Given

Vapor compression heat pump cycle operating at steady state

Find

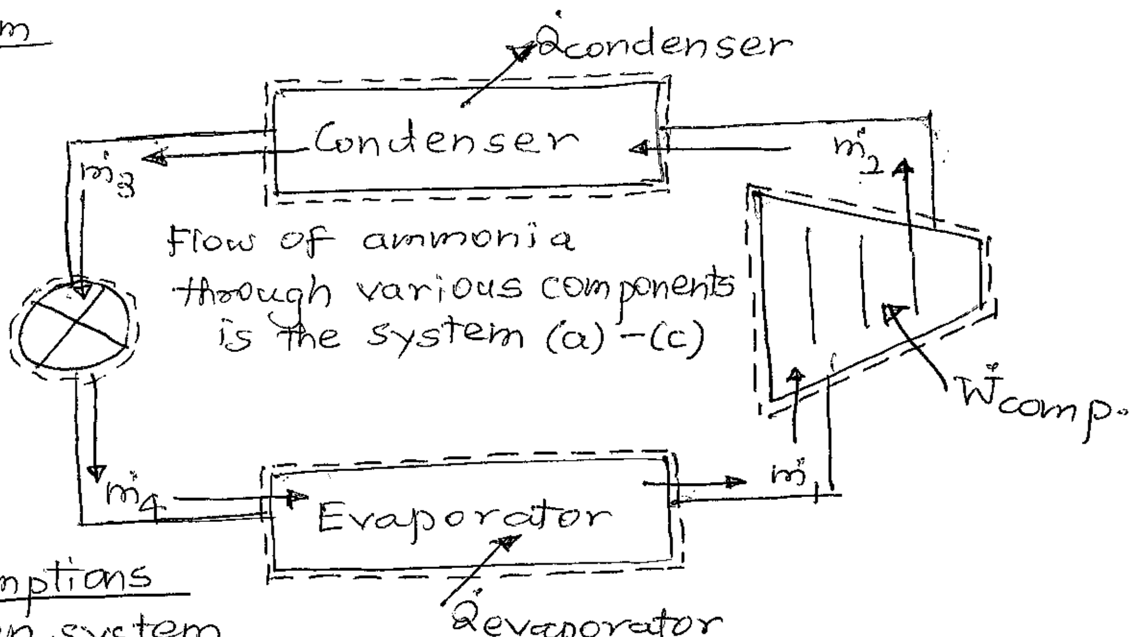
(a) Heat transfer rate (kW) for the condenser

(b) Coefficient of performance for the cycle

(c) Rate of entropy generation $\left(\frac{kW}{K}\right)$ for each component in the cycle

(d) Rate of entropy generation $\left(\frac{kW}{K}\right)$ for the entire cycle

System



Assumptions

- Open system
- Steady state, steady flow
- 1-D, uniform flow
- Ignore change in KE and PE
- Neglect any piping effects between components
- Evaporator and condenser: $\dot{W}=0$
- Compressor: $\dot{Q}=0$
- Throttling valve: $\dot{Q}=0$, $\dot{W}=0$

Basic Equations

$$\cancel{\frac{dm}{dt}}^{\,0}\bigg|_{system} = \sum_{in}\dot{m}_{in} - \sum_{out}\dot{m}_{out}$$

$$\cancel{\frac{dE}{dt}}^{\,0}\bigg|_{system} = \dot{Q} - \dot{W} + \sum_{in}\dot{m}_{in}(h+\cancel{ke}+\cancel{pe})_{in} - \sum_{out}\dot{m}_{out}(h+\cancel{ke}+\cancel{pe})_{out}$$

$$\left.\frac{dS}{dt}\right|_{system} = \sum_{in} \dot{m}_{in} s_{in} - \sum_{out} \dot{m}_{out} s_{out} + \sum_j \frac{\dot{Q}_j}{T_{j,boundary}} + \dot{\sigma}_{generation}$$

Solution

(a) Condenser:

Mass balance: $0 = \dot{m}_2 - \dot{m}_3 \Rightarrow \dot{m}_2 = \dot{m}_3 = \dot{m}_{NH_3}$

Energy balance: $0 = \dot{Q}_{condenser} - 0 + \dot{m}_2 h_2 - \dot{m}_3 h_3$

$$\dot{Q}_{condenser} = \dot{m}_3 h_3 - \dot{m}_2 h_2 = \dot{m}_{NH_3}(h_3 - h_2)$$
$$= 0.01 \frac{kg}{s} * (376.52 - 1696.8) \frac{kJ}{kg}$$

$\boxed{\dot{Q}_{condenser} = -13.2 \text{ kW}}$ $\dot{Q} < 0$ heat transfer out of the system

(b) Coefficient of performance for the heat pump cycle:

$$COP_{HP} = \frac{Q_H}{W_{net,in}} = \frac{|\dot{Q}_{condenser}|}{|\dot{W}_{comp.}|}$$

Compressor:

Mass balance: $0 = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}_{NH_3}$

Energy balance: $0 = 0 - \dot{W}_{comp.} + \dot{m}_1 h_1 - \dot{m}_2 h_2$

$$\dot{W}_{comp.} = \dot{m}_1 h_1 - \dot{m}_2 h_2 = \dot{m}_{NH_3}(h_1 - h_2)$$
$$= 0.01 \frac{kg}{s} * (1446.0 - 1696.8) \frac{kJ}{kg} = -2.51 \text{ kW}$$

$$COP_{HP} = \frac{|-13.2 \text{ kW}|}{|-2.51 \text{ kW}|} \Rightarrow \boxed{COP_{HP} = 5.26}$$

(c) Compressor:

Entropy balance: $0 = \dot{m}_1 s_1 - \dot{m}_2 s_2 + 0 + \dot{\sigma}_{comp.}$

$$\dot{\sigma}_{comp.} = \dot{m}_2 s_2 - \dot{m}_1 s_1 = \dot{m}_{NH_3}(s_2 - s_1)$$
$$= 0.01 \frac{kg}{s} * (5.5022 - 5.3773) \frac{kJ}{kg\text{-}K}$$

$$\boxed{\dot{\sigma}_{comp.} = +0.001249 \frac{kW}{K}}$$

Condenser:

Entropy balance: $0 = \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dfrac{\dot{Q}_{condenser}}{T_{boundary}} + \dot{\sigma}_{condenser}$

$\dot{\sigma}_{condenser} = \dot{m}_3 s_3 - \dot{m}_2 s_2 - \dfrac{\dot{Q}_{condenser}}{T_{boundary}}$

$= \dot{m}_{NH_3}(s_3 - s_2) - \dfrac{\dot{Q}_{condenser}}{T_{boundary}}$

$= 0.01 \frac{kg}{s} * (1.3737 - 5.5022) \frac{kJ}{kg\text{-}K} - \dfrac{-13.2\ kW}{(30+273)K}$

$\boxed{\dot{\sigma}_{condenser} = +0.002279 \frac{kW}{K}}$

Throttling valve:

Entropy balance: $0 = \dot{m}_3 s_3 - \dot{m}_4 s_4 + 0 + \dot{\sigma}_{throttle}$

$\dot{\sigma}_{throttle} = \dot{m}_4 s_4 - \dot{m}_3 s_3 = \dot{m}_{NH_3}(s_4 - s_3)$

$= 0.01 \frac{kg}{s} * (1.4351 - 1.3737) \frac{kJ}{kg\text{-}K}$

$\boxed{\dot{\sigma}_{throttle} = +0.000614 \frac{kW}{K}}$

Evaporator:

Entropy balance: $0 = \dot{m}_4 s_4 - \dot{m}_1 s_1 + \dfrac{\dot{Q}_{evap.}}{T_{boundary}} + \dot{\sigma}_{evap.}$

$\dot{\sigma}_{evap} = \dot{m}_1 s_1 - \dot{m}_4 s_4 - \dfrac{\dot{Q}_{evap.}}{T_{boundary}}$

$= \dot{m}_{NH_3}(s_1 - s_4) - \dfrac{\dot{Q}_{evap.}}{T_{boundary}}$

Energy balance: $0 = \dot{Q}_{evap.} - 0 + \dot{m}_4 h_4 - \dot{m}_1 h_1$

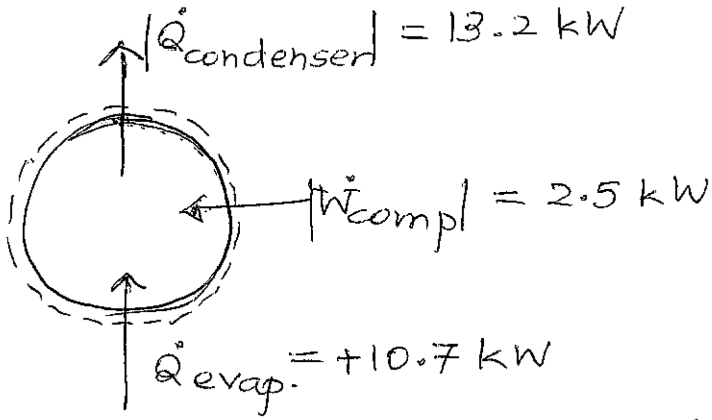
$\dot{Q}_{evap.} = \dot{m}_1 h_1 - \dot{m}_4 h_4 = \dot{m}_{NH_3}(h_1 - h_4)$

$= 0.01 \frac{kg}{s} * (1446.0 - 376.52) \frac{kJ}{kg} = +10.7\ kW$

$\dot{\sigma}_{evap.} = 0.01 \frac{kg}{s} * (5.3773 - 1.4351) \frac{kJ}{kg\text{-}K} - \dfrac{+10.7\ kW}{(0+273)K}$

$\boxed{\dot{\sigma}_{evap.} = +0.000228 \frac{kW}{K}}$

(d) For the heat pump cycle:



Entropy balance: $0 = 0 - 0 + \left(\frac{\dot{Q}_{evap.}}{T_{evap.}} + \frac{\dot{Q}_{condenser}}{T_{condenser}}\right) + \dot{\sigma}_{cycle}$

Rate of entropy generation for the cycle:

$$\dot{\sigma}_{cycle} = -\left(\frac{+10.7\text{ kW}}{273\text{ K}} + \frac{-13.2\text{ kW}}{303\text{ K}}\right)$$

$$\boxed{\dot{\sigma}_{cycle} = +0.00437\ \frac{\text{kW}}{\text{K}}}$$

Note

$\dot{\sigma}_{comp.} + \dot{\sigma}_{condenser} + \dot{\sigma}_{throttle} + \dot{\sigma}_{evap}$

$= +0.001249\ \frac{\text{kW}}{\text{K}} + 0.002279\ \frac{\text{kW}}{\text{K}} + 0.000614\ \frac{\text{kW}}{\text{K}} + 0.000228\ \frac{\text{kW}}{\text{K}}$

$= +0.00437\ \frac{\text{kW}}{\text{K}}$

$= \dot{\sigma}_{cycle}$

Given

Air-standard Otto cycle

Find

(a) Pressure (bar) and temperature (K) at all states in the cycle

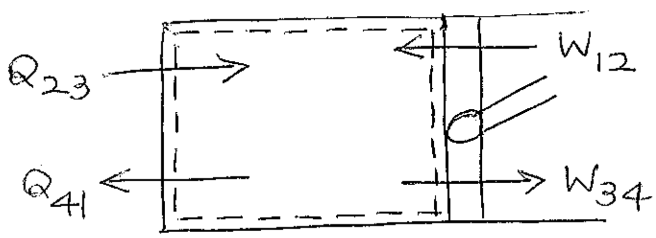
(b) Thermal efficiency (%) of the cycle

(c) Mean effective pressure (bar) of the cycle

(d) Total specific entropy generation $\left(\frac{kJ}{kg\text{-}K}\right)$ for the energy rejection process

(e) P-v and T-s diagrams

System



Assumptions

- Closed system (m=constant)
- Internally reversible process
- Neglect friction between piston and cylinder wall
- Ignore change in KE and PE
- Only air is the working fluid
- Air behaves as an ideal gas

Basic Equations

$$Q - W = \Delta E = \Delta U + \cancel{\Delta KE}^{\sim 0} + \cancel{\Delta PE}^{\sim 0}$$

$$W \to (W_{boundary} + \cancel{W_{other}}^{0})$$

$$\frac{Q}{T_{boundary}} + \sigma_{generation} = \Delta S$$

Solution

(a) State 1 $P_1 = 1\text{ bar}$, $T_1 = 300\text{ K}$

State 2 $s_2 = s_1$ (isentropic)

$$\frac{v_{r_2}}{v_{r_1}} = \frac{v_2}{v_1} \Rightarrow \frac{v_{r_2}}{621.2} = \frac{1}{10} \Rightarrow v_{r_2} = 62.12$$

$$\boxed{T_2 \cong 730\text{ K}}$$

$$\frac{P_2}{P_1} = \frac{P_{r_2}}{P_{r_1}} \Rightarrow \frac{P_2}{1\text{ bar}} = \frac{33.72}{1.386} \Rightarrow \boxed{P_2 = 24.3\text{ bar}}$$

State 3 $T_3 = 2000\text{ K}$

$$\frac{P_3}{P_2} = \frac{T_3}{T_2} \Rightarrow \frac{P_3}{24.3\text{ bar}} = \frac{2000\text{ K}}{730\text{ K}} \Rightarrow \boxed{P_3 = 66.6\text{ bar}}$$

State 4 $s_4 = s_3$ (isentropic)

$$\frac{v_{r_4}}{v_{r_3}} = \frac{v_4}{v_3} \Rightarrow \frac{v_{r_4}}{2.776} = \frac{10}{1} \Rightarrow v_{r_4} = 27.76$$

Interpolating: $\boxed{T_4 = 967.5\text{ K}}$

$$\frac{P_4}{P_3} = \frac{P_{r_4}}{P_{r_3}} \Rightarrow \frac{P_4}{66.6\text{ bar}} = \frac{100}{2068} \Rightarrow \boxed{P_4 = 3.22\text{ bar}}$$

Alternatively

$$\frac{P_4}{P_1} = \frac{T_4}{T_1} \Rightarrow \frac{P_4}{1\text{ bar}} = \frac{967.5\text{ K}}{300\text{ K}} \Rightarrow P_4 = 3.22\text{ bar}$$

(b) Considering energy balance for adiabatic compression: $\cancel{q_{12}}^{\,0} - w_{12} = (u_2 - u_1)$

$T_1 = 300\text{ K}$, $u_1 = 214.1\ \frac{\text{kJ}}{\text{kg}}$

$T_2 = 730\text{ K}$, $u_2 = 536.1\ \frac{\text{kJ}}{\text{kg}}$

$$\Rightarrow w_{12} = -322\ \frac{\text{kJ}}{\text{kg}}$$

Considering energy balance for adiabatic expansion: $\cancel{q_{34}}^{0} - w_{34} = (u_4 - u_3)$

$T_3 = 2000\ K,\ u_3 = 1678\ \frac{kJ}{kg}$
$T_4 = 967.5\ K,\ u_4 = 731.3\ \frac{kJ}{kg}$ $\Rightarrow w_{34} = +946.7\ \frac{kJ}{kg}$

$w_{23} = \int_2^3 P\, d\cancel{v}^{0} = 0\ ,\quad w_{41} = \int_4^1 P\, d\cancel{v}^{0} = 0$

$w_{net} = w_{12} + \cancel{w_{23}}^{0} + w_{34} + \cancel{w_{41}}^{0} = +624.7\ \frac{kJ}{kg}$

Considering energy balance for energy addition:

$q_{23} - \cancel{w_{23}}^{0} = (u_3 - u_2) = +1141.9\ \frac{kJ}{kg}$

Considering energy balance for energy rejection:

$q_{41} - \cancel{w_{41}}^{0} = (u_1 - u_4) = -517.2\ \frac{kJ}{kg}$

$q_{net} = \cancel{q_{12}}^{0} + q_{23} + \cancel{q_{34}}^{0} + q_{41} = +624.7\ \frac{kJ}{kg}$

Note $\quad q_{net} = w_{net}$ for the cycle!

Thermal efficiency: $\eta_{thermal} = \frac{w_{net}}{q_{in}} = \frac{w_{net}}{q_{23}}$

$\eta_{thermal} = \frac{624.7\ \frac{kJ}{kg}}{1141.9\ \frac{kJ}{kg}} = 0.547 \Rightarrow \boxed{\eta_{thermal} = 54.7\%}$

(c) Mean effective pressure : $MEP = \frac{w_{net}}{v_1 - v_2}$

$P_1 v_1 = R_{air} T_1$

$R_{air} = \frac{\bar{R}}{M_{air}} = \frac{8.314\ \frac{kJ}{kmol-K}}{28.97\ \frac{kg}{kmol}} = 0.287\ \frac{kJ}{kg-K}$

$v_1 = \frac{R_{air} T_1}{P_1} = \frac{0.287\ \frac{kJ}{kg-K} * 300\ K}{(1 \times 100)\ kPa} = 0.861\ \frac{m^3}{kg}$

$$v_2 = \frac{v_1}{10} = 0.0861\ \frac{m^3}{kg}$$

$$MEP = \frac{624.7\ \frac{kJ}{kg}}{(0.861 - 0.0861)\ \frac{m^3}{kg}} = 806\ kPa \Rightarrow \boxed{MEP = 8.06\ bar}$$

(d) Considering entropy balance for energy rejection:

$$\sigma_{total} = (s_1 - s_4) - \frac{q_{41}}{T_b = T_{surr}} = (s_1^o - s_4^o - R_{air} \ln \frac{P_1}{P_4}) - \frac{q_{41}}{T_{surr}}$$

$T_1 = 300\ K$, $s_1^o = 1.703\ \frac{kJ}{kg-K}$

$T_4 = 967.5\ K$, $s_4^o = 2.931\ \frac{kJ}{kg-K}$

$$\sigma_{total} = (1.703 - 2.931 - 0.287 \ln \frac{1\ bar}{3.22\ bar})\frac{kJ}{kg-K} - \frac{-517.2\ \frac{kJ}{kg}}{(20+273.15)\ K}$$

$$\boxed{\sigma_{total} = +0.872\ \frac{kJ}{kg-K}}$$

(e)