ME 200 – Thermodynamics 1
Chapter 9 In-Class Notes
for Spring 2023

Gas Power Cycles

- Air-Standard Cycles
- Internal Combustion (IC) Engines
  - Otto and Diesel Cycles
- Gas Turbine (GT) Engines
  - Brayton Cycles

Lectures 37 and 38

- Definitions
- Air-Standard Assumptions
- Introduction to IC Engines
- Otto, Diesel, Dual Cycles

Definitions

- Gas Power Cycle: utilizes a gas that does not change phase as the working fluid

- Internal Combustion Engine: fuel and air are burned within the boundaries of a system (e.g., an automotive engine)

- External Combustion Engine: heat is supplied to the working fluid through a heat exchanger (e.g., an electric steam power plant cycle).

- Open Cycles: the working fluid is exhausted after each cycle (e.g., internal combustion engines)

- Closed Cycles: the working fluid is recirculated within the cycle (e.g., external combustion engines)

- Ideal Engine Cycles: cycles that have no internal irreversibilites (note that Carnot cycles have no internal and external irreversibilities → more "realistic" ideal cycles have external heat transfer irreversibilities)

Air-Standard Assumptions & Models

- More realistic & specific ideal cycle models for gas power cycles (separate models for different engine types) than Carnot cycle

- Open cycles are replaced with closed cycle equivalents
  - Internal combustion processes are replaced by heat addition processes from external sources (external combustion processes)
  - Exhaust processes are replaced by heat rejection processes to external sinks

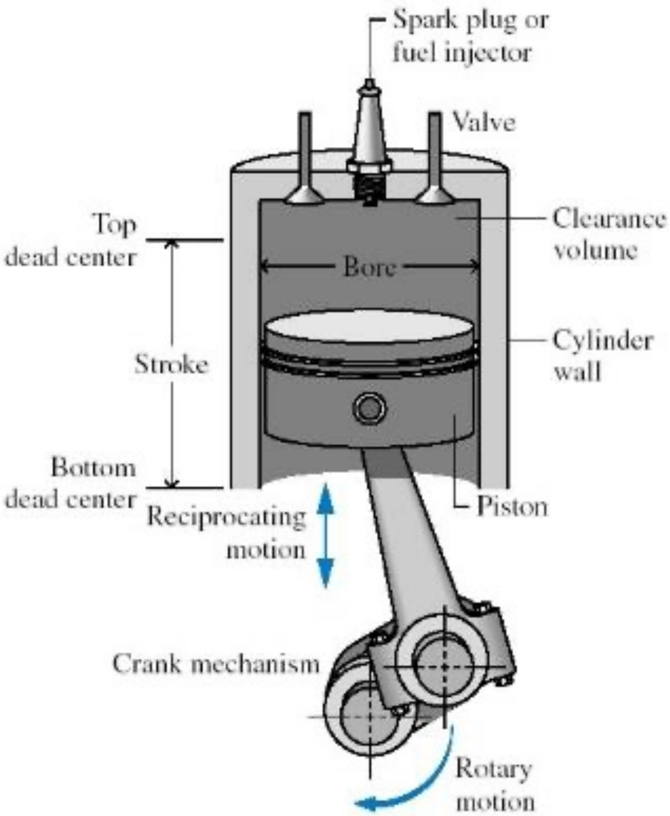
- Incorporates non-isothermal heat addition and rejection processes → has external heat transfer irreversibilities

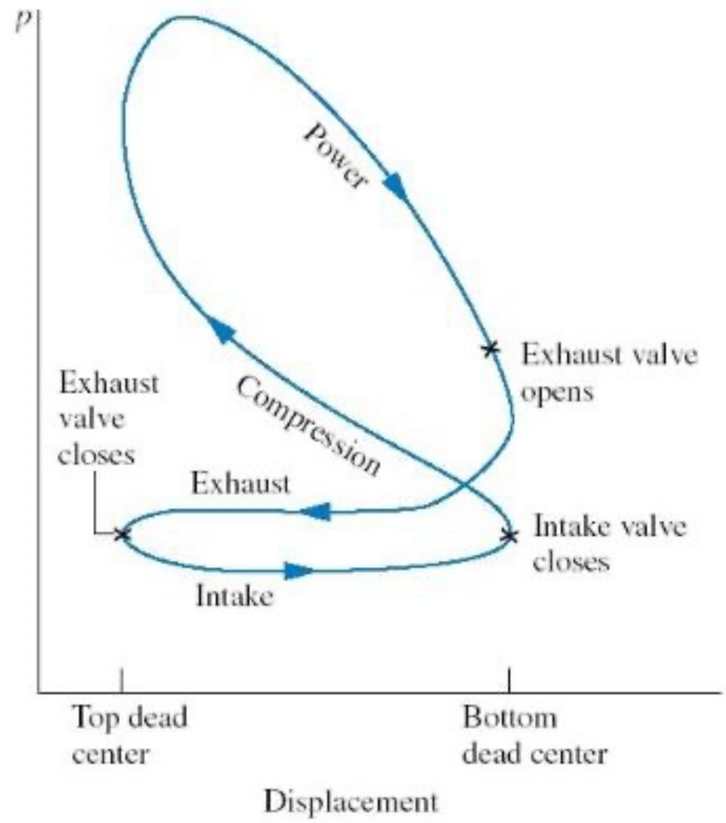
- All processes are assumed to be internally reversible

- Working fluid is air that acts as an ideal gas

- Often employ **cold-air-standard** assumptions which utilize constant specific heats evaluated at room temperature → **cold air-standard cycle**

Internal Combustion Engines

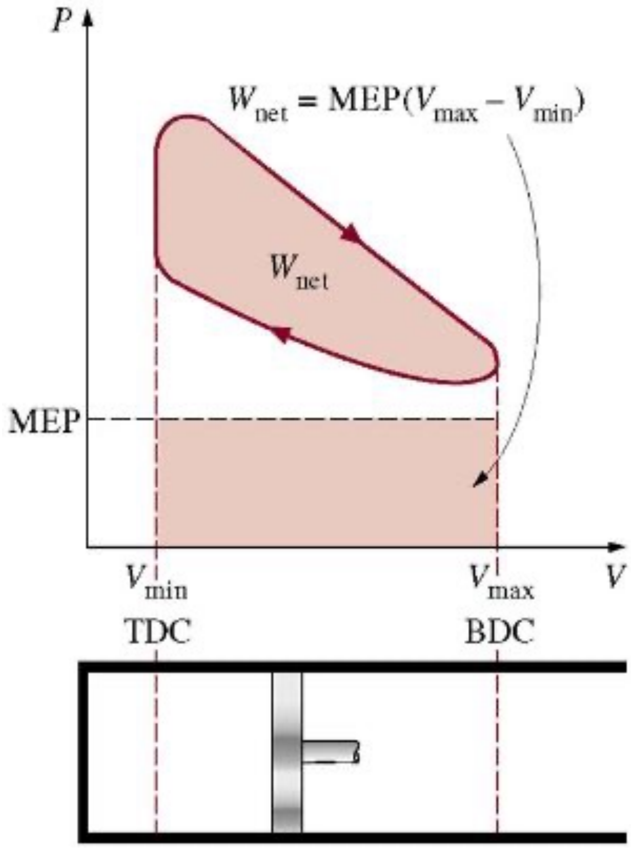




- Spark ignition (SI):
  - combustion initiated by spark
  - air and fuel can be added together

- Compression ignition (CI):
  - combustion initiated by auto ignition
  - requires fuel injection to control ignition

Mean Effective Pressure



MEP produces same net work with constant pressure as for actual cycle (includes both expansion and compression)

$$MEP = \frac{work\ for\ one\ cycle}{displacement\ volume}$$

$$MEP = \frac{W_{net}}{V_{\max} - V_{\min}}$$

Compression Ratio

$$r = \frac{V_{\max}}{V_{\min}} = \frac{V_{BDC}}{V_{TDC}}$$

Trends

- want high MEP for high power density
- increasing r leads to higher MEP
- r is limited by fuel ignition properties

<u>Fuel Ratings</u>

Octane Rating: composition of iso-octane and heptane mixture that gives same auto-ignition temperature as actual fuel

Gasoline: ~85 to 93 — to avoid auto-ignition

Diesel: ~60 — to have auto-ignition

Higher octane fuel <u>allows</u> higher compression ratios (and therefore greater efficiency and power density)

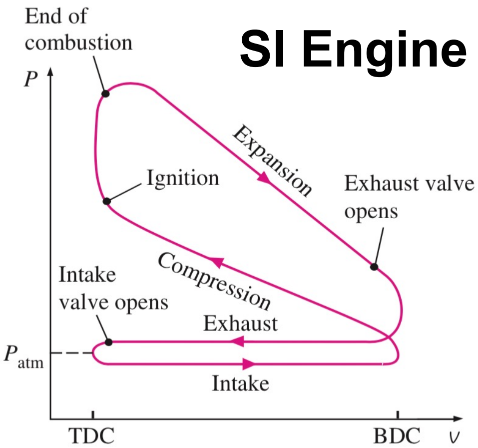
There is no benefit in using higher octane fuel in a low compression ratio engine.

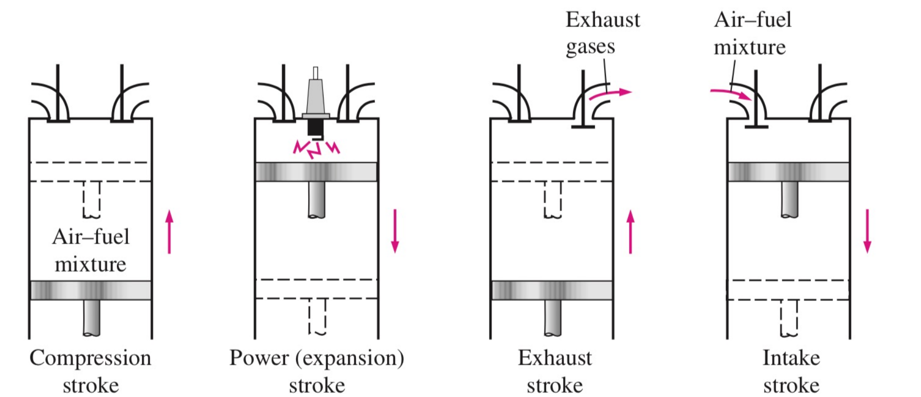
Otto Cycle

- air standard model for an SI engine

Actual SI Engine vs. Otto Cycle

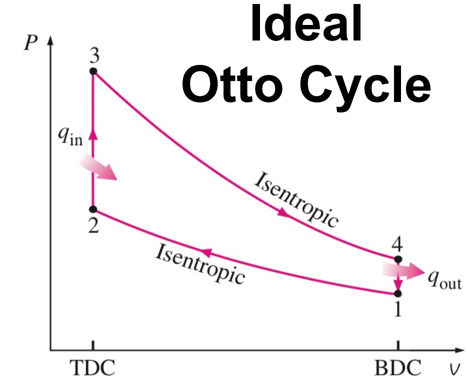
SI Engine

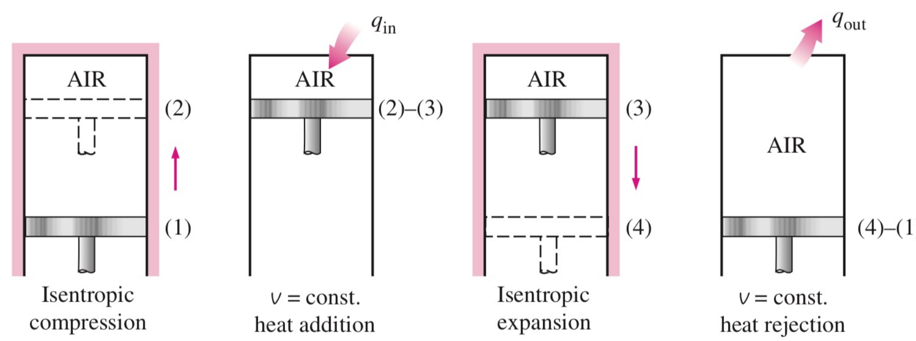




(*a*) Actual four-stroke spark-ignition engine

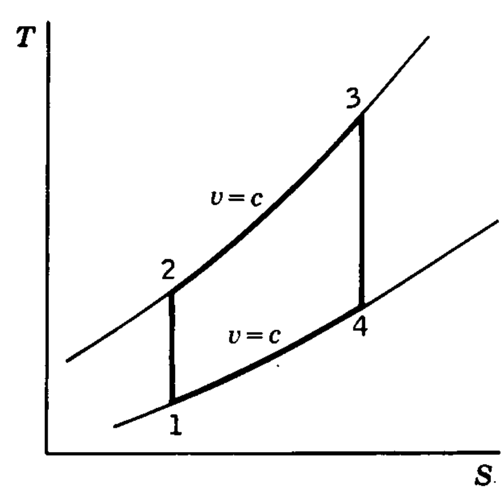
Ideal Otto Cycle





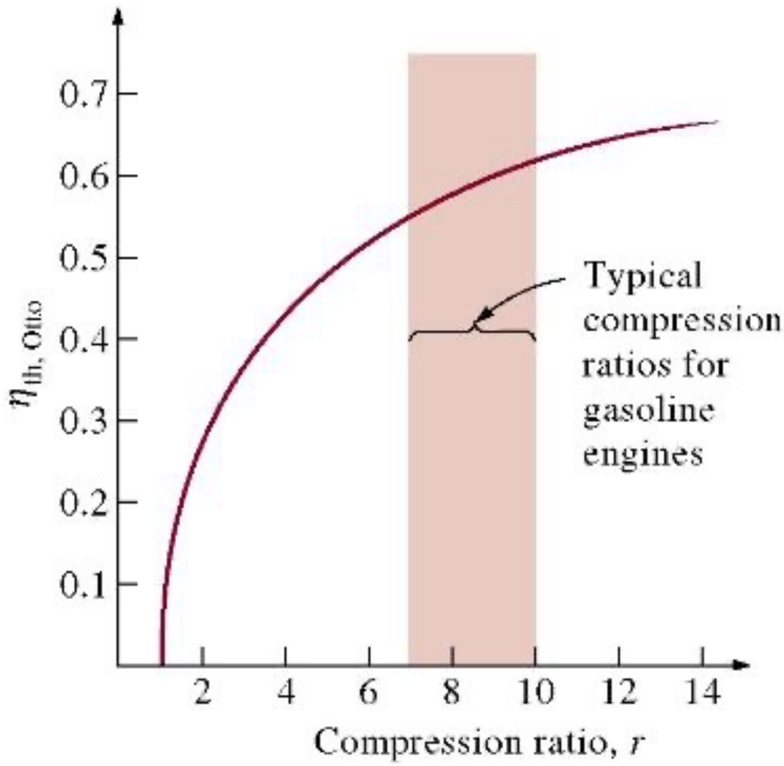
(*b*) Ideal Otto cycle

T-S Diagram



How does the Otto cycle compare with the Carnot Cycle?

Otto Cycle Trends – Compression Ratio



$$r = \frac{V_1}{V_2} = \frac{V_{BDC}}{V_{TDC}}$$

Why not use higher compression ratios?

Use P-V and T-S diagrams to show why η increases with compression ratio

<u>Otto Cycle Efficiency Analysis</u>

<u>Assumptions</u>: closed system, $\Delta KE = \Delta PE = 0$, ideal gas behavior

1st Law applied to gas in the cylinder for each process

$$1 \to 2: \quad \cancelto{0}{Q_{12}} - W_{12} = m(u_2 - u_1)$$

$$2 \to 3: \quad Q_{23} - \cancelto{0}{W_{23}} = m(u_3 - u_2)$$

$$3 \to 4: \quad \cancelto{0}{Q_{34}} - W_{34} = m(u_4 - u_3)$$

$$4 \to 1: \quad Q_{41} - \cancelto{0}{W_{41}} = m(u_1 - u_4)$$

For each cycle

$$Q_H = Q_{23}, \quad Q_L = -Q_{41}, \quad W_{net} = Q_H - Q_L$$

$$\eta_{th} = \frac{W_{net}}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

So

$$\eta_{th} = 1 - \frac{u_4 - u_1}{u_3 - u_2}$$

*** use ideal gas tables to evaluate properties ****

Otto Cycle Example

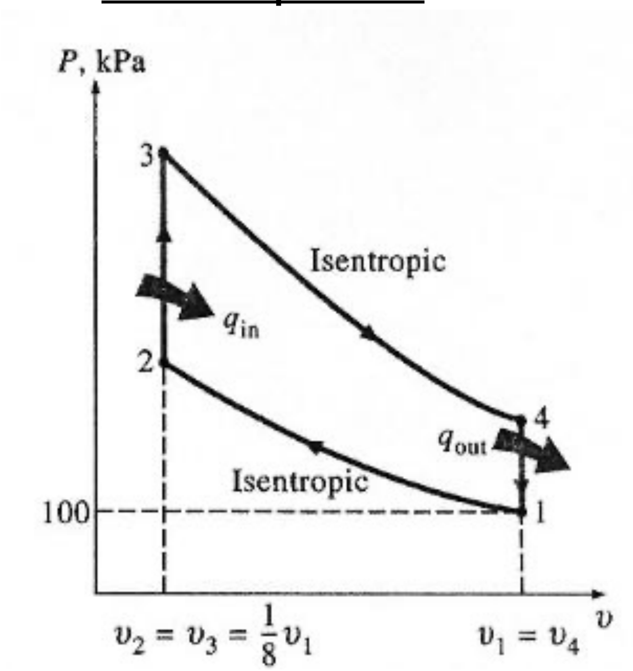
Given: Ideal Otto cycle, r=8, V_1 = 550 cm^3

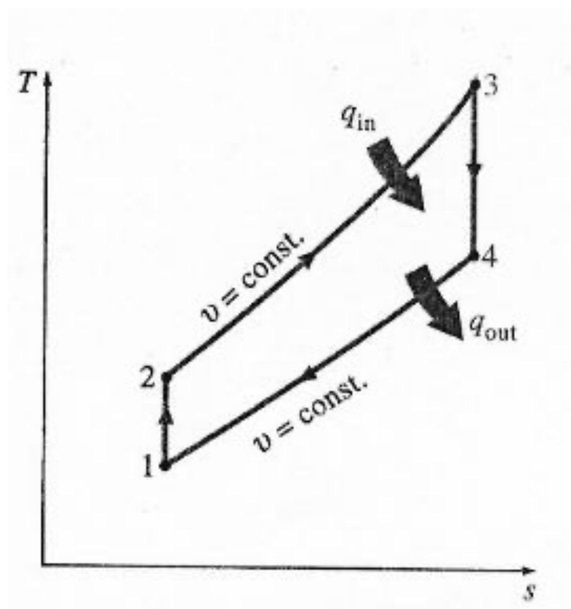
State 1: P_1 =1 atm, T_1 =300 K

State 3: T_3 = 2000 K

Find: T, P at each state, η, W_{cycle}, MEP

Assumptions: Variable specific heats, Otto Cycle





State Table

State	V(cm^3)	P(bar)	T(K)	u(kJ/kg)
1	550	1	300	
2				
3			2000	
4				

Process 1→2 (isentropic compression)

$$\frac{v_{r2}}{v_{r1}} = \frac{V_2}{V_1} = \frac{1}{r} \quad \left(\begin{array}{c}\text{isentropic compression,} \\ v_r \text{ tabulated}\end{array}\right)$$

Using data from air ideal gas tables

$$\text{At } T_1 = 300\ K, u_1 = 214.07\ \frac{kJ}{kg}$$

$$v_{r2} = \frac{v_{r1}}{r} = \frac{621.2}{8} = 77.65 \Rightarrow T_2 = 673.1\ K$$

$$u_2 = 483.4\ \frac{kJ}{kg}$$

Using the ideal gas law for fixed mass

$$m = \frac{P_2V_2}{RT_2} = \frac{P_1V_1}{RT_1} \Rightarrow P_2 = P_1\frac{V_1}{V_2}\frac{T_2}{T_1}$$

$$P_2 = 1 \text{ bar} \times 8 \times \frac{673.1}{300} = 17.95 \text{ bar}$$

Process 2→3 (constant volume heat addition)

$$V_3 = V_2$$

$$m = \frac{P_2 V_2}{RT_2} = \frac{P_3 V_3}{RT_3} \Rightarrow P_3 = P_2 \frac{T_3}{T_2}$$

$$P_3 = 17.95 \text{ bar} \times \frac{2000}{673.1} = 53.34 \; bar$$

$$u_3 = 1678.7 \; \frac{kJ}{kg}$$

Process 3→4 (isentropic expansion)

$$v_{r4} = \frac{V_4}{V_3} v_{r3} = r \cdot v_{r3} = 8 \cdot 2.776 = 22.21$$

$$\Rightarrow T_4 = 1042.9 \; K, \; u_4 = 795.9 \; kJ/kg$$

Process 4→1 (constant volume heat rejection)

$$V_1 = V_4$$

$$m = \frac{P_1 V_1}{RT_1} = \frac{P_4 V_4}{RT_4} \Rightarrow P_4 = P_1 \frac{T_4}{T_1}$$

$$P_4 = 1 \text{ bar} \times \frac{1042.9}{300} = 3.48 \text{ bar}$$

Work for Each Cycle

$$W_{net} = Q_H - Q_L = m\cdot\left(u_3 - u_2\right) - m\cdot\left(u_4 - u_1\right)$$

but,

$$m = \frac{P_1V_1}{RT_1} = \frac{\left(100\ kN/m^2\right)\cdot\left(550\ cm^3\right)}{\left(0.287\ kJ/kg-K\right)\cdot\left(300\ K\right)}\cdot\frac{1\ m^3}{(100\ cm)^3}$$

$$= 6.388\times10^{-4}\ \text{kg}$$

$$W_{net} = 6.388x10^{-4}\,kg\begin{bmatrix}\left(1678.7-483.4\right)\\ -\left(795.9-214.08\right)\end{bmatrix}kJ/\text{kg}$$

$$= 0.392\ kJ/cycle$$

Mean Effective Pressure (MEP)

$$W_{net} = m\cdot w_{net} = MEP\quad\overbrace{\left(V_1 - V_2\right)}^{displacement\ \ volume}$$

$$\Rightarrow MEP = \frac{W_{net}}{V_1 - V_2} = \frac{0.392\ kJ}{\left(550-\dfrac{550}{8}\right)\dfrac{1}{10^6}\,m^3}$$

$$MEP = 814.6\ kPa = 8.15\ bar$$

Cycle Efficiency

$$\eta_{th} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{u_4 - u_1}{u_3 - u_2} = 1 - \frac{795.9 - 214.07}{1678.7 - 483.4}$$

$$\eta_{th} = 0.51$$

Comparison to Carnot

$$\eta_{carnot} = 1 - \frac{T_L}{T_H}, \; assume\; T_L = T_1,\; T_H = T_3$$

$$\Rightarrow \eta_{Carnot} = 1 - \frac{300}{2000} \;\Rightarrow\; \eta_{Carnot} = 0.85$$

Relation to Real Engine

$$V_{disp} = V_1 - V_2 \sim 500\; cm^3 \text{ (e.g., motorcycle)}$$

For 3000 cycles/min (3000 rpm for 2-stroke engine)

$$\dot{W}_{net} = N \cdot W_{net} \sim 26\; hp$$

Diesel Cycle

- air standard model for a CI engine

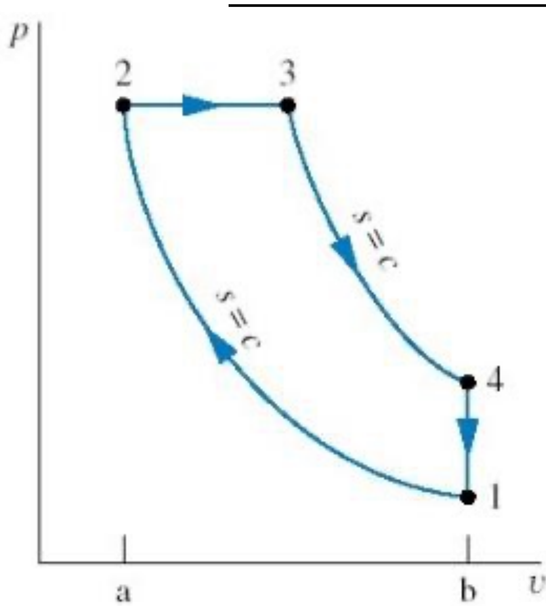
SI engines

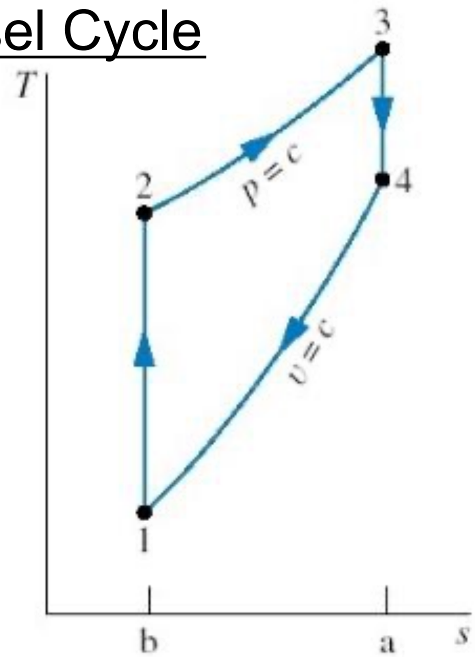
- Air-fuel mixture compressed to below fuel auto-ignition temperature
- Limits compression ratio and efficiency to avoid knocking
- Combustion initiated by a spark near TDC
- Combustion process modeled as constant volume

Diesel engines

- Air compressed above fuel auto-ignition temperature
- Fuel injection via liquid spray near TDC; evaporates & ignites on contact with hot compressed air, e.g. compression ignition
- Combustion process modeled as constant pressure

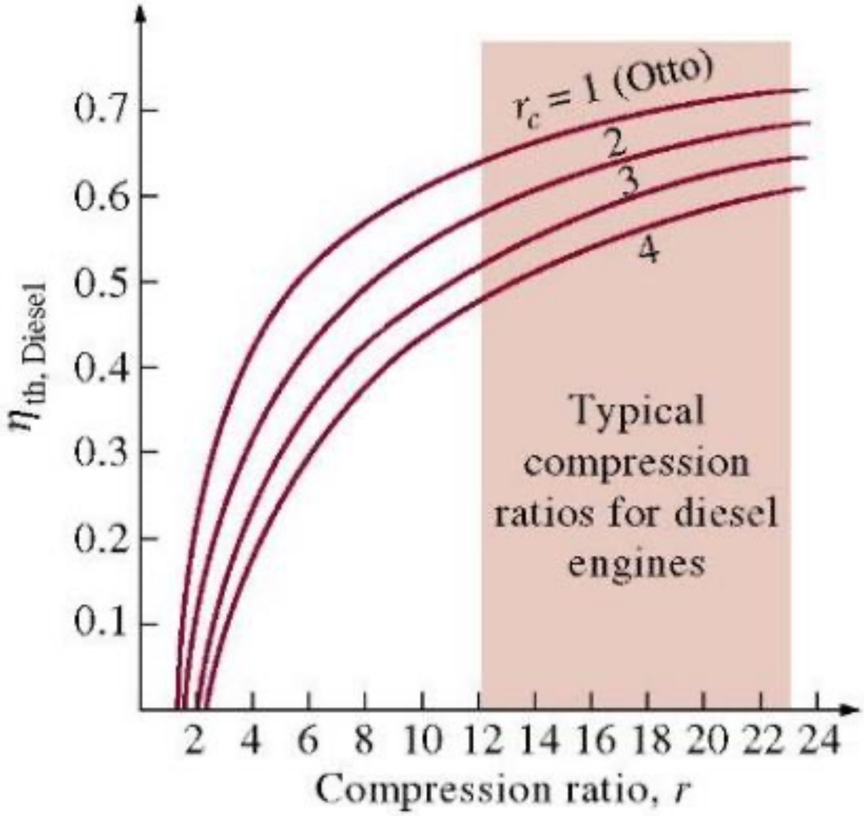
Air-Standard Diesel Cycle





$$r = \frac{V_1}{V_2} = compression\ ratio,\ r_c = \frac{V_3}{V_2} = cutoff\ \ ratio$$

Diesel versus Otto Cycle Efficiency



Why do current diesel engines typically have higher efficiencies than gasoline engines

Use P-V and T-S diagrams to show why the Otto cycle is more efficient than the Diesel cycle for the same compression ratio

Diesel Cycle Efficiency Analysis

Assumptions: closed system, $\Delta KE = \Delta PE = 0$, ideal gas behavior

1st Law applied to gas in the cylinder for each process

$$1 \rightarrow 2: \quad \cancelto{0}{Q_{12}} - W_{12} = m(u_2 - u_1)$$

$$2 \rightarrow 3: \quad Q_{23} - W_{23} = m(u_3 - u_2)$$

$$3 \rightarrow 4: \quad \cancelto{0}{Q_{34}} - W_{34} = m(u_4 - u_3)$$

$$4 \rightarrow 1: \quad Q_{41} - \cancelto{0}{W_{41}} = m(u_1 - u_4)$$

For each cycle

$$Q_H = Q_{23} = m(\Delta u_{23} + P\Delta v_{23}) = m\Delta h_{23},$$

$$Q_L = -Q_{41}, \quad W_{net} = Q_H - Q_L$$

So

$$\eta_{th} = \frac{W_{net}}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

$$\eta_{th} = 1 - \frac{u_4 - u_1}{h_3 - h_2}$$

*** use ideal gas tables to evaluate properties ****

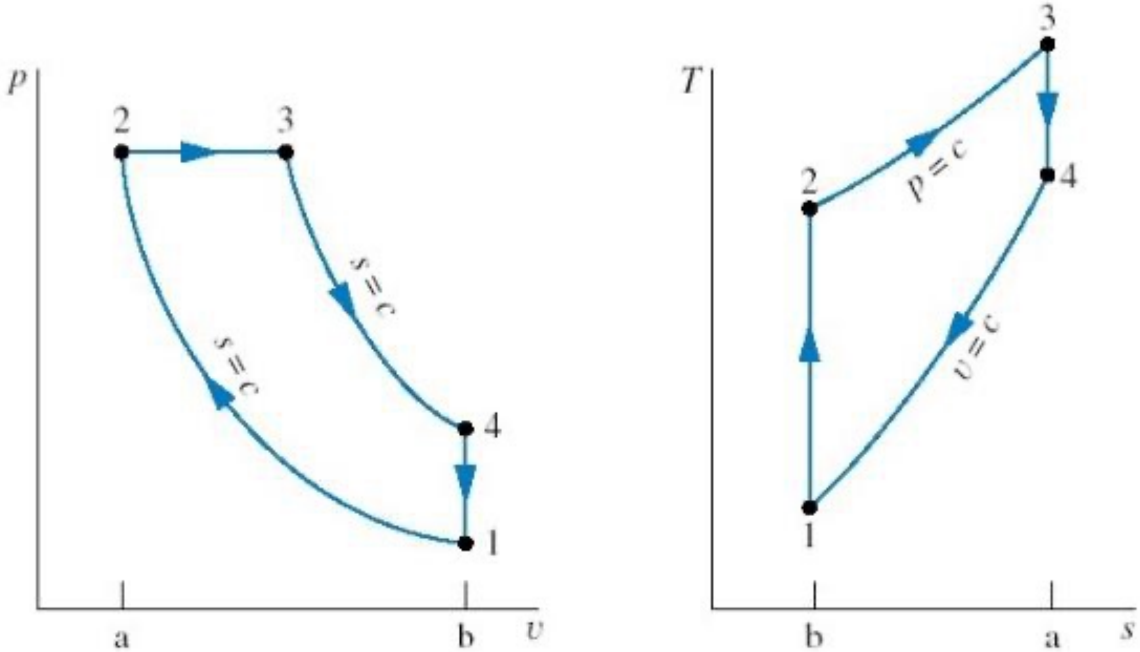
Diesel Cycle Example

Given: Ideal Diesel Cycle, r = 18, r_c = 2

P_1 = 0.1 MPa, T_1 = 300 K

Find: T and P for states 2, 3, and 4, η_{th}, w_{net}, & MEP

Assumptions: Diesel Cycle with variable spec. heats



Process 1→2 (isentropic compression)

$$\frac{v_{r2}}{v_{r1}} = \frac{V_2}{V_1} = \frac{1}{r} \quad \left(\begin{array}{l}\text{isentropic compression,} \\ \quad v_r \text{ tabulated}\end{array}\right)$$

Using data from ideal gas tables for air

$$\text{At } T_1 = 300\ K,\ u_1 = 214.1\ \frac{kJ}{kg}$$

$$v_{r2} = \frac{v_{r1}}{r} = \frac{621.2}{18} = 34.51 \Rightarrow T_2 = 898.3\ K$$

$$u_2 = 673.1\ \frac{kJ}{kg},\ h_2 = 930.9\ \frac{kJ}{kg}$$

Using the ideal gas law for a closed system (constant m)

$$\frac{PV}{T} = mR \Rightarrow P_2 = P_1 \frac{T_2}{T_1} \cdot \frac{V_1}{V_2} = 0.1 \cdot \frac{898.3}{300} \cdot 18 = 5.39 \text{ MPa}$$

Process 2→3 (constant pressure heat addition)

$$P_3 = P_2 = \ 5.39 \ MPa$$

$$m = \frac{P_2 V_2}{RT_2} = \frac{P_3 V_3}{RT_3} \Rightarrow T_3 = \frac{V_3}{V_2} T_2 = r_c \cdot T_2$$

$$T_3 = 1796.6 \ K$$

$$u_3 = 1490.1 \ \frac{kJ}{kg}, \ h_3 = 1999.1 \ \frac{kJ}{kg} \ \ \text{(ideal gas tables)}$$

Process 3→4 (isentropic expansion)

$$v_{r4} = \frac{V_4}{V_3} v_{r3} = \frac{V_4}{V_2} \frac{V_2}{V_3} v_{r3} = r \cdot \frac{1}{r_c} \cdot v_{r3} = 18 \cdot \frac{1}{2} \cdot 3.97 = 35.73$$

$$\Rightarrow T_4 = 887.7 \ K, \ u_4 = 664.3 \frac{kJ}{kg} \ \text{(Table A-22)}$$

$$P_4 = P_3 \frac{T_4}{T_3} \cdot \frac{V_3}{V_4} = P_3 \frac{T_4}{T_3} \cdot \frac{V_2}{V_4} \cdot \frac{V_3}{V_2} = P_3 \frac{T_4}{T_3} \cdot \frac{1}{r} \cdot r_c$$

$$P_4 = 5.39 \ MPa \cdot \frac{887.7}{1796.6} \cdot \frac{1}{18} \cdot 2 = 0.296 \ MPa$$

Cycle Efficiency

$$\eta_{th} = 1 - \frac{q_L}{q_H} = 1 - \frac{u_4 - u_1}{h_3 - h_2} = 1 - \frac{664.3 - 214.1}{1999.1 - 930.9}$$

$$\eta_{th} = 0.578$$

Mean Effective Pressure (MEP)

$$W_{net} = m \cdot w_{net} = MEP \quad \overbrace{\left(V_1 - V_2\right)}^{displacement\ volume}$$

$$\Rightarrow MEP = \frac{w_{net}}{v_1 - v_2} = \frac{w_{net}}{v_1\left(1 - \frac{1}{r}\right)}$$

From the ideal gas law

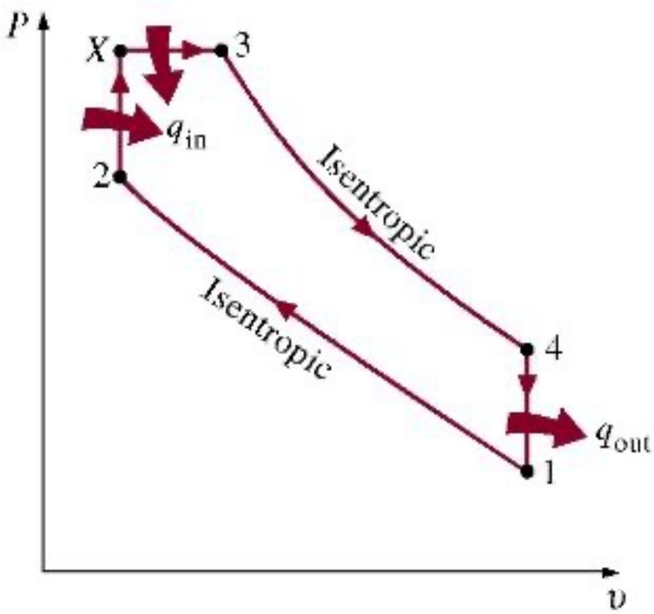
$$v_1 = \frac{RT_1}{P_1} = \frac{8314}{28.97}\frac{Nm}{kgK} \cdot \frac{300K}{1x10^5\ \frac{N}{m^2}} = 0.861\frac{m^3}{kg}$$

Then

$$MEP = \frac{617.9\frac{kJ}{kg}}{0.861\frac{m^3}{kg}\left(1 - \frac{1}{18}\right)} \cdot \frac{10^3\,Nm}{1\ kJ} \cdot \frac{1MPa}{10^6\ \frac{N}{m^2}}$$

$$MEP = 0.76\ MPa$$

Dual Cycle



- Combustion process in actual IC engines is neither constant volume nor constant pressure
- Better model is combined part constant volume/part constant pressure combustion process
- This results in dual cycle
- Adjust X to match actual cycle

Cycle Parameters

$$r = \frac{V_1}{V_2} = \frac{V_{BDC}}{V_{TDC}} = compression\ ratio$$

$$r_c = \frac{V_3}{V_x} = \frac{V_3}{V_2} = cutoff\ \ ratio\ (1\, for\ Otto\ Cycle)$$

$$r_p = \frac{P_x}{P_2} = \frac{P_3}{P_2} = pressure\ ratio\ (1\, for\ Diesel\ Cycle)$$

Dual Cycle Efficiency Analysis

Assumptions: closed system, $\Delta KE = \Delta PE = 0$, ideal gas behavior

1st Law applied to gas in the cylinder for each process

$$1 \rightarrow 2: \quad Q_{12} - W_{12} = m(u_2 - u_1)$$

$$2 \rightarrow x: \quad Q_{2x} - W_{2x} = m(u_x - u_2)$$

$$x \rightarrow 3: \quad Q_{x3} - W_{x3} = m(u_3 - u_x)$$

$$3 \rightarrow 4: \quad Q_{34} - W_{34} = m(u_4 - u_3)$$

$$4 \rightarrow 1: \quad Q_{41} - W_{41} = m(u_1 - u_4)$$

For each cycle

$$Q_H = Q_{2x} + Q_{x3} = m\Delta u_{2x} + m(\Delta u_{x3} + P\Delta v_{x3})$$

$$Q_H = m\left(\Delta u_{2x} + \Delta h_{x3}\right), \; Q_L = -Q_{41}, \; W_{net} = Q_H - Q_L$$

$$\eta_{th} = \frac{W_{net}}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

So

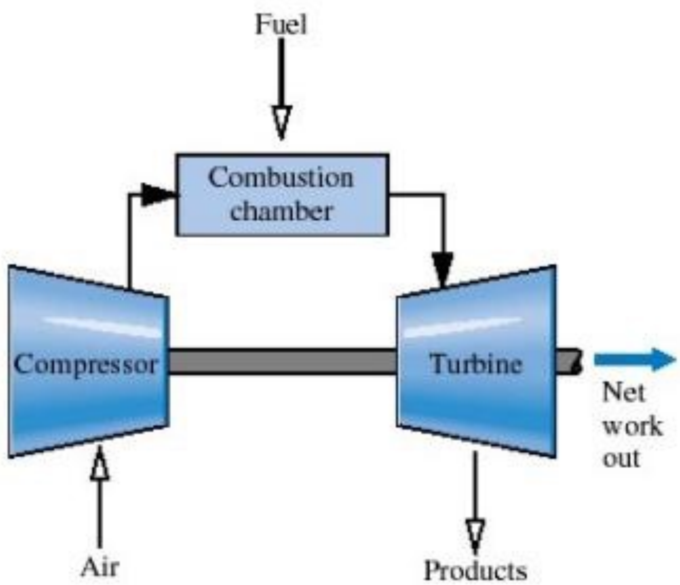
$$\eta_{th} = 1 - \frac{u_4 - u_1}{\left(u_x - u_2\right) + \left(h_3 - h_x\right)}$$

*** use ideal gas tables to evaluate properties ****

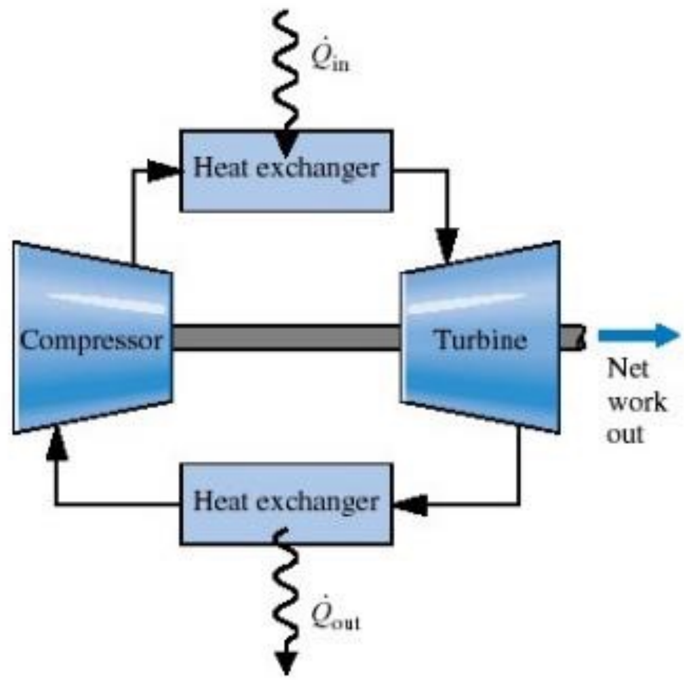
Lectures 39 & 40 - Brayton Cycle

- air standard model for gas turbine engines

<u>Gas Turbine Engine</u>



Open Cycle

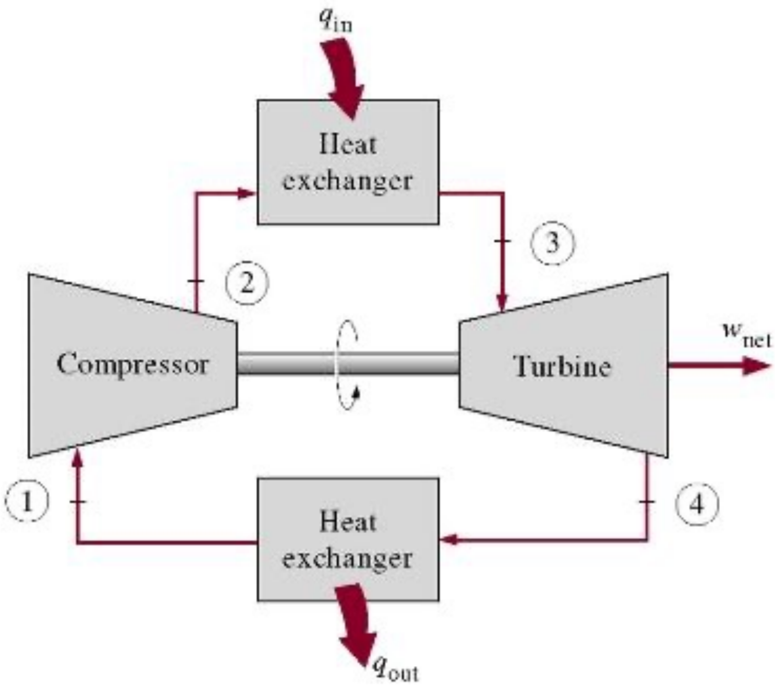


Closed Cycle

<u>Common Applications</u>

- Aircraft propulsion
- Stationary power plant (marine, industrial, peaking)
- Coupled with Rankine Cycle for very high efficiencies (Combined Cycle Plant)

Brayton Cycle



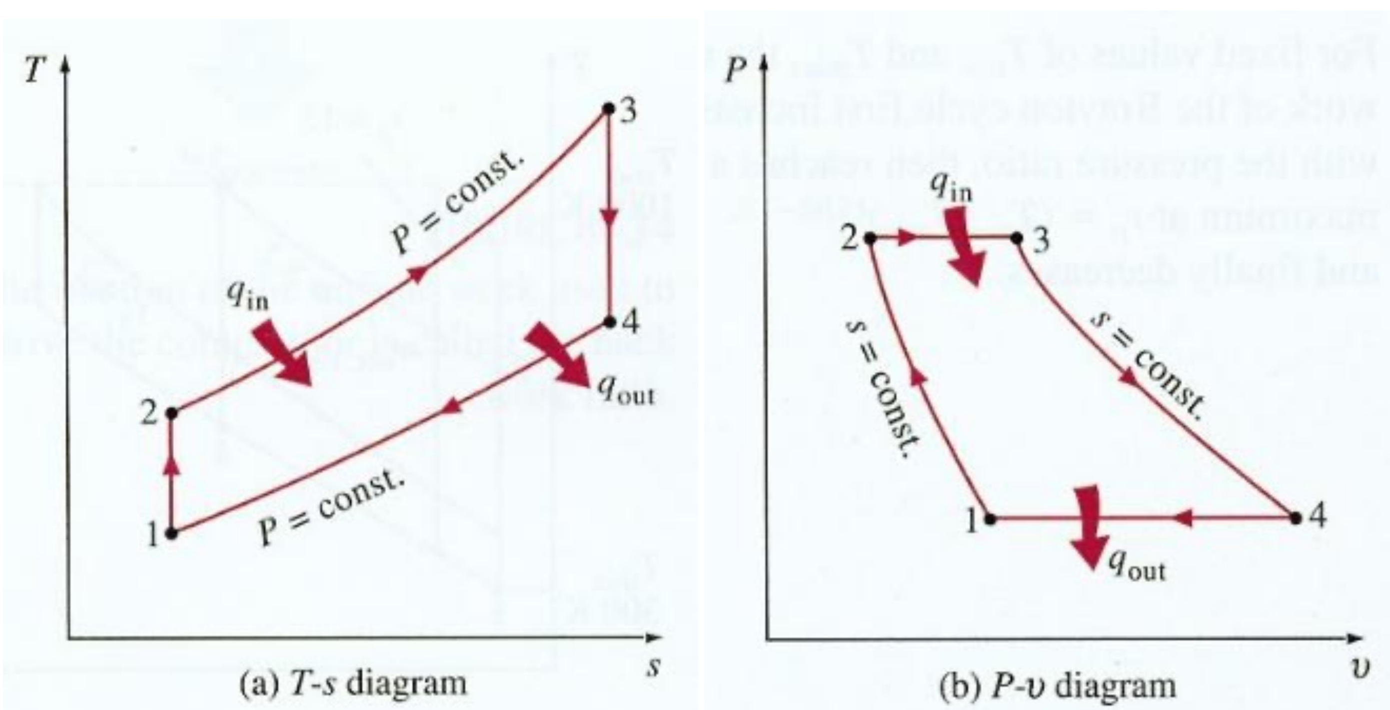
Idealized closed cycle

1-2: Isentropic comp.

2-3: Constant-pressure heat addition

3-4: Isentropic exp.

4-1: Constant-pressure heat rejection



Brayton Cycle Efficiency Analysis

Assumptions: SSSF components, Δke = Δpe = 0, ideal gas behavior

1st Law applied to gas in each device

$$1 \rightarrow 2: \quad \cancelto{0}{\dot{Q}_{12}} - \dot{W}_{12} = \dot{m}(h_2 - h_1)$$

$$2 \rightarrow 3: \quad \dot{Q}_{23} - \cancelto{0}{\dot{W}_{23}} = \dot{m}(h_3 - h_2)$$

$$3 \rightarrow 4: \quad \cancelto{0}{\dot{Q}_{34}} - \dot{W}_{34} = \dot{m}(h_4 - h_3)$$

$$4 \rightarrow 1: \quad \dot{Q}_{41} - \cancelto{0}{\dot{W}_{41}} = \dot{m}(h_1 - h_4)$$

At steady-steady state

$$\dot{Q}_H = \dot{Q}_{23}, \quad \dot{Q}_L = -\dot{Q}_{41}, \quad \dot{W}_{net} = \dot{Q}_H - \dot{Q}_L$$

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{\dot{Q}_H - \dot{Q}_L}{\dot{Q}_H} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H}$$

So

$$\eta_{th} = 1 - \frac{h_4 - h_1}{h_3 - h_2}$$

Back-Work Ratio:

$$r_{BW} = \frac{\dot{W}_C}{\dot{W}_T} = \frac{h_2 - h_1}{h_3 - h_4}$$

Cold-Air-Standard Efficiency

- Brayton cycle with constant specific heats
- useful for illustrating performance trends

$$\eta_{th} = 1 - \frac{h_4 - h_1}{h_3 - h_2} = 1 - \frac{C_p\left(T_4 - T_1\right)}{C_p\left(T_3 - T_2\right)}$$

$$= 1 - \frac{T_1}{T_2}\frac{\left(T_4/T_1 - 1\right)}{\left(T_3/T_2 - 1\right)}$$

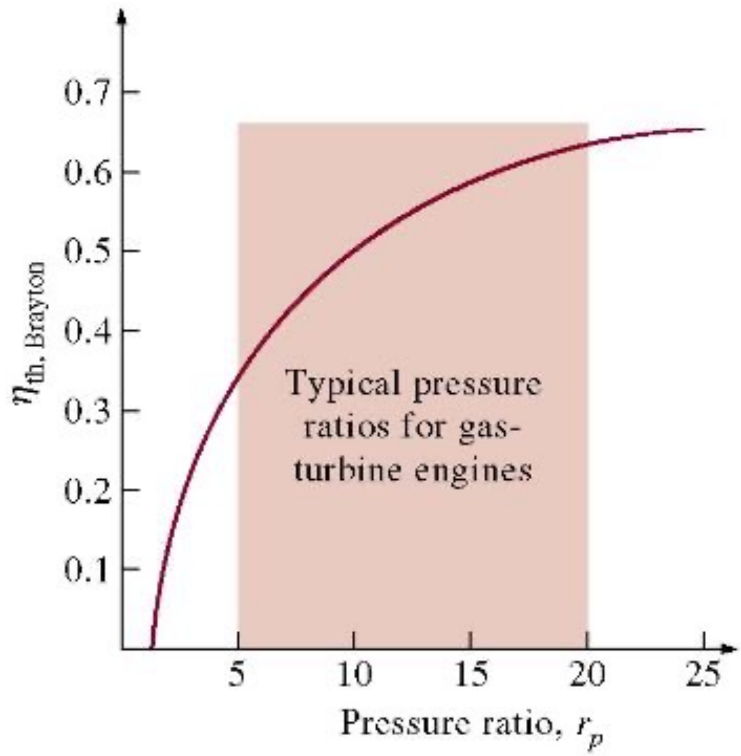
But for ideal gas & isentropic process & constant k

$$\left.\begin{aligned} \frac{T_2}{T_1} &= \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = r_p^{(k-1)/k} \\ \frac{T_3}{T_4} &= \left(\frac{P_3}{P_4}\right)^{(k-1)/k} = r_p^{(k-1)/k} \end{aligned}\right\} \quad \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

So

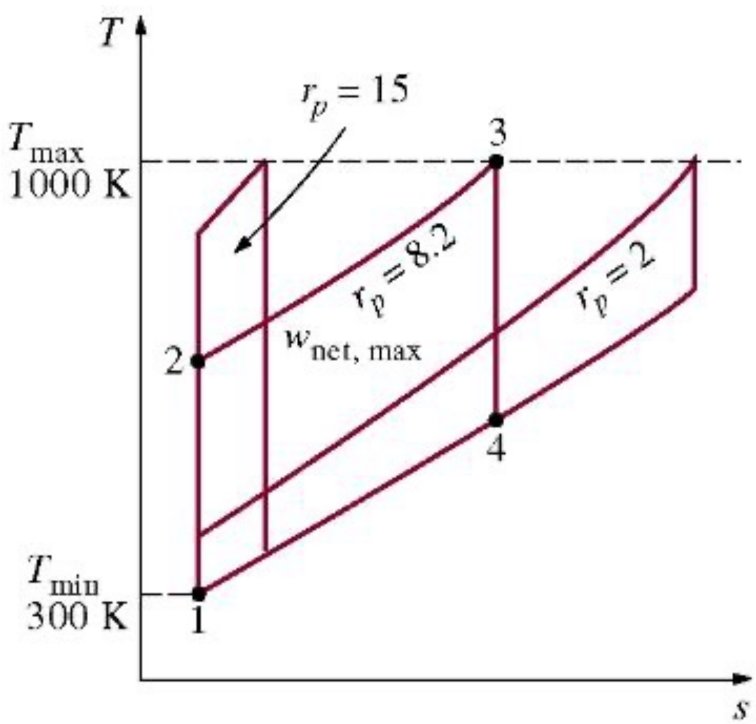
$$\eta_{th} = 1 - \frac{T_1}{T_2} \quad or \quad \eta_{th} = 1 - \frac{1}{r_p^{(k-1)/k}}$$

Brayton Cycle Performance Trends



cold-air-standard assumption with k = 1.4

Effect of pressure ratio for fixed min/max temperatures



Which cycle has the greater specific work output?

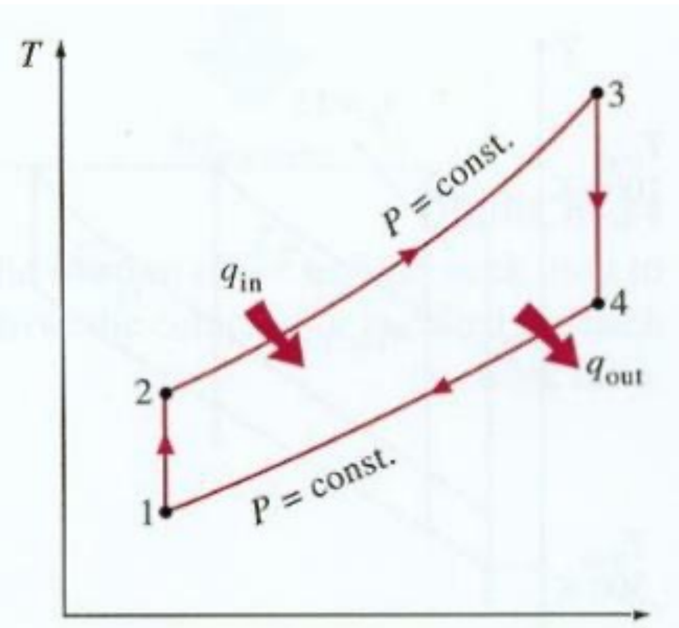
Which cycle has the greater efficiency?

Brayton Cycle Example

Given: Ideal Brayton Cycle, r_p = 10
P_1 = 0.1 MPa, T_1 = 300 K, $\dot{V}_1 = 5\ m^3/s$
T_3 = 1400 K ,

Find: η_{th}, $\dot{W}_{net}$, $\dot{W}_C / \dot{W}_T$

Assumptions: Ideal Brayton Cycle with var. spec. heats



(a) T-s diagram

State	P (kPa)	T(K)	h (kJ/kg)
1	100	300	
2	1000		
3	1000	1400	
4	100		

Process 1→2 (isentropic compression)

$$\frac{P_{r2}}{P_{r1}} = \left(\frac{P_2}{P_1}\right)_{s_2=s_1} = r_p \quad \left(\begin{array}{c}\text{isentropic compression,} \\ P_r \text{ tabulated}\end{array}\right)$$

Using data from ideal gas tables for air

$$\text{At } T_1 = 300\ K,\ P_{r1} = 1.386,\ \ h_1 = 300.2\ \frac{kJ}{kg}$$

$$P_{r2} = r_p \cdot P_{r1} = 10 \cdot 1.386 = 13.86\ \Rightarrow T_2 = 574\ K$$

$$h_2 = 579.9\ \frac{kJ}{kg}$$

<u>Process 3→4 (isentropic expansion)</u>

$$\frac{P_{r4}}{P_{r3}} = \left(\frac{P_4}{P_3}\right)_{s_4 = s_3} = \frac{1}{r_p}$$

Using data from ideal gas tables for air

$$\text{At } T_3 = 1400\ K,\ P_{r3} = 450.5,\ \ h_3 = 1515.4\ \frac{kJ}{kg}$$

$$P_{r4} = P_{r3} / r_p = 450.5 / 10 = 45.05\ \Rightarrow T_4 = 788\ K$$

$$h_4 = 808.3\ \frac{kJ}{kg}$$

Cycle Efficiency, Back-work Ratio, Net Power

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_T - w_c}{q_H}$$

$$w_T = \frac{\dot{W}_T}{\dot{m}} = h_3 - h_4 = 707.1 \text{ kJ/kg}$$

$$w_C = \frac{\dot{W}_C}{\dot{m}} = h_2 - h_1 = 279.7 \text{ kJ/kg}$$

$$q_H = \frac{\dot{Q}_H}{\dot{m}} = h_3 - h_2 = 935.5 \text{ kJ/kg}$$

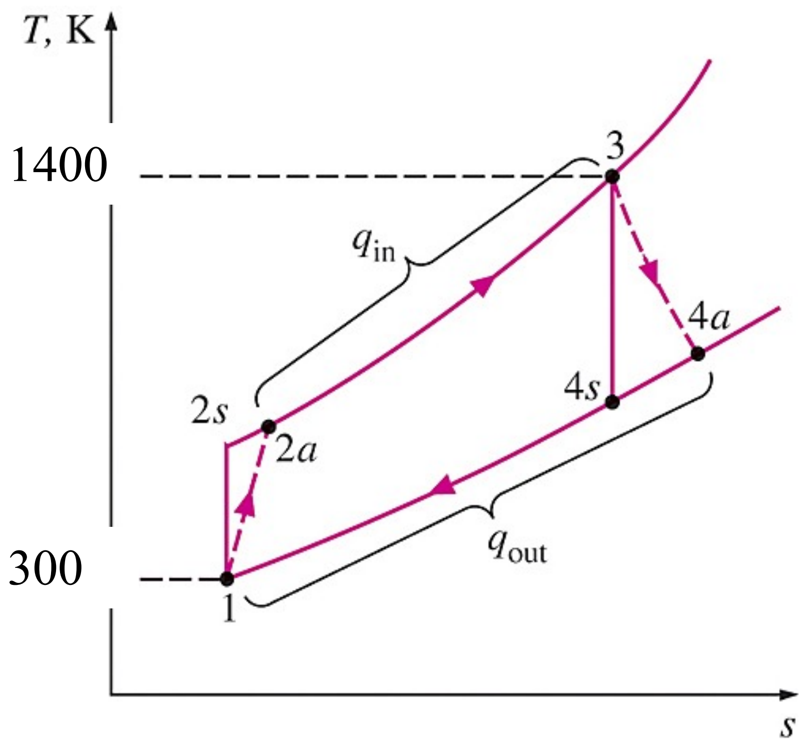
$$\Rightarrow \boxed{\eta = 0.46, \quad \frac{\dot{W}_C}{\dot{W}_T} = \frac{w_C}{w_T} = 0.4}$$

$$\dot{W}_{net} = \dot{m}\left(w_T - w_C\right)$$

$$\text{but} \quad \dot{m} = \frac{\dot{V}_1}{v_1} \quad \text{with } v_1 = \frac{RT_1}{P_1} = 0.86\ m^3/kg$$

$$\Rightarrow \dot{m} = 5.81\ kg/s \ \text{ and } \ \boxed{\dot{W}_{net} = 2483 \text{ kW}}$$

Part 2 – Now assume $\eta_C = 0.8$, $\eta_T = 0.8$



$$\eta_C = \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

$$\eta_T = \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$$

$$\frac{\dot{W}_C}{\dot{m}} = \frac{h_{2s} - h_1}{\eta_C} = \frac{279.7 \text{ kJ/kg}}{0.8} = 349.6 \text{ kJ/kg}$$

$$\frac{\dot{W}_T}{\dot{m}} = \eta_T \cdot \left(h_3 - h_{4s}\right) = 0.8 \cdot 707.1 = 565.7 \text{ kJ/kg}$$

$$\frac{\dot{Q}_H}{\dot{m}} = h_3 - h_2$$

$$\text{but} \quad h_2 = h_1 + \frac{\dot{W}_C}{\dot{m}} = 649.8 \ \ kJ/kg$$

$$\Rightarrow \frac{\dot{Q}_H}{\dot{m}} = 1515.4 - 649.8 = 856.6 \ kJ/kg$$

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{w_T - w_c}{q_H} = \frac{565.7 - 349.6}{856.6} \Rightarrow \boxed{\eta = 0.25}$$

$$\boxed{\frac{\dot{W}_C}{\dot{W}_T} = \frac{w_C}{w_T} = 0.62}$$

Notes:

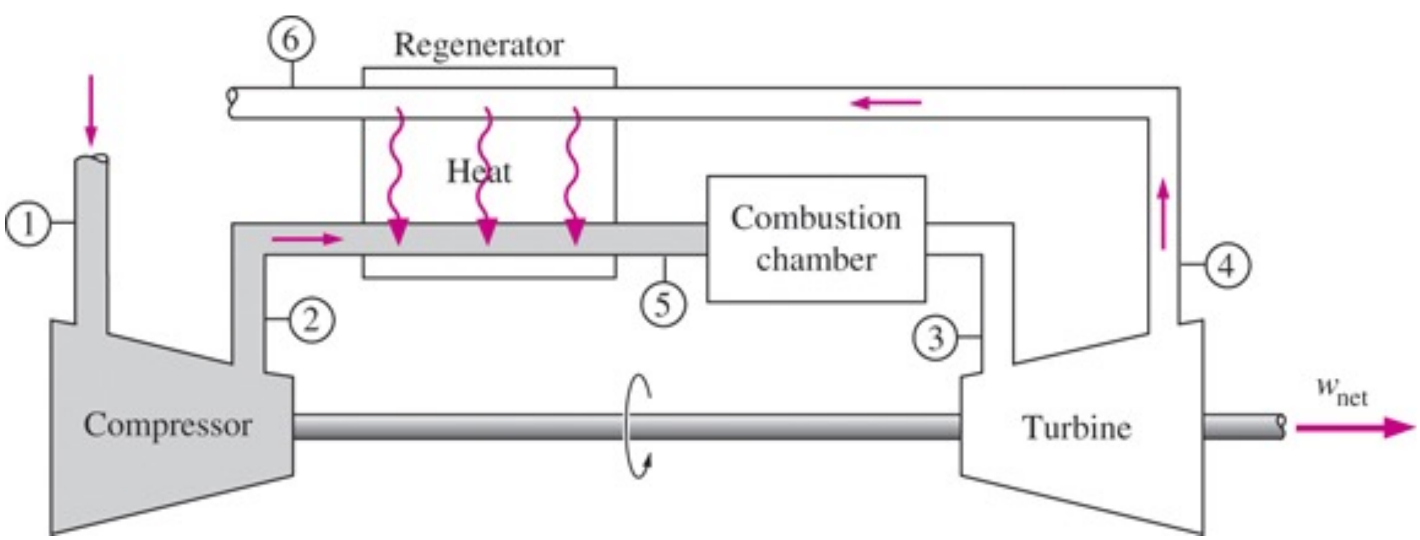
- The overall efficiency is reduced by about 45%!
- Gas turbine performance is very sensitive to turbine and compressor efficiencies!

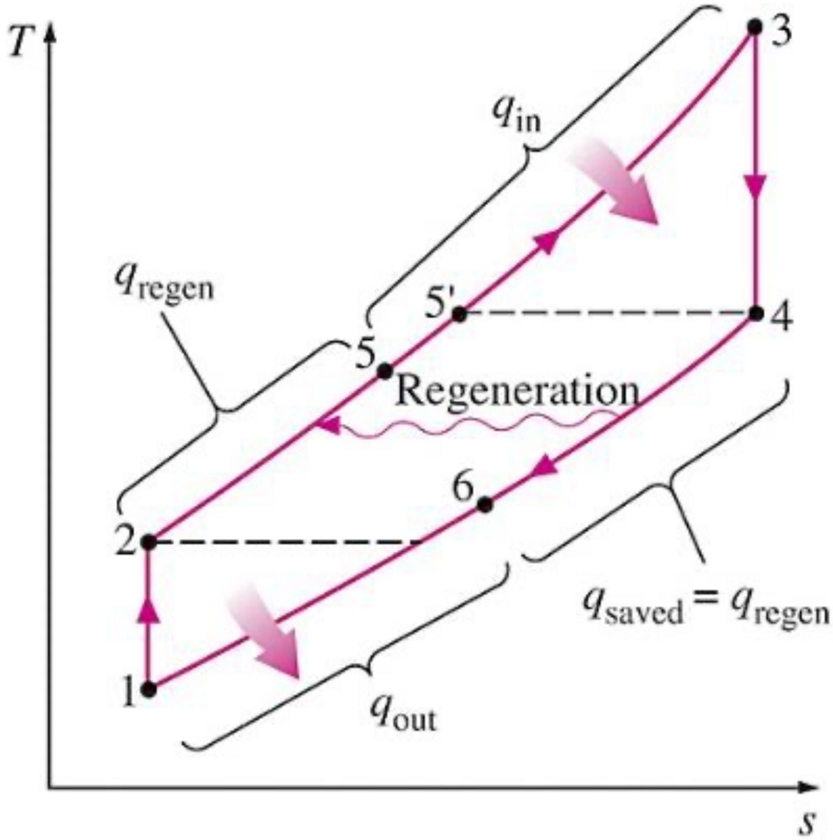
Gas Turbine Improvements

One way to improve efficiency is to raise the average temperature for heat addition from the source and/or reduce the average temperature for heat rejection to the sink through: 1) regeneration and/or 2) reheat.

Another approach to improve efficiency is to reduce the back-work ratio through compressor cooling or intercooling.

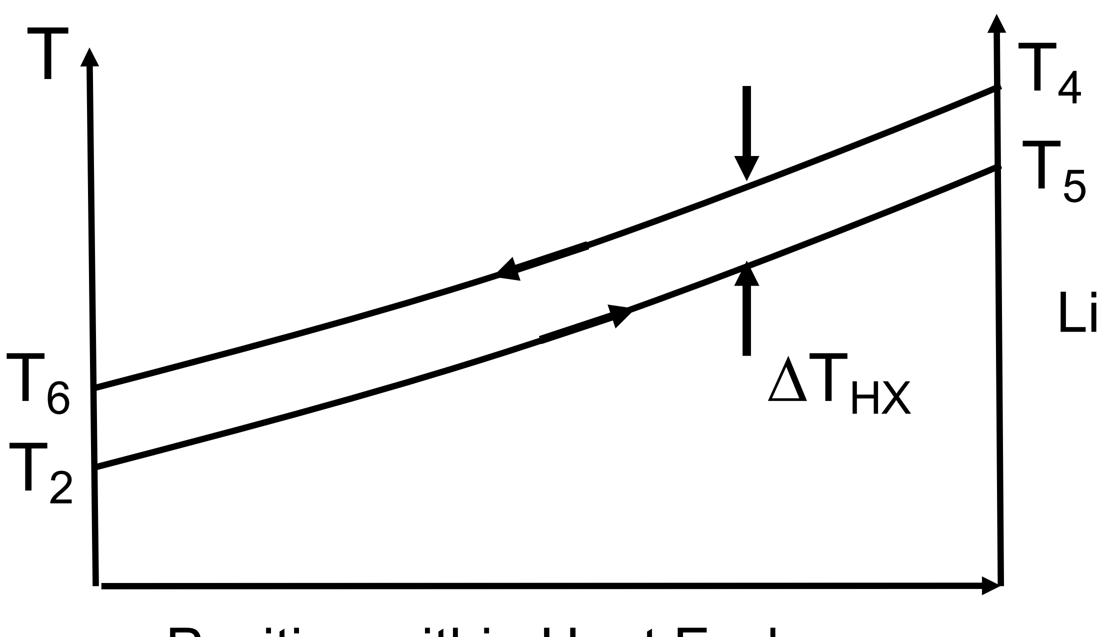
Regeneration



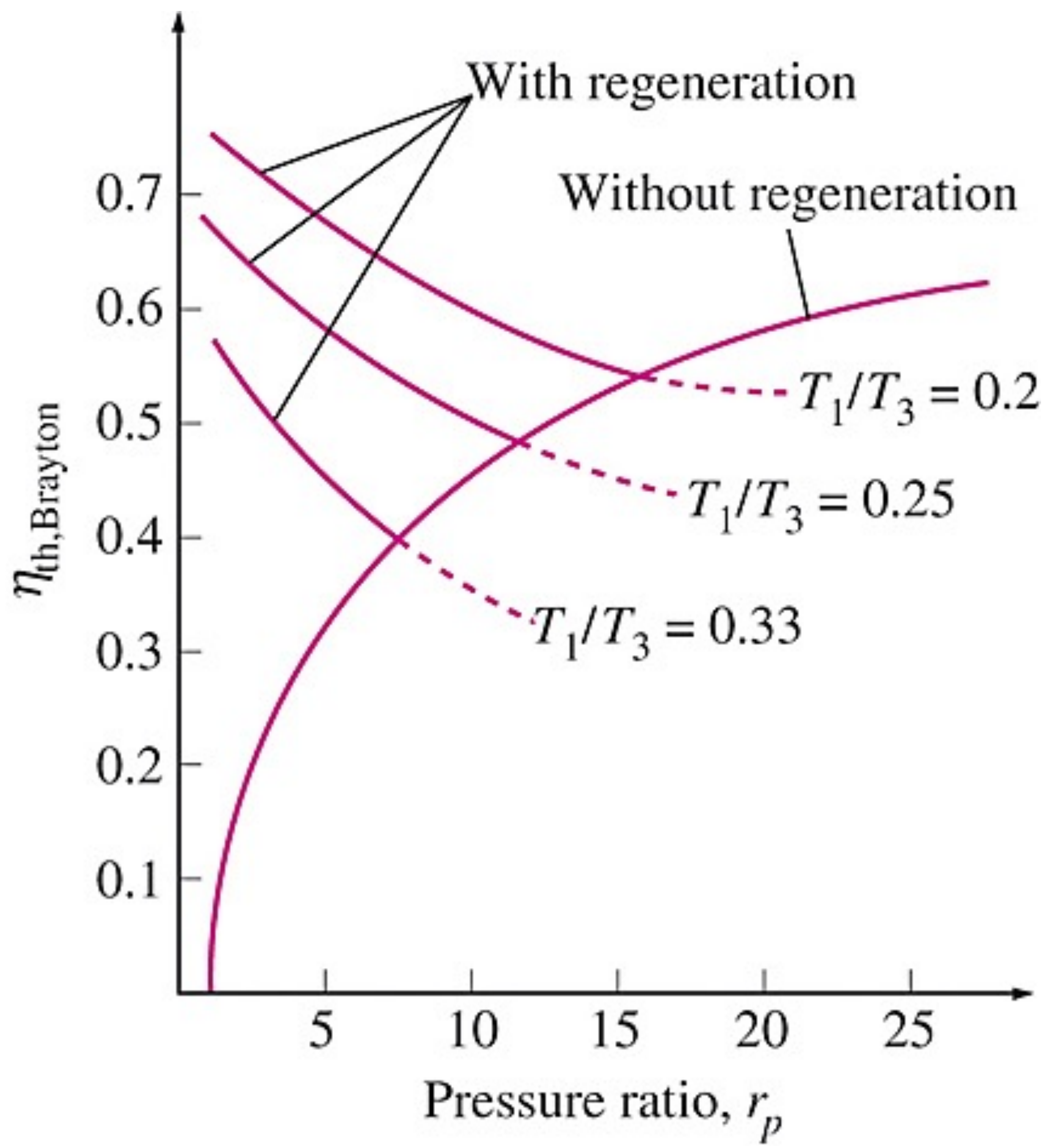


T

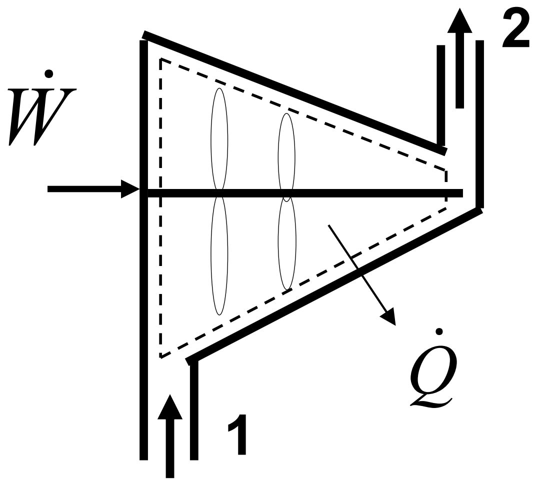
Heat Transfer Limitations



Limiting States?


With regeneration
Without regeneration
0.7
0.6
0.5
0.4
0.3
0.2
0.1
$\eta_{th,Brayton}$
$T_1/T_3 = 0.2$
$T_1/T_3 = 0.25$
$T_1/T_3 = 0.33$
5
10
15
20
25
Pressure ratio, r_p

Compressor Cooling

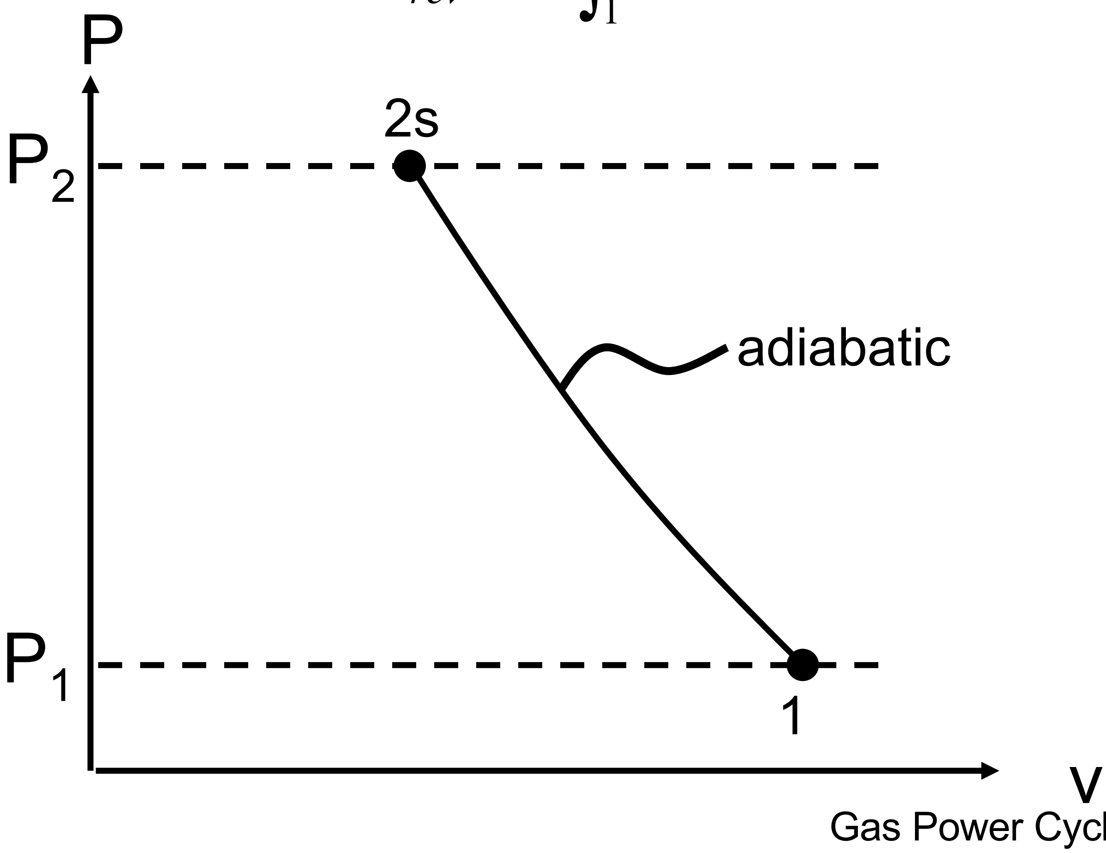


If the compressor is cooled then the 1st Law for steady-state and neglibible Δke and Δpe gives

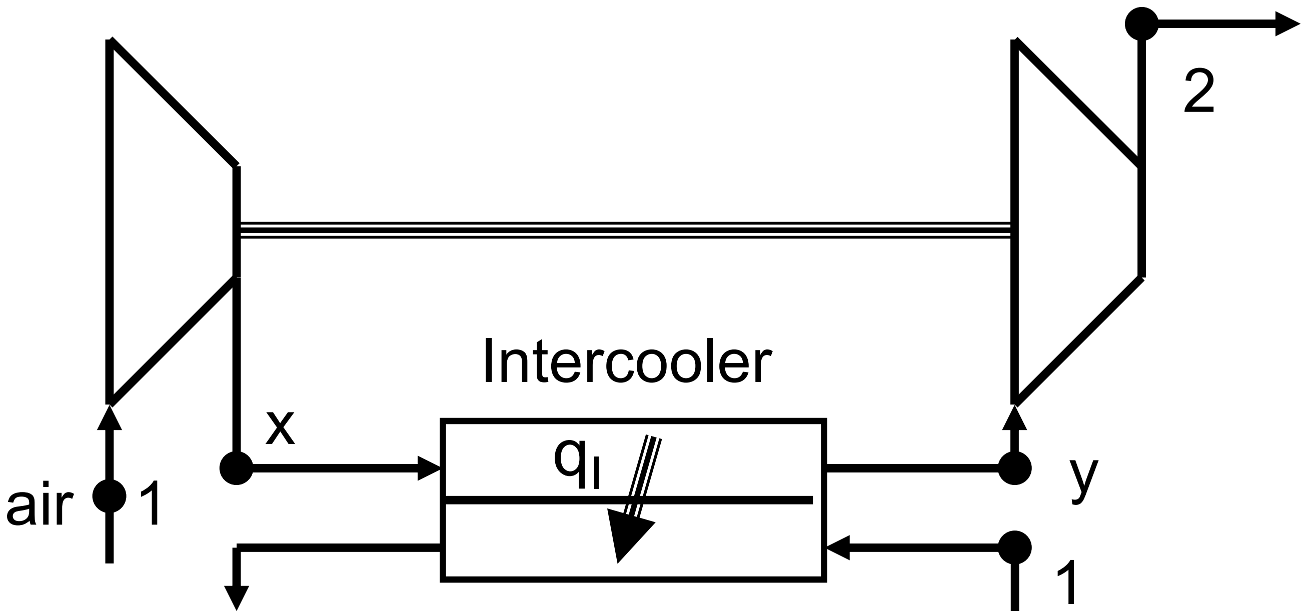
$$w = h_1 - h_2 + q$$

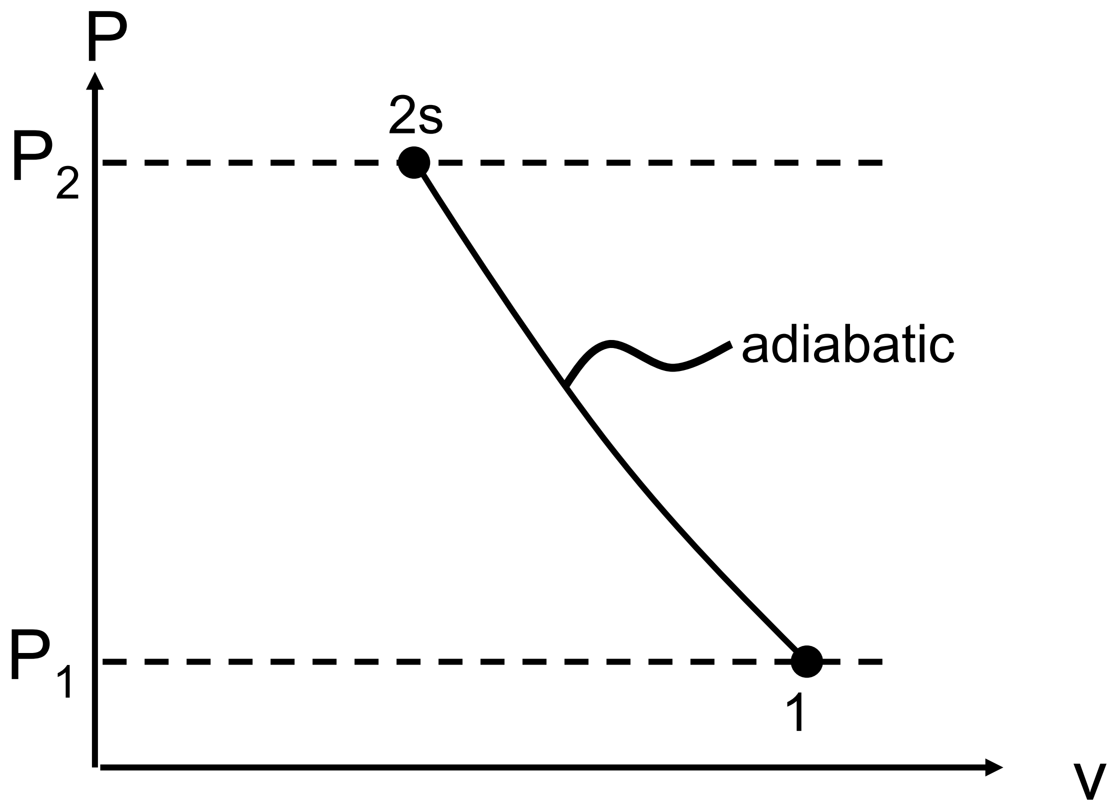
Recall that for an internally reversible, steady state process with neglibible Δke and Δpe

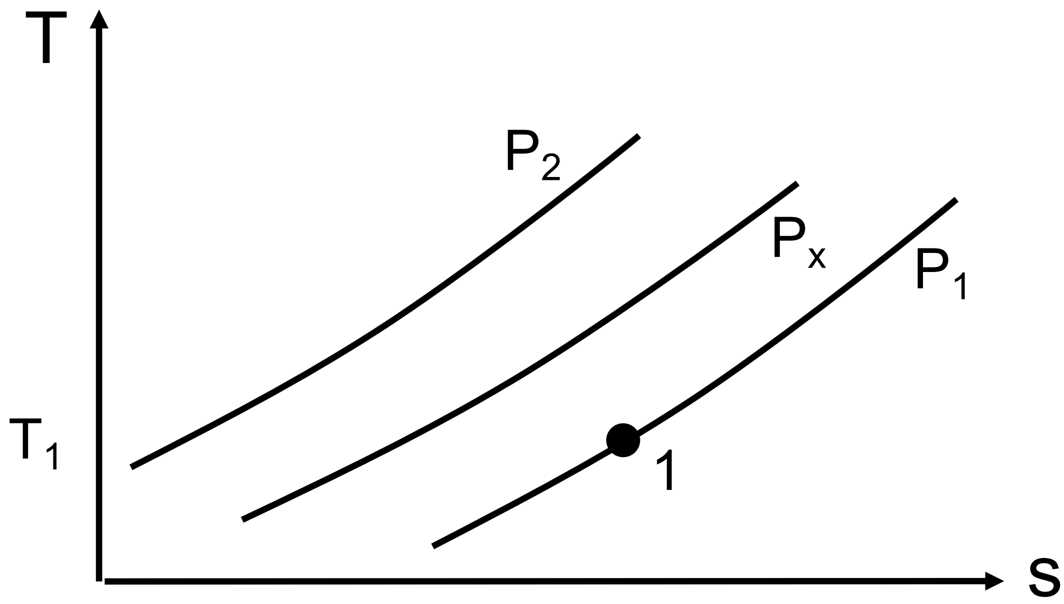
$$w_{rev} = -\int_1^2 v dP$$



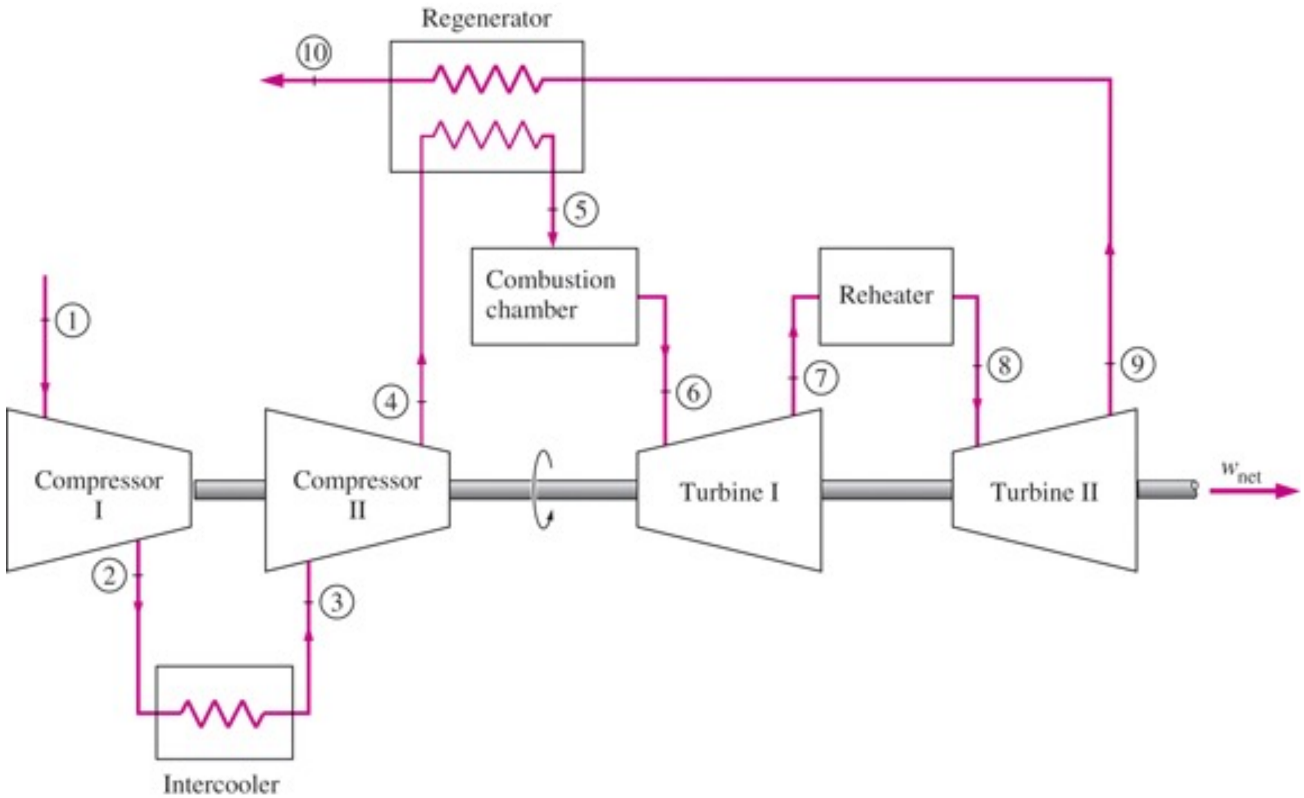
Compressor Intercooling

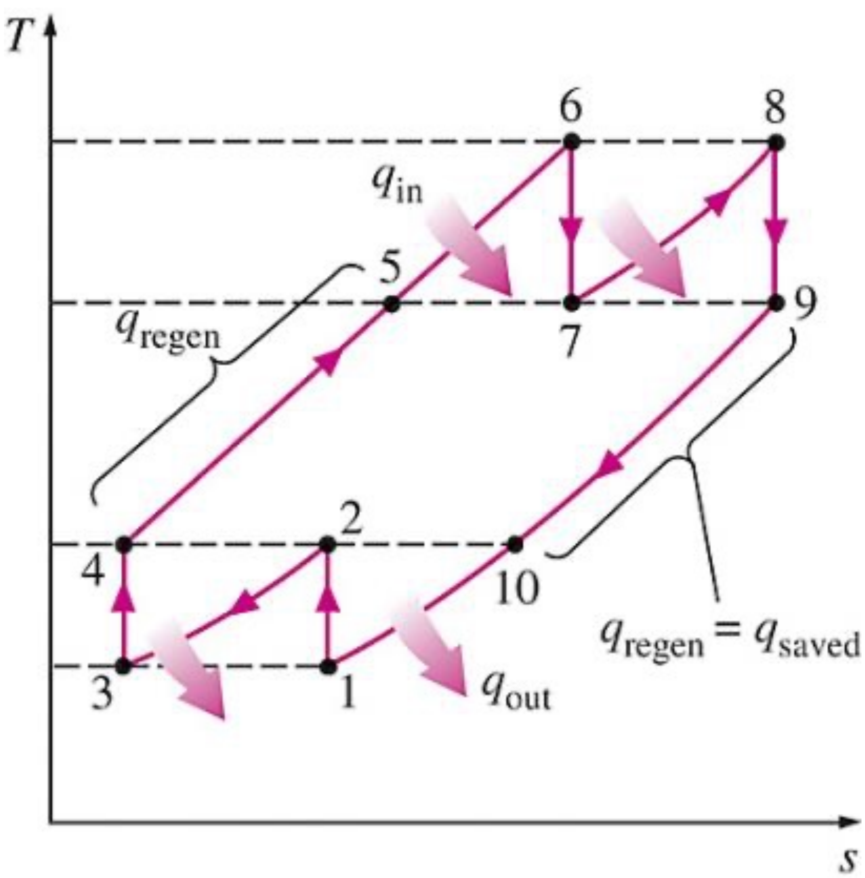





T
P2
Px
P1
T1
1
s

Intercooling, Reheat, & Regeneration



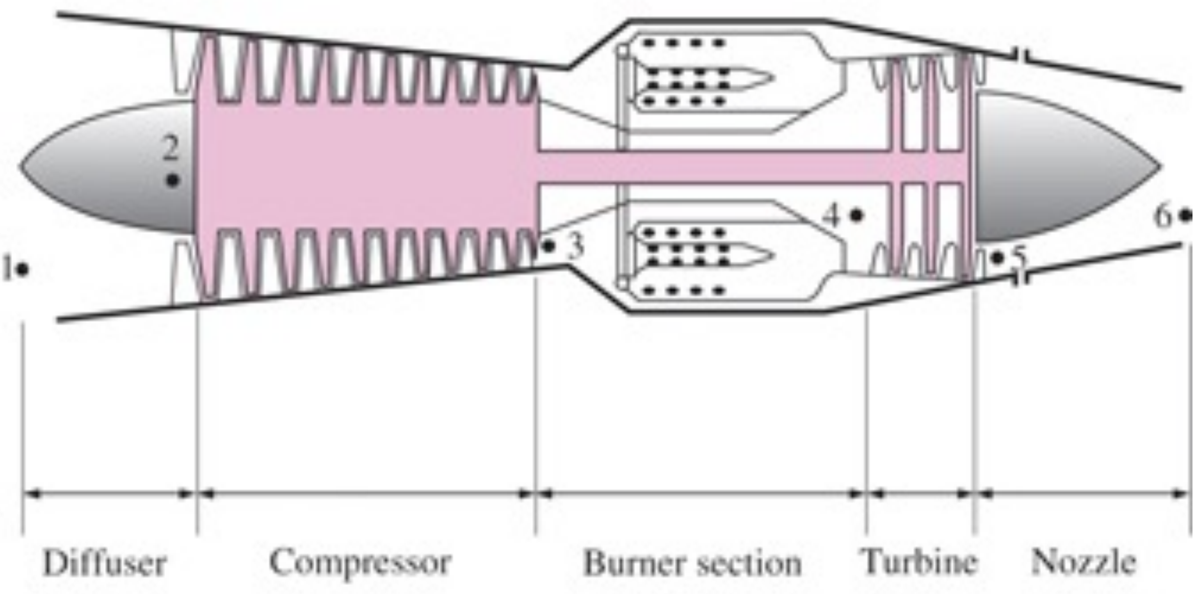


Example Results

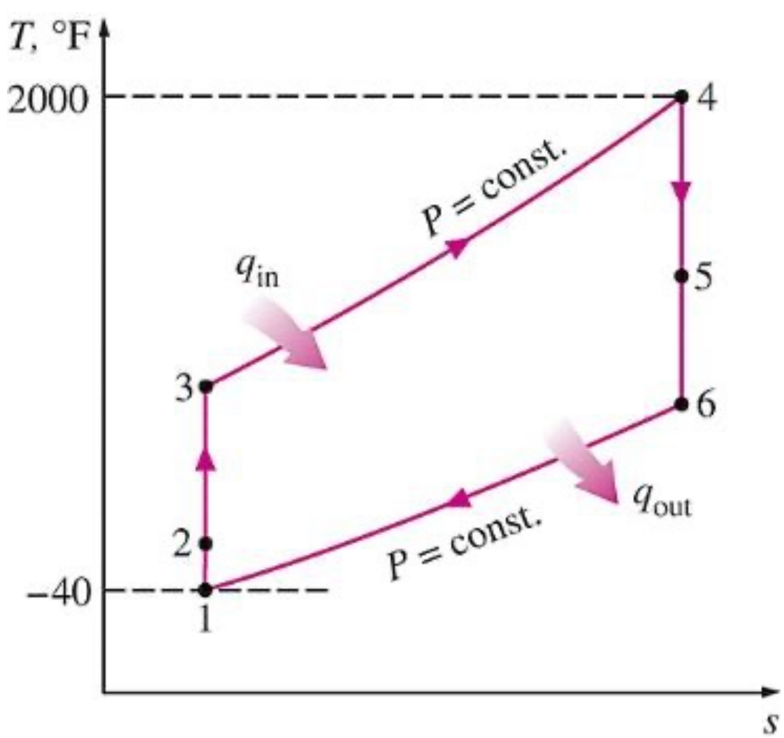
T_1 = 295 K (22^{o}C), P_1 = 0.95 bars, $r_p = P_2/P_1 = 6$,
T_H = 1100 K

System	η_{th}
1.) Ideal Brayton Cycle	0.385
2.) Brayton cycle with η_C = 0.82, η_T = 0.85	0.233
3.) #1 with ideal regenerator (η_{HX} = 1.0)	0.562
4.) #2 with real regen. (η_{HX} = 0.7)	0.318
5.) #4 with ideal intercooler and reheater	0.370

Lecture 41 - Jet Propulsion



<u>Ideal Air-Standard Cycle</u>



1-2: decelerate air & increase T & P

2-3: compression

3-4: constant P heat addition

4-5: work output (to drive compressor & auxiliaries)

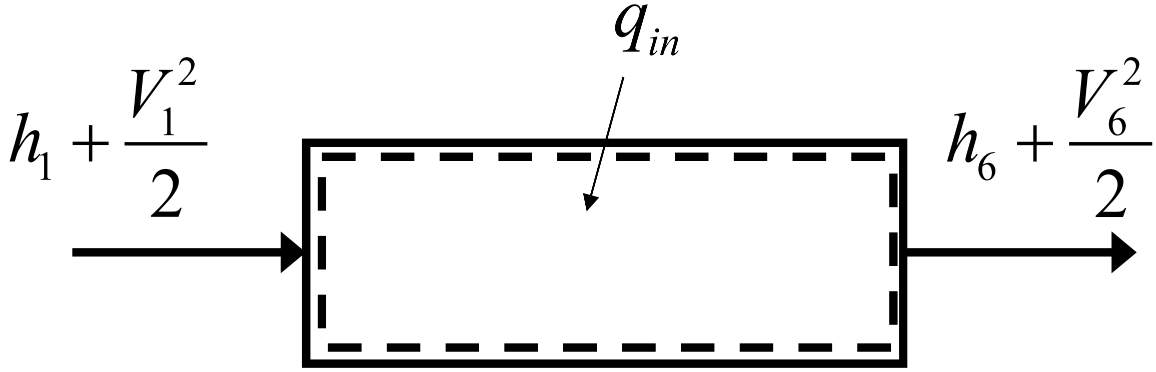
5-6: accelerate air & decrease T & P (thrust)

Comparison to Stationary Gas Turbine

- Higher pressure ratios (no regeneration): 10:1 up to 25:1
- No net shaft work output (only auxiliary equipment or turboprop engines, where turbine drives propeller)
- Exhaust gases do not expand back to ambient pressure at turbine outlet (final expansion occurs in nozzle)
- Diffuser in front of compressor increases pressure and decreases kinetic energy (ram effect)

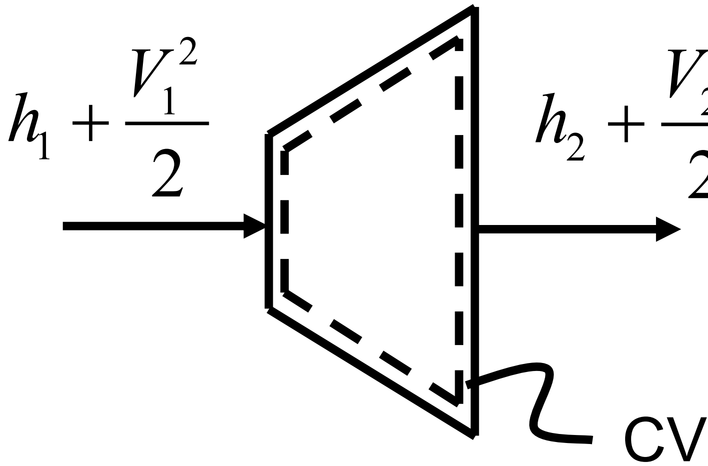
Overall Performance

- Treat combustion process as external heat input with air as the working fluid (air-standard model)
- steady-state, steady-flow process
- Use engine as the reference frame for air velocities



$$h_1 + \frac{V_1^2}{2} + q_{in} = h_6 + \frac{V_6^2}{2}$$

Diffuser Analysis (Engine Reference Frame)



Assumptions:

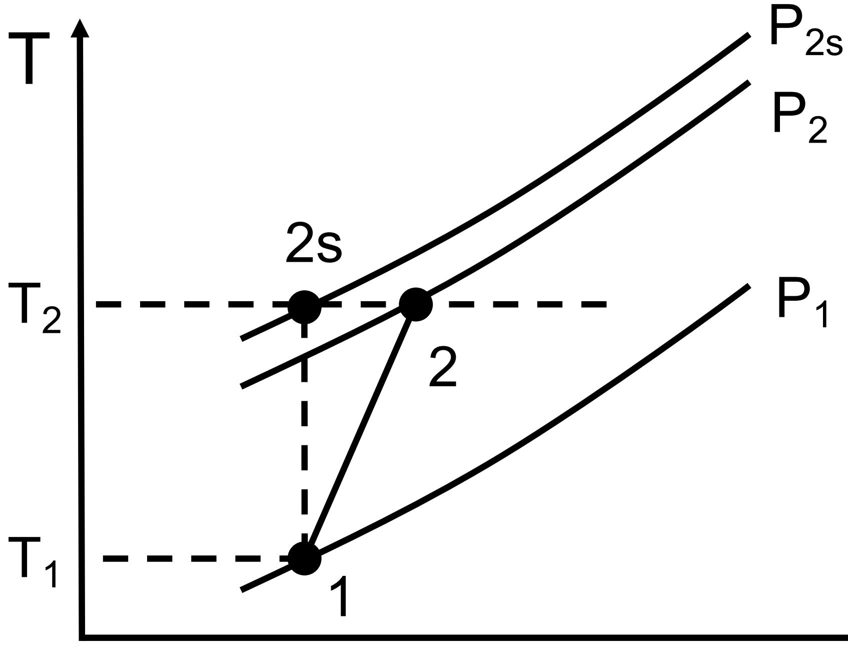
1. adiabatic, q = 0
2. no work, w = 0
3. SSSF
4. Δpe = 0

$$h_2 = h_1 + \frac{V_1^2 - V_2^2}{2} \qquad V_1^2 \gg V_2^2$$

So

$$h_2 \approx h_1 + \frac{V_1^2}{2}$$

Diffuser converts kinetic energy to enthalpy



Use pressure rise as a basis for diffuser efficiency

$$k_P = \frac{actual\ \ pressure\ rise}{isentropic\ \ pressure\ rise\ \ for\ actual\ \Delta T} = \frac{P_2 - P_1}{P_{2s} - P_1}$$

Then the actual exit pressure is determined with

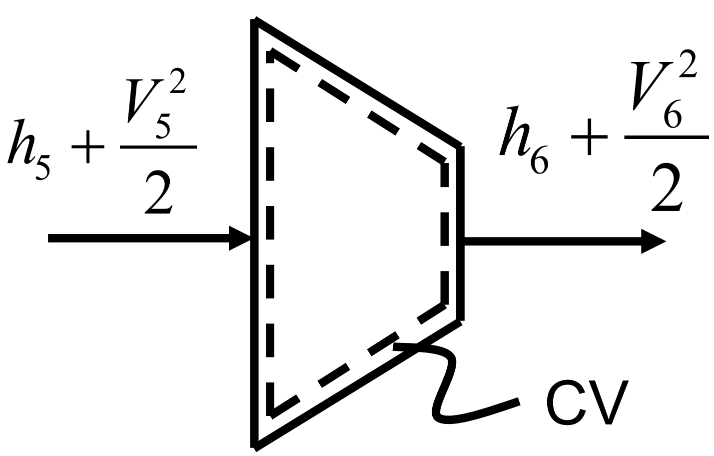
$$P_2 = P_1 + k_P\left(P_{2s} - P_1\right) \qquad\qquad 0 < k_P < 1$$

Compressor, Heat Addition, Turbine

- Same analysis as for stationary gas turbine, except

  turbine work (w_T) = compressor work (w_c)

Nozzle (Engine Reference Frame)



Assumptions:

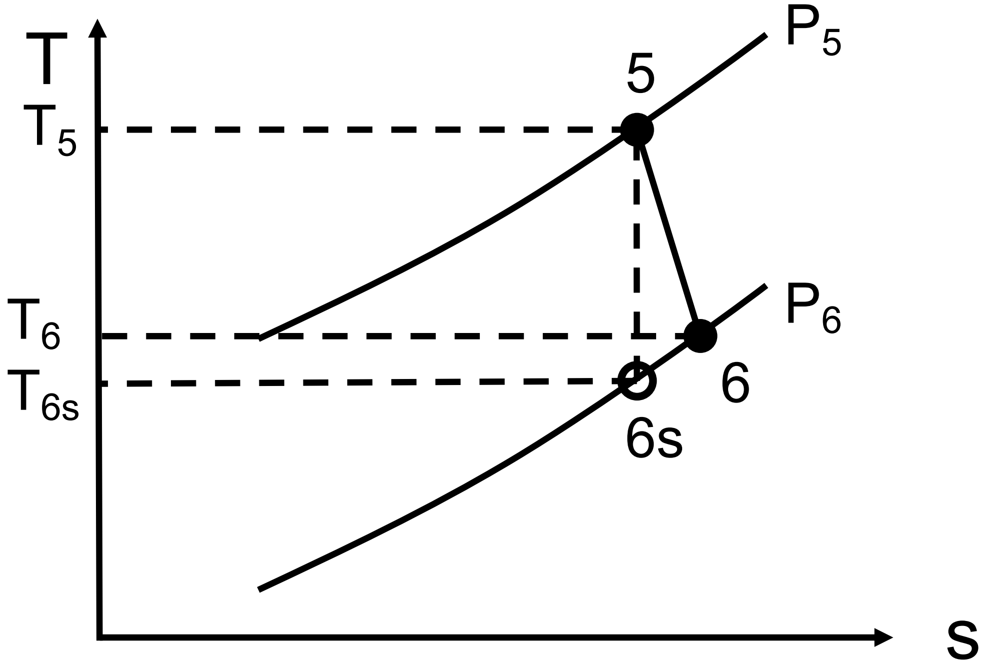
1. adiabatic, q = 0
2. no work, w = 0
3. SSSF
4. Δpe = 0

$$\frac{V_6^2 - V_5^2}{2} = h_5 - h_6 \qquad\qquad V_6^2 \gg V_5^2$$

So

$$\frac{V_6^2}{2} \approx h_5 - h_6$$

Nozzle converts enthalpy to kinetic energy



Define nozzle efficiency

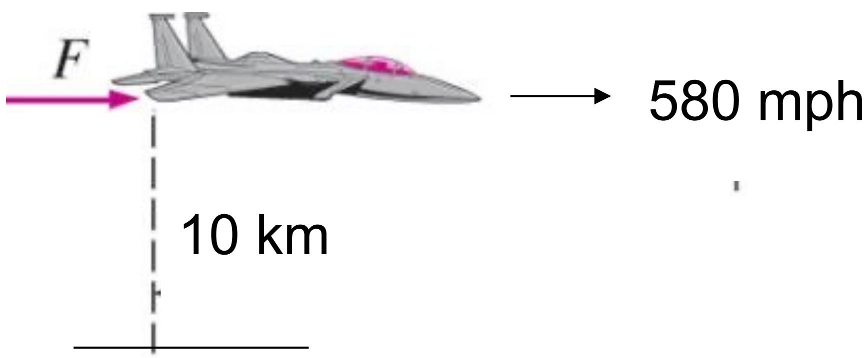
$$\eta_N = \frac{V_6^2 / 2}{V_{6s}^2 / 2} \approx \frac{h_5 - h_6}{h_5 - h_{6s}} \quad \Rightarrow \quad \frac{V_6^2}{2} \approx \eta_N \left(h_5 - h_{6s} \right)$$

Example

Given: Jet aircraft flying at 580 mph with an altitude of 10 km, ambient temperature (T_1) and pressure (P_1) of 200 K and 0.3 bar with other information given below

Find: Engine air flow rate, exit air velocity, heat input

Assumptions: air-standard engine



T_1 = 200 K

P_1 = 0.30 bar

P_3/P_2 = 10

T_4 = 1300 K

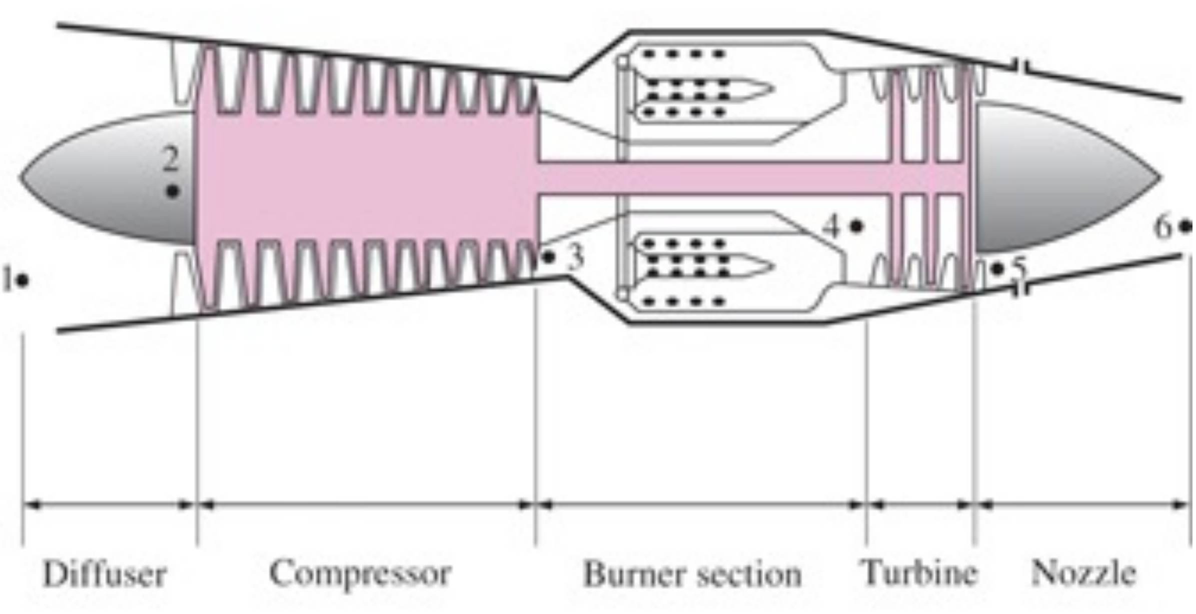
A_1 = 0.2 m^2

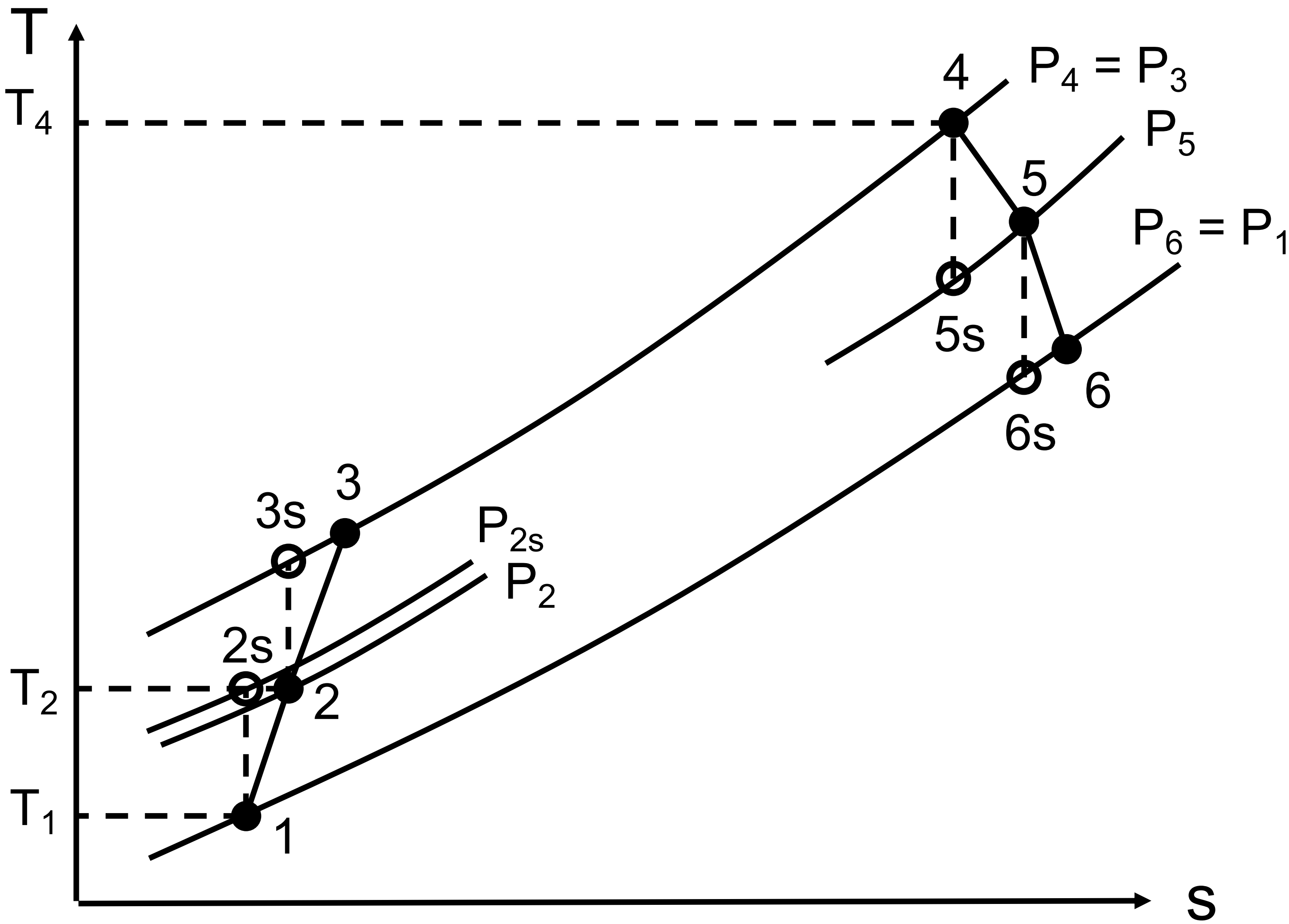
k_P = 0.92

η_C = 0.82

η_T = 0.86

η_N = 0.95





Entering air velocity approximately the jet velocity

$$V_1 = 580\ mph = 260\ m/s$$

So the entering air flow rate is

$$\dot{m}_a = \rho_1 \cdot A_1 \cdot V_1 = \frac{P_1}{RT_1} \cdot A_1 \cdot V_1 = 27.2\ kg/s$$

In order to determine the exit velocity (V_6), we need to evaluate the states from the inlet to the outlet of the gas turbine.

<u>Diffuser</u>

$$h_2 \approx h_1 + \frac{V_1^2}{2} = 199.97\frac{kJ}{kg} + \frac{260^2}{2}\frac{m^2}{s^2}\frac{1^{kJ}/_{kg}}{1000^{m^2}/_{s^2}}$$

$$= 233.8\ kJ/kg$$

From the air tables

$$T_2 = 233.8K \quad and \quad P_{r2} = 0.577$$

For an isentropic diffuser process (ideal gas)

$$\frac{P_{r2}}{P_{r1}} = \frac{P_{2s}}{P_1} \Rightarrow P_{2s} = 0.515\ bar$$

Then, using the diffuser efficiency

$$P_2 = P_1 + k_P\left(P_{2s} - P_1\right) = 0.5\ bar$$

<u>Compressor</u>

$$w_c = h_3 - h_2 = \frac{1}{\eta_c}\left(h_{3s} - h_2\right)$$

$$P_3 = 10P_2 = 5\ bar = P_4$$

For an isentropic compression process (ideal gas)

$$P_{r3} = P_{r2}\frac{P_3}{P_2} = 5.77$$

Use the air tables to get h_{3s} = *452 kJ/kg*. Then

$$w_c = 266.1\ kJ/kg\ \ and\ \ h_3 = 499.9\ kJ/kg$$

<u>Combustor</u>

$$T_4 = 1300K \quad \Rightarrow \quad h_4 = 1396\ kJ/kg$$

$$q_{in} = h_4 - h_3 = 896.1\ kJ/kg$$

<u>Turbine</u> $\quad w_T = h_4 - h_5 = \eta_T\left(h_4 - h_{5s}\right) = w_c$

$$\Rightarrow h_5 = h_4 - w_c = 1129.2\ kJ/kg$$

$$and\ \ h_{5s} = h_4 - \frac{w_c}{\eta_T} = 1086.6\ kJ/kg$$

For isentropic expansion (ideal gas)

$$P_5 = P_4 \cdot \frac{P_{r5}}{P_{r4}} = 1.98\ bar$$

Nozzle

$$\frac{V_6^2}{2} = h_5 - h_6 = \eta_N \left(h_5 - h_{6s} \right)$$

For an isentropic process with an ideal gas

$$P_{r6} = P_{r5} \cdot \frac{P_6}{P_5} = P_{r5} \cdot \frac{P_1}{P_5} = 22.9$$

and from the air tables (@P_{r6}=*22.9*)

$$h_{6s} = 668.5\ kJ/kg$$

So the exit air velocity is

$$V_6 = 921.4\ m/s$$

Finally, the required rate of heat input is

$$\dot{Q}_{in} = \dot{m}_a \cdot q_{in} = \left(27.2\ kg/s\right) \cdot \left(896.1\ kJ/kg\right)$$

$$\dot{Q}_{in} = 24.4\ MW$$