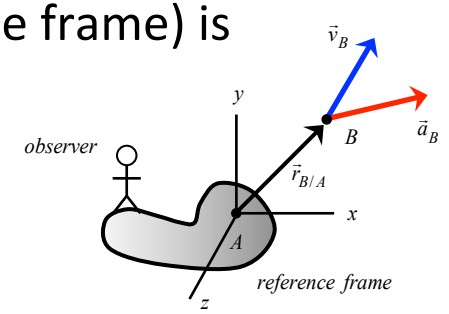
Summary: 3D Moving Reference Frame Kinematics 1

PROBLEM: A person attached to a moving body (reference frame) is observing the motion of point B.



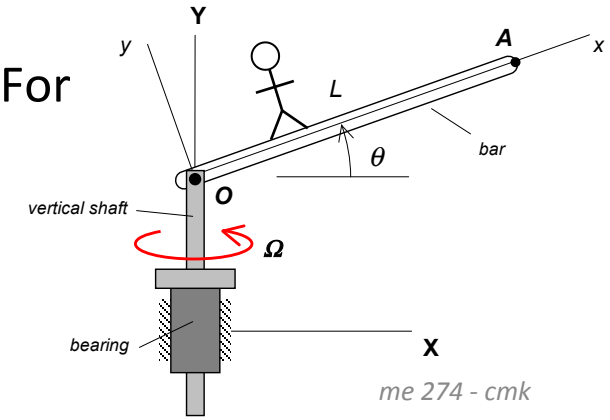
$$\vec{v}_B = \vec{v}_A + \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \left(\vec{a}_{B/A}\right)_{rel} + \vec{\alpha} \times \vec{r}_{B/A} + 2\vec{\omega} \times \left(\vec{v}_{B/A}\right)_{rel} + \vec{\omega} \times \left(\vec{\omega} \times \vec{r}_{B/A}\right)$$

where:

- $\vec{\omega}$ and $\vec{\alpha}$ are the angular velocity/acceleration <u>*of the observer*</u> (*no exceptions*).
- $\left(\vec{v}_{B/A}\right)_{rel}$ and $\left(\vec{a}_{B/A}\right)_{rel}$ are the velocity/acceleration of B <u>*as seen by the observer*</u>.
- A is <u>*ANY*</u> point on the <u>*same*</u> reference frame as the observer.
- Generally, <u>*you*</u> are free to choose your observer.

QUESTION: How is this different from the 2D case? For observer on arm OA:



$$\vec{\omega} = \Omega\hat{J} + \dot{\theta}\hat{k}$$

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} = \dot{\Omega}\hat{J} + \Omega\dot{\hat{J}} + \ddot{\theta}\hat{k} + \dot{\theta}\dot{\hat{k}} = \dot{\Omega}\hat{J} + \ddot{\theta}\hat{k} + \dot{\theta}\left(\vec{\omega} \times \hat{k}\right)$$

($\Omega\dot{\hat{J}} = 0$)