Summary: Vibrations - damped free response

EOM: For free response of damped system:

$$m\ddot{x} + c\dot{x} + kx = 0$$

STANDARD FORM OF EOM: Divide EOM by "m" to get:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$$

where

$$2\zeta\omega_n = c/m \;\Rightarrow\; \zeta = c/2\sqrt{km} = damping\ ratio$$

$$\omega_n = \sqrt{k/m} = natural\ \ frequency$$

SOLUTION OF EOM: For $0 \le \zeta < 1 \left(UNDERdamped\right)$

$$x(t) = e^{-\zeta\omega_n t}\left(C\cos\omega_d t + S\sin\omega_d t\right) \quad ; \quad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

HOW TO FIND THE RESPONSE COEFFICIENTS, C AND S?

Enforce the initial conditions on the solution:

$$x(0) = x_0 = e^0\left(C\cos 0 + S\sin 0\right) \;\Rightarrow\; C = x_0$$

$$\dot{x}(0) = \dot{x}_0 = -\zeta\omega_n e^0\left(C\cos 0 + S\sin 0\right) + \omega_d e^0\left(-C\sin 0 + S\cos 0\right) \;\Rightarrow\; S = \frac{\dot{x}_0 + \zeta\omega_n x_0}{\omega_d}$$