



$$\sum M_C = -(mg \sin \theta)R - (kx)R - (2kx)R = I_C \ddot{\phi}$$

where $I_C = I_O + mR^2 = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2$ (PAT)

and $\ddot{x} = R\ddot{\phi}$ (kinematics)

EOM in x: $\frac{3}{2}m\ddot{x} + 3kx = mg \sin \theta$

Static equilibrium: $\ddot{x} = 0 \Rightarrow x_{static} = \frac{mg \sin \theta}{3k}$

Therefore: $z = x - x_{static} \Rightarrow \frac{3}{2}m\ddot{z} + 3kz = 0$