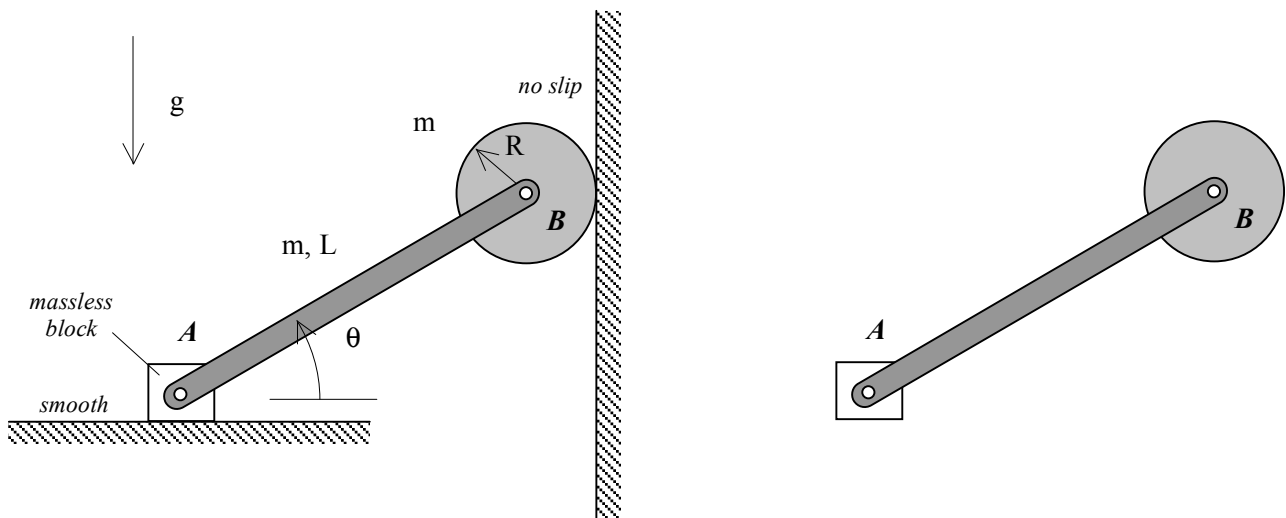
Angular Impulse Momentum

Given: System released from rest with $\theta = 36.87°$. Consider the bar and the disk to be homogeneous.

Find: Determine the *angular velocity* of the bar when $\theta = 0$. Use: $m = 100 \ kg$, $L = 2 \ meters$ and $R = 0.4 \ meters$.

Please clearly indicate the four steps in a neat and orderly presentation of your work.


Work – Energy

Given: Homogeneous bar OA (of length L and mass m) is pinned to ground at end O. End A of the bar is connected to a cable that is wrapped around a homogeneous disk (of mass m and outer radius R), with the disk being pinned to ground at its center C. Assume that the cable does not slip on the disk. The system is released from rest with OA being horizontal and the cable being vertical.

Find: Determine the *angular acceleration of the disk* on release. Use the following: $m = 10kg$, $L = 4\,meters$ and $R = 2\,meters$.

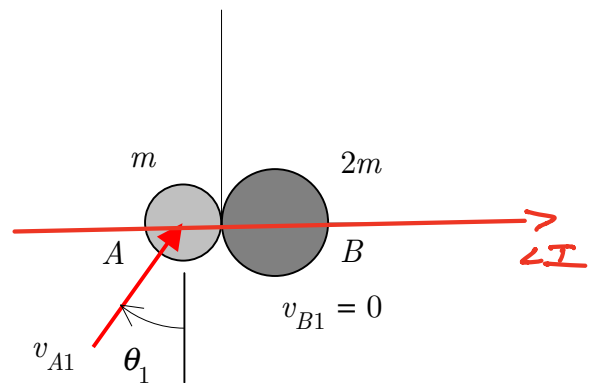Please clearly indicate the four steps in a neat and orderly presentation of your work.
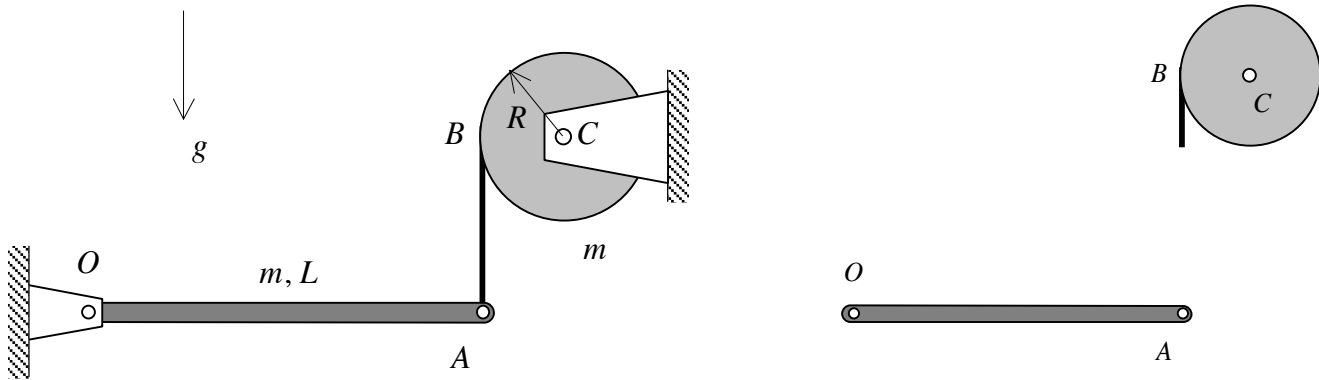
Newton – Euler

Given: Particle A (of mass m) is traveling with a speed of v_{A1} in the direction shown below when it strikes a stationary particle B (of mass 2m). The coefficient of restitution for the impact of A with B is known to be $e = 0.5$.

Find: If $\theta_1 = 25°$, what is the direction of travel of particle A *AFTER* impacting B? Provide a mathematical justification for your answer.

$$m \qquad\qquad 2m$$

$$A \qquad\qquad B$$

$$v_{B1} = 0$$

$$v_{A1} \quad \theta_1$$

System
Linear Impulse
Coefficient of
restitution

Mass A
Linear Impulse

Mass B
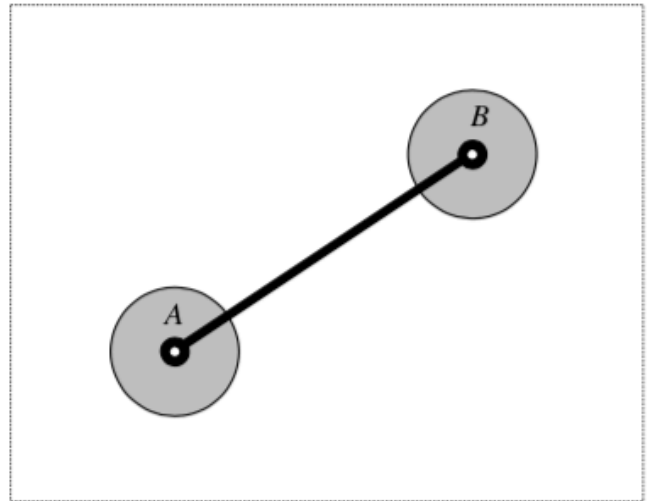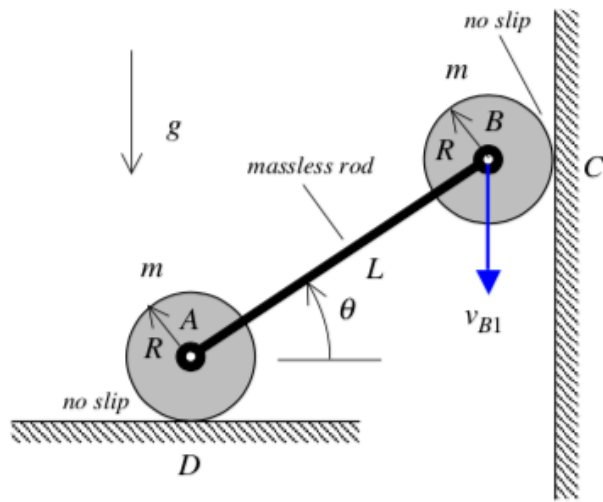Linear Impulse

Given: System released at $\theta = 53.13°$ with the center of the disk moving downward with a speed of v_{B1}. Consider the bar and the disk to be homogeneous.

Find: Determine the *angular velocity* of disk B when $\theta = 0$. Use: $m = 100\ kg$, $L = 3\ meters$, $v_{B1} = 4\ m/sec$ and $R = 0.5\ meters$.

Please clearly indicate the four steps in a neat and orderly presentation of your work.

Work – Energy

**Given:** Particle P (of mass m) slides along a smooth horizontal surface with a speed of v. P then strikes end A of a stationary thin, homogeneous bar (of length L and mass M) that is pinned to ground at O. The coefficient of restitution for the impact of P with end A of the bar is known to be *e*.

Find: Determine the angular speed of bar OA immediately after the impact. Use the following parameters in your analysis: $m = 10kg$, $M = 30\,kg$, $L = 2\,meters$, $v = 40\,m\,/\,sec$ and $e = 0.5$.

Please clearly indicate the four steps in a neat and orderly presentation of your work.
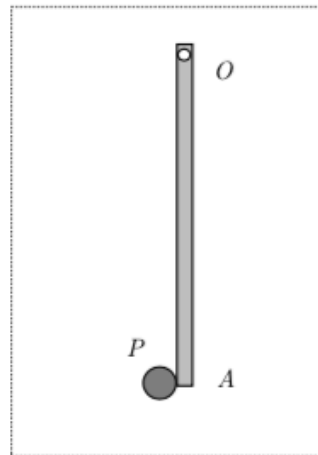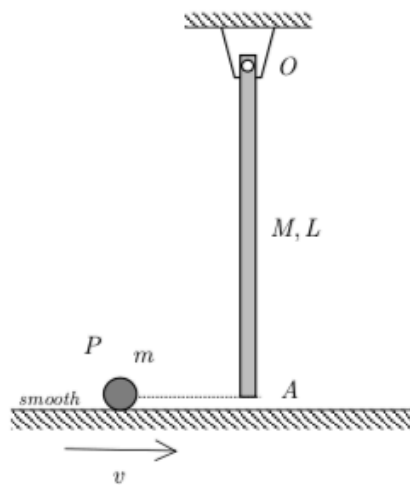
Angular Impulse Momentum

Coefficient of Restitution

Given: The system shown below is released from rest. The bar is homogeneous with a mass of *m* and a length of *L*. The center of the mass of the bar, G, is constrained to move within a smooth vertical guide. End A of the bar is constrained to move on a smooth horizontal surface.

Find: Determine the *angular acceleration* of the bar on release. Use the following parameter values in your work: $m = 100 \ kg$, $L = 2 \ meters$ and $\theta = 53.13°$.

Please clearly indicate the four steps in a neat and orderly presentation of your work.
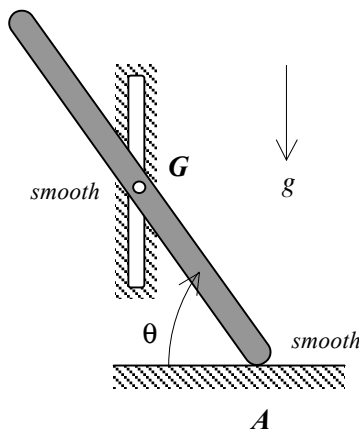

Newton – Euler

Given: Particle A (of mass m) slides upon a *smooth* HORIZONTAL surface. A flexible, inextensible cord is connected to A at one end and has a constant force $\vec{F}$ acting to the left on the other end. Initially, when A is at a radial distance of $R = R_1$ from O, the cord is in contact with a small, smooth peg at O. At this instant, A is moving perpendicular of line OA with a speed of v_{A1}, as shown in the figure.

Find: When A is at a radial distance of $R = R_2$ from O, determine the speeds of ends A and B of the cord. Use the following parameter values in your work:

$m = 10kg$, $R_1 = 2\ meters$, $R_2 = 3\ meters$, $\left|\vec{F}\right| = 280\ N$ and

$v_{A1} = 15\ m\ /\ sec$.

Please clearly indicate the four steps in a neat and orderly presentation of your work.
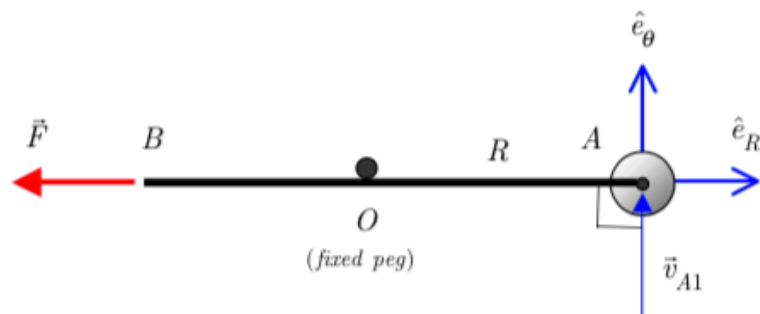


HORIZONTAL SURFACE

Angular Impulse Momentum

Work Energy

Given: Particle A (of mass 2m) is attached to a rigid bar of negligible mass. Particle A is also connected to a cable that is wrapped around two pulleys and connected to particle B on its other end. The system is released from rest with OA being horizontal and with section AC of the cable being vertical. Assume that the radii of the pulleys to negligible.

Find: Determine the *angular velocity* of the bar at Position 2 where it has rotated 90° CW to a vertical orientation. (At Position 2, section AC of the cable is horizontal.) Use the following parameter values in your work: $m = 10\,kg$ and $L = 4\,meters$.

Please clearly indicate the four steps in a neat and orderly presentation of your work.


Position 1

Position 2
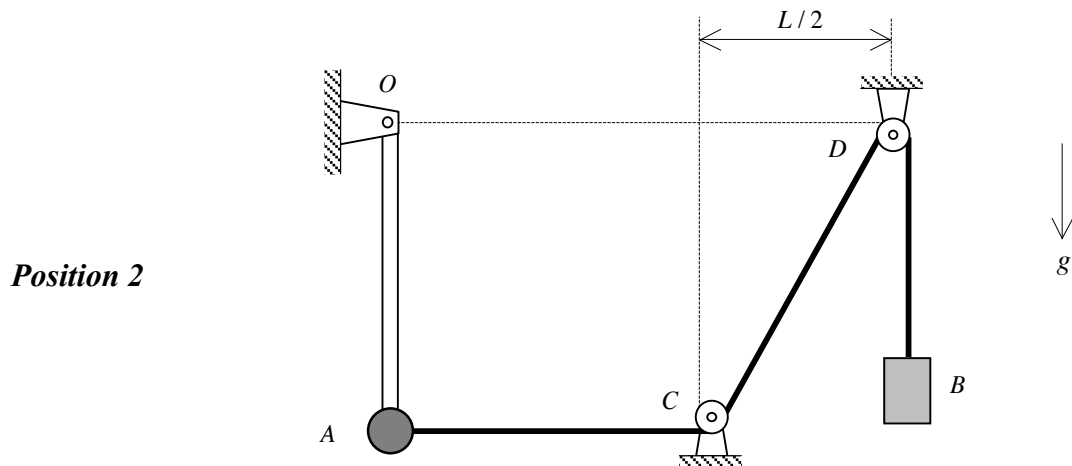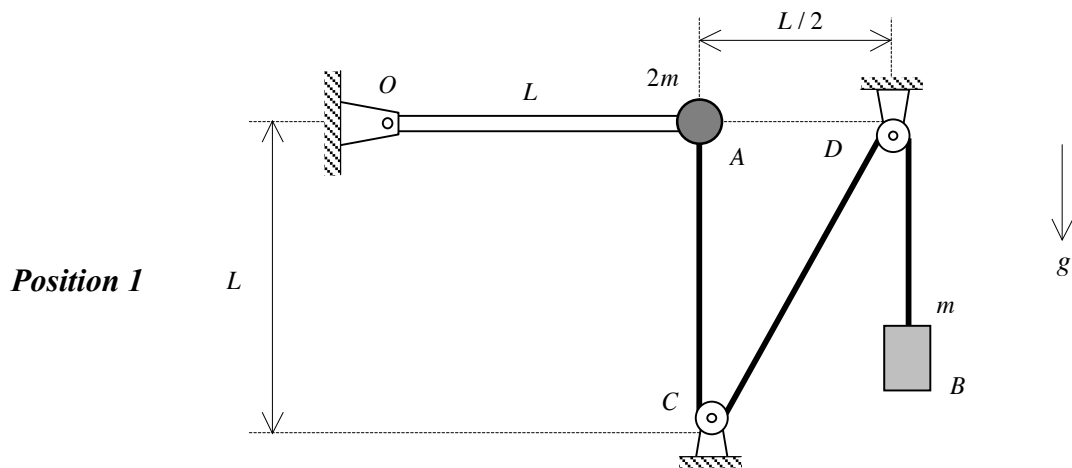
PLEASE START YOUR ANALYSIS ON THE NEXT PAGE.

Work – Energy

Given: A homogeneous disk with an outer radius of R and of mass M is pinned to
ground at its center O. Rigid link AP, of negligible mass and length 2R, is
pinned to the outer perimeter of the disk at A. Particle P (of mass m) is
attached to the free end of link AP. At the instant shown below: A is directly
to the right of O: P is directly below A; link AP is rotating in the
counterclockwise sense with an angular speed of ω_{AP}; and, the disk is
rotating in the counterclockwise sense with an angular speed of ω_d.

Find: For the position shown, determine the angular acceleration of the disk and the
angular acceleration of link AP. Write your answers as vectors. Leave your
answers in terms of: m, M, R, g, ω_d and ω_{AP}.

Please clearly indicate the four steps in a neat and orderly presentation of your work.
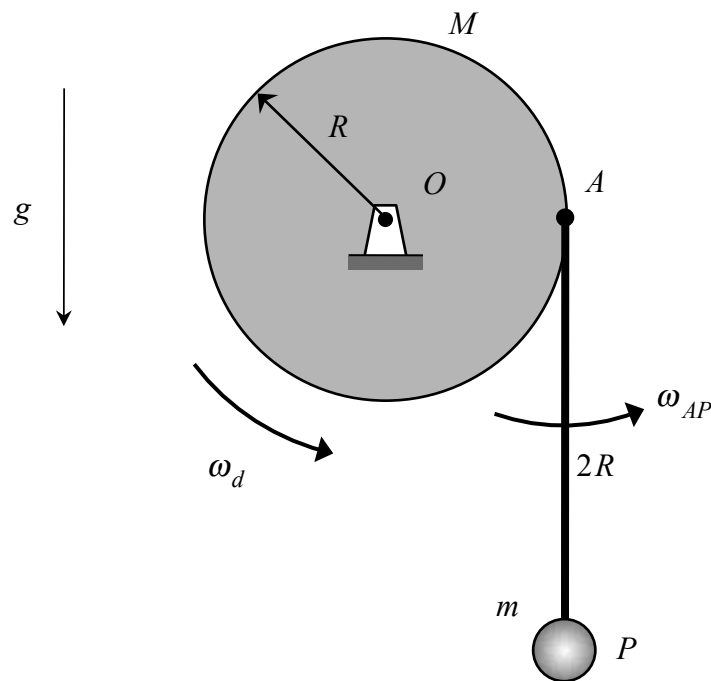
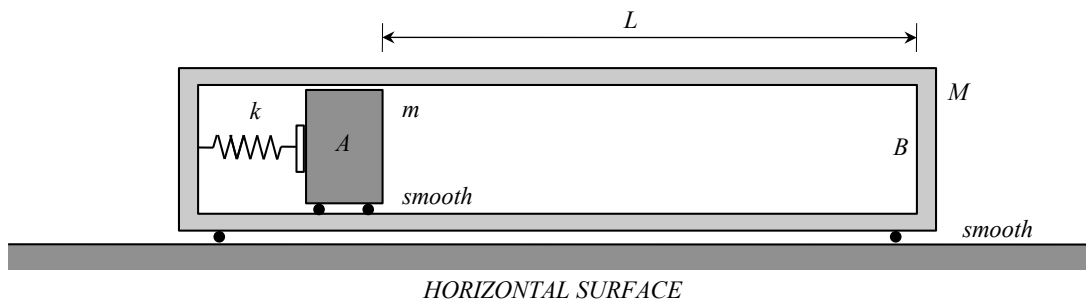Work appearing above this line will NOT be graded.

Newton – Euler

Given: A box having a mass of M is constrained to move along a smooth, horizontal surface. Block A (having a mass of m) is able to slide along smooth, horizontal surface inside the box, as shown in the figure below. Block A is pressed against a spring having a stiffness of k. Initially, the system is at rest and the spring is compressed by an amount Δ_1. The coefficient of restitution between A and the box at end B is e.

Find: After the spring is released:

    a) Determine the speed of the box and the speed of block A immediately BEFORE A impacts the box at B. Write your answers as vectors.

    b) Determine the speed of the box and the speed of block A immediately AFTER A impacts the box at B. Write your answers as vectors.

Use the following: $m = 3kg$, $M = 5kg$, $k = 12,000N/m$, $e = 0.5$, $\Delta_1 = 0.3m$ and $L = 2m$.

Please clearly indicate the four steps in a neat and orderly presentation of your work.

HORIZONTAL SURFACE

Work appearing above this line will NOT be graded.

a) Work-Energy & Linear Impulse Momentum

b) Linear Impulse Momentum & Coefficient of Restitution

Given: Link OC and the homogeneous circular disk are released from rest when OC is horizontal. The disk is pinned at its center to end C of link OC. Link OC has a mass of m and a radius of gyration about its center of mass G of k_G. The disk has a mass of M and an outer radius of r. Assume all joints to be smooth.

Find: Determine the angular speed of link OC when OC has rotated to a vertical orientation.

Use the following: $m = 10kg$, $L = 1.2\ meters$, $k_G = 0.3$, $M = 8\ kg$ and $r = 0.4\ meters$.

Please clearly indicate the four steps in a neat and orderly presentation of your work.


0.3L   0.7L
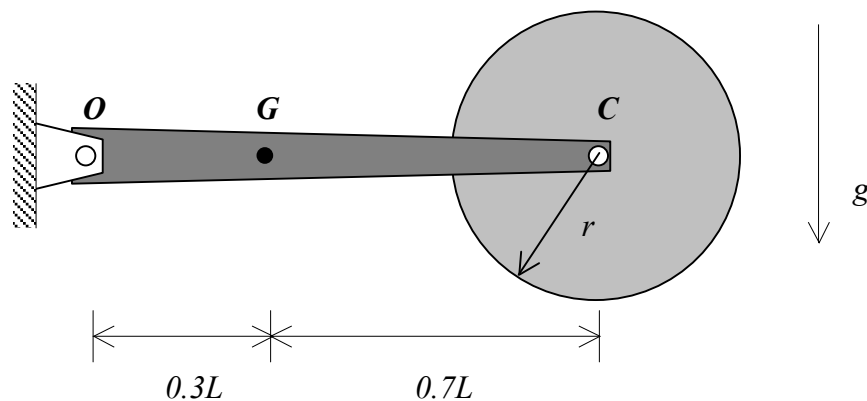
Work appearing above this line will NOT be graded.

Work Energy

Given: Blocks A and B (having masses of m and 2m, respectively) are connected by rigid bar AB, with bar AB having negligible mass. Block A is constrained to move along a smooth, vertical wall, whereas block B moves along a smooth horizontal surface. The system is released from rest with $\theta = 53.13°$.

Find: Determine the speeds of blocks A and B at the position where $\theta = 0$.

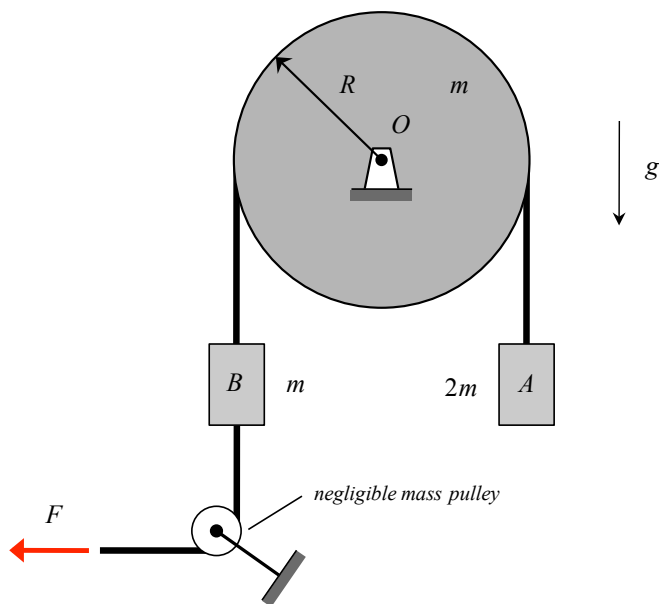Use the following parameter values: $L = 2\ m$ and $m = 10\ kg$.
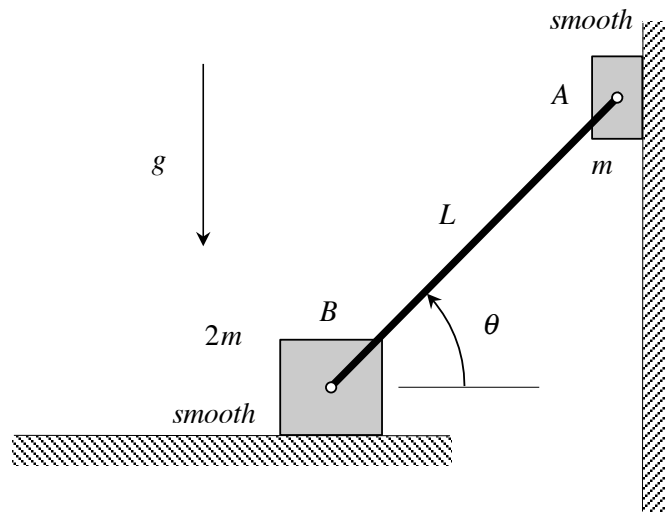
Work – Energy

Given: A homogeneous disk of mass m and outer radius R is attached to a horizontal shaft at the disk's center O. An inextensible cable is wrapped around the outer radius of the disk, as shown. Block A (having a mass of 2m) is attached to one end of the cable, and block B (having a mass of m) is attached to the other end of the cable. A second cable is attached to block B, with a force F applied to the other end of this cable. The system is released from rest. Assume that the cable does not slip on the disk, and that the pulley around which the second cable is wrapped has negligible mass.

Find: For this problem:

   a) Draw individual free body diagrams of the disk, block A and block B.

   b) Determine the angular acceleration of the disk on release. Write your answer as a vector.

Newton – Euler