Given

A turbine and a nozzle operating at steady state

Find

(a) Specific entropy generation $\left(\frac{kJ}{kg-K}\right)$ for the turbine

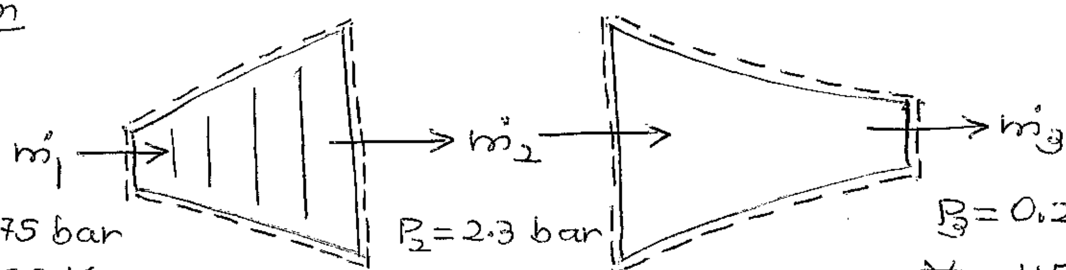
(b) Isentropic efficiency (%) of the turbine

(c) Specific entropy generation $\left(\frac{kJ}{kg-K}\right)$ for the nozzle

(d) Isentropic efficiency (%) of the nozzle

(e) T-s diagram

System



Flow of air through the turbine and nozzle is the system

Assumptions

- Open system
- Steady state, steady flow
- 1-D, uniform flow
- Ignore change in PE
- Neglect change in KE (turbine)
- Turbine : $\dot{Q} = 0$
- Nozzle : $\dot{Q} = 0$, $\dot{W} = 0$
- Air behaves as an ideal gas

Basic Equations

$$\cancel{\frac{dm}{dt}\Big)_{system}}^{0} = \sum_{in} \dot{m}_{in} - \sum_{out} \dot{m}_{out}$$

$$\cancel{\frac{dE}{dt}\Big)_{system}}^{0} = \sum_{in} \dot{m}_{in}(h+ke+\cancel{pe}) - \sum_{out} \dot{m}_{out}(h+ke+\cancel{pe})_{out} + \cancel{\dot{Q}} - \dot{W}$$

$$\cancel{\frac{dS}{dt}\Big)_{system}}^{0} = \sum_{in} \dot{m}_{in} s_{in} - \sum_{out} \dot{m}_{out} s_{out} + \cancel{\sum_j \frac{\dot{Q}_j}{T_j}\Big)_{boundary}} + \dot{\sigma}_{generation}$$

Solution

(a) Turbine

Mass balance: $0 = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}$

Entropy balance: $0 = \dot{m}_1 s_1 - \dot{m}_2 s_2 + 0 + \dot{\sigma}_{turbine}$

$\dot{\sigma}_{turbine} = \dot{m}(s_2 - s_1)$

Specific entropy generation for the turbine:

$$\sigma_{turbine} = \frac{\dot{\sigma}_{turbine}}{\dot{m}} = s_2 - s_1 = s_2^\circ - s_1^\circ - R_{air} \ln \frac{P_2}{P_1}$$

State 1 $T_1 = 1600\ K$, $s_1^\circ = 3.523\ \frac{kJ}{kg-K}$

State 2 $T_2 = 1340\ K$, $s_2^\circ = 3.309\ \frac{kJ}{kg-K}$

$$\sigma_{turbine} = \left(3.309 - 3.523 - 0.287 \ln \frac{2.3\ bar}{6.75\ bar}\right) \frac{kJ}{kg-K}$$

$\boxed{\sigma_{turbine} = +0.095\ \frac{kJ}{kg-K}}$ $\sigma_{generation} > 0$ ✓

(b) Isentropic efficiency of the turbine:

$$\eta_{turbine} = \frac{w_{actual}}{w_{isentropic}} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

State 1 $T_1 = 1600\ K$, $h_1 = 1758\ \frac{kJ}{kg}$, $P_1 = 6.75\ bar$

State 2 $T_2 = 1340\ K$, $h_2 = 1444\ \frac{kJ}{kg}$, $P_2 = 2.3\ bar$

State 2s $P_{2s} = 2.3\ bar$, $s_{2s} = s_1$

$$\frac{Pr_{2s}}{Pr_1} = \frac{P_{2s}}{P_1} \Rightarrow Pr_{2s} = 791.2 * \frac{2.3\ bar}{6.75\ bar} = 269.6$$

Interpolating: $T_{2s} = 1237\ K$, $h_{2s} = 1321.4\ \frac{kJ}{kg}$

Alternatively

$$s_{2s} - s_1 = 0 \Rightarrow s_{2s}^\circ - s_1^\circ - R_{air} \ln \frac{P_{2s}}{P_1} = 0$$

$$s_{2s}^\circ = 3.523\ \frac{kJ}{kg-K} + 0.287\ \frac{kJ}{kg-K} \ln \frac{2.3\ bar}{6.75\ bar} = 3.214\ \frac{kJ}{kg-K}$$

Interpolating: $T_{2s} = 1237\ K$, $h_{2s} = 1321.4\ \frac{kJ}{kg}$

$$\eta_{turbine} = \frac{314\ \frac{kJ}{kg}}{436.6\ \frac{kJ}{kg}} = 0.719$$

$\boxed{\eta_{turbine} = 71.9\%}$

(c) Nozzle

Mass balance: $0 = \dot{m}_2 - \dot{m}_3 \Rightarrow \dot{m}_2 = \dot{m}_3 = \dot{m}$

Entropy balance: $0 = \dot{m}_2 s_2 - \dot{m}_3 s_3 + 0 + \dot{\sigma}_{nozzle}$

$\dot{\sigma}_{nozzle} = \dot{m}(s_3 - s_2)$

Specific entropy generation for the nozzle:

$$\sigma_{nozzle} = \frac{\dot{\sigma}_{nozzle}}{\dot{m}} = s_3 - s_2 = s_3^o - s_2^o - R_{air} \ln \frac{P_3}{P_2}$$

Energy balance: $0 = \dot{m}_2\left(h_2 + \cancel{\frac{V_2^2}{2}}^{\,0}\right) - \dot{m}_3\left(h_3 + \frac{V_3^2}{2}\right) + 0 - 0$

$$\cancel{\dot{m}_3}\left(h_3 + \frac{V_3^2}{2}\right) = \cancel{\dot{m}_2} h_2 \Rightarrow h_3 = h_2 - \frac{V_3^2}{2} = 1444 \frac{kJ}{kg} - \frac{(1150)^2}{2*1000} \frac{kJ}{kg}$$

$h_3 = 782.75 \frac{kJ}{kg}$

Interpolating: $T_3 = 764.2\ K$, $s_3^o = 2.6689 \frac{kJ}{kg-K}$

~~State~~ $\sigma_{nozzle} = \left(2.6689 - 3.3090 - 0.287 \ln \frac{0.23\ bar}{2.3\ bar}\right) \frac{kJ}{kg-K}$

$\boxed{\sigma_{nozzle} = +0.020742 \frac{kJ}{kg-K}}$ $\quad \sigma_{generation} > 0$ ✓

(d) Isentropic efficiency of the nozzle:

$$\eta_{nozzle} = \frac{\Delta KE_{actual}}{\Delta KE_{isentropic}} = \frac{\frac{V_3^2 - \cancel{V_2^2}^{\,0}}{2}}{\frac{V_{3S}^2 - \cancel{V_2^2}^{\,0}}{2}} = \frac{h_2 - h_3}{h_2 - h_{3S}}$$

<u>State 2</u> $T_2 = 1340 K$, $h_2 = 1444 \frac{kJ}{kg}$, $P_2 = 2.3$ bar

<u>State 3</u> $T_3 = 764.2K$, $h_3 = 782.75 \frac{kJ}{kg}$, $P_3 = 0.23$ bar

<u>State 3S</u> $P_{3S} = 0.23$ bar, $s_{3S} = s_2$

$$\frac{P_{r_{3S}}}{P_{r_2}} = \frac{P_{3S}}{P_2} \Rightarrow P_{r_{3S}} = 375.3 * \frac{0.23\ bar}{2.3\ bar} = 37.53$$

Interpolating: $T_{3S} = 750.9\ K$, $h_{3S} = 768.3 \frac{kJ}{kg}$

<u>Alternatively</u>

$$s_{3S} - s_2 = 0 \Rightarrow s_{3S}^o - s_2^o - R_{air} \ln \frac{P_{3S}}{P_2} = 0$$

$$s_{3S}^o = 3.309 \frac{kJ}{kg-K} + 0.287 \frac{kJ}{kg-K} \ln \frac{0.23\ bar}{2.3\ bar} = 2.6482 \frac{kJ}{kg-K}$$

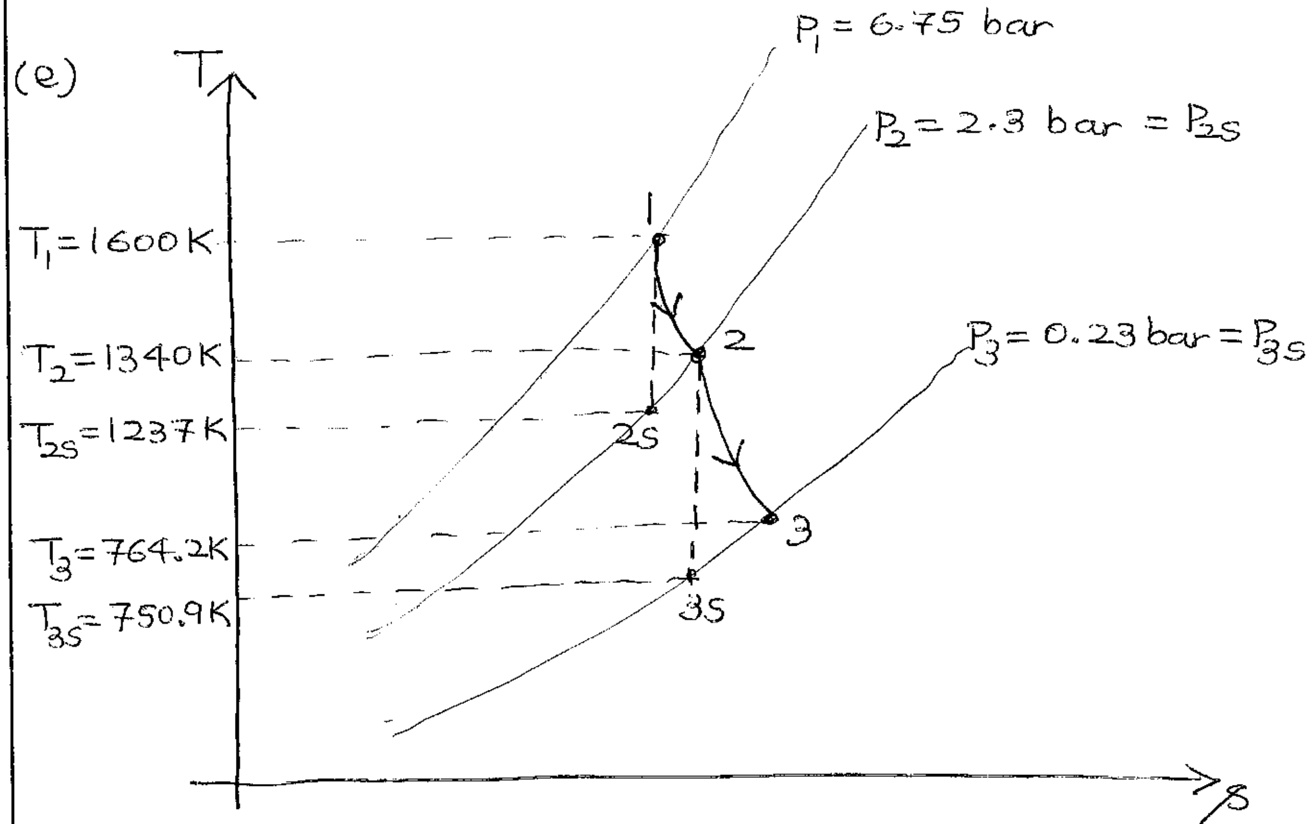
Interpolating: $T_{3S} = 750.9\ K$, $h_{3S} = 768.3 \frac{kJ}{kg}$

$$\eta_{nozzle} = \frac{661.25 \frac{kJ}{kg}}{675.70 \frac{kJ}{kg}} = 0.978 \qquad \boxed{\eta_{nozzle} = 97.8\%}$$

Note

$V_3 = 1150 \frac{m}{s}$, $V_{3S} = 1162 \frac{m}{s}$

(e)



Given

Compression of saturated liquid water through an adiabatic pump

Find

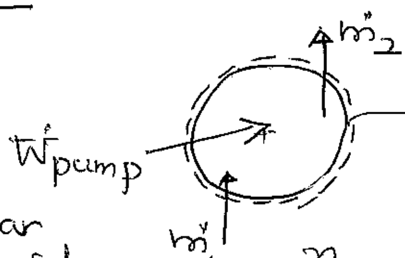
(a) Specific work $\left(\frac{kJ}{kg}\right)$ for the pump

(b) Specific entropy generation $\left(\frac{kJ}{kg-K}\right)$ for the pump

(c) T-s diagram

System

$P_2 = 25$ bar



Flow of liquid water through the pump is the system

$P_1 = 1$ bar
sat. liquid

$\eta_{pump} = 85\%$

Assumptions

- Open system
- Steady state, steady flow
- 1-D, uniform flow
- Ignore change in KE and PE
- Adiabatic : $\dot{Q} = 0$

Basic Equations

$$\cancel{\left.\frac{dm}{dt}\right|_{system}}^{0} = \cancel{\sum_{in}} \dot{m}_{in} - \cancel{\sum_{out}} \dot{m}_{out}$$

$$\cancel{\left.\frac{dE}{dt}\right|_{system}}^{0} = \cancel{\sum_{in}} \dot{m}_{in}(h + \cancel{ke} + \cancel{pe})_{in} - \cancel{\sum_{out}} \dot{m}_{out}(h + \cancel{ke} + \cancel{pe})_{out} + \cancel{\dot{Q}}^{0} - \dot{W}$$

$$\cancel{\left.\frac{dS}{dt}\right|_{system}}^{0} = \sum_{in} \dot{m}_{in} s_{in} - \sum_{out} \dot{m}_{out} s_{out} + \cancel{\sum_j \frac{\dot{Q}_j^{\,0}}{T_{j,boundary}}} + \dot{\sigma}_{generation}$$

Solution

State 1 $P_1 = 1$ bar, sat. liquid

$$T_1 = T_{sat}(P_1) = 99.61^\circ C$$

$$h_1 = h_f(P_1) = 417.50 \frac{kJ}{kg}$$

$$s_1 = s_f(P_1) = 1.3028 \frac{kJ}{kg-K}$$

State 2s $P_{2s} = 25$ bar, $s_{2s} = s_1 = 1.3028 \frac{kJ}{kg-K}$

$s_{2s} < s_f(P_{2s}) \Rightarrow$ compressed liquid

Interpolating: $T_{2s} = 99.78°C$, $h_{2s} = 420.05 \frac{kJ}{kg}$

State 2

$$\eta_{pump} = \frac{|w_{isentropic}|}{|w_{actual}|} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

$$\Rightarrow 0.85 = \frac{420.05 - 417.50}{h_2 - 417.50} \Rightarrow h_2 = 420.50 \frac{kJ}{kg}$$

$P_2 = 25$ bar, $h_2 = 420.50 \frac{kJ}{kg}$

$h_2 < h_f(P_2) \Rightarrow$ compressed liquid

Interpolating: $T_2 = 99.88°C$, $s_2 = 1.3039 \frac{kJ}{kg-K}$

(Double Interpolation

(a) Mass balance: $0 = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}$

Energy balance: $0 = 0 - \dot{W}_{pump} + \dot{m}_1 h_1 - \dot{m}_2 h_2$

$\dot{W}_{pump} = \dot{m}(h_1 - h_2)$

Specific work for the pump: $w_{pump} = \frac{\dot{W}_{pump}}{\dot{m}} = h_1 - h_2$

$$\boxed{w_{pump} = -3 \frac{kJ}{kg}}$$ $w < 0$ work input

(b) Entropy balance: $0 = \dot{m}_1 s_1 - \dot{m}_2 s_2 + 0 + \dot{\sigma}_{pump}$

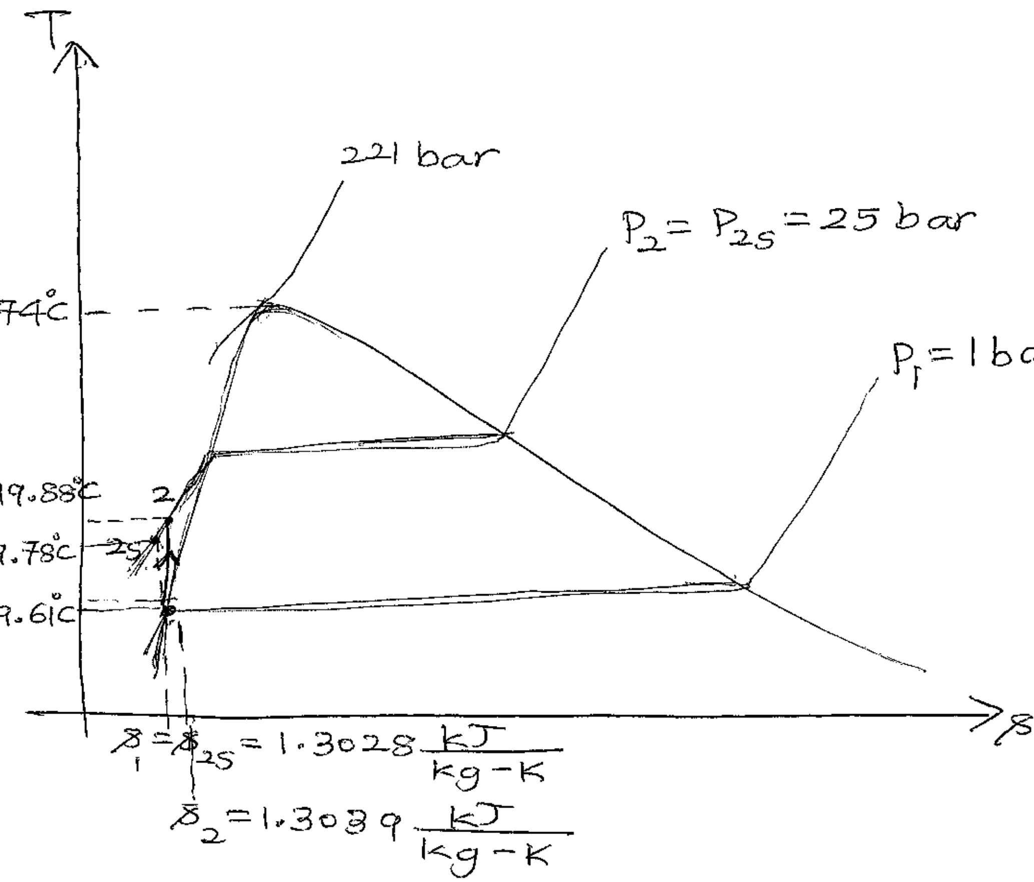
$\dot{\sigma}_{pump} = \dot{m}(s_2 - s_1)$

Specific entropy generation for the pump:

$$\sigma_{pump} = \frac{\dot{\sigma}_{pump}}{\dot{m}} = s_2 - s_1$$

$$\boxed{\sigma_{pump} = +0.0011 \frac{kJ}{kg-K}}$$ $\sigma_{generation} > 0$ ✓

(c)



Note

$$w_{\substack{int.rev\\flow}} = -\int v\,dP + \Delta \cancel{ke}^{\sim 0} + \Delta \cancel{pe}^{\sim 0}$$

Assuming liquid water to be incompressible

$$v_1 = v_f(P_1) = 0.0010432 \frac{m^3}{kg} \simeq v_2$$

$$h_1 - h_{2s} = -v_1 (P_{2s} - P_1)$$

$$h_{2s} = h_1 + v_1 (P_{2s} - P_1) = 417.50 \frac{kJ}{kg} + 0.0010432 \frac{m^3}{kg} * (25-1) \times 100 \text{ kPa}$$

$$h_{2s} = 420 \frac{kJ}{kg}$$

$$s_{2s} - s_1 = 0 = C_{water} \ln \frac{T_{2s}}{T_1} \Rightarrow T_{2s} = T_1 = 99.61°C$$

Given

Compression of saturated water vapor through an adiabatic compressor

Find

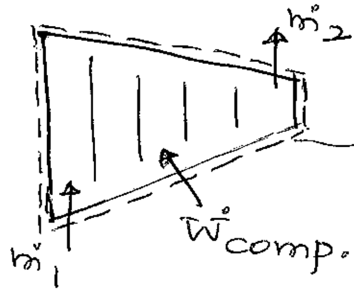
(a) Specific work $\left(\frac{kJ}{kg}\right)$ for the compressor

(b) Specific entropy generation $\left(\frac{kJ}{kg-K}\right)$ for the compressor

(c) T-s diagram

System

$P_2 = 25$ bar



Flow of water vapor through the compressor is the system

$P_1 = 1$ bar
sat. vapor

$\eta_{comp.} = 85\%$

Assumptions

- Open system
- Steady state, steady flow
- 1-D, uniform flow
- Ignore change in KE and PE
- Adiabatic: $\dot{Q} = 0$

Basic Equations

$$\cancel{\left.\frac{dm}{dt}\right|_{system}} = \sum_{in} \dot{m}_{in} - \sum_{out} \dot{m}_{out}$$

$$\cancel{\left.\frac{dE}{dt}\right|_{system}}^{\,0} = \sum_{in} \dot{m}_{in}(h + \cancel{ke} + \cancel{pe})_{in} - \sum_{out} \dot{m}_{out}(h + \cancel{ke} + \cancel{pe})_{out} + \cancel{\dot{Q}}^{\,0} - \dot{W}$$

$$\cancel{\left.\frac{dS}{dt}\right|_{system}}^{\,0} = \sum_{in} \dot{m}_{in} s_{in} - \sum_{out} \dot{m}_{out} s_{out} + \cancel{\sum_j \frac{\dot{Q}_j}{T_{j,boundary}}}^{\,0} + \dot{\sigma}_{generation}$$

Solution

State 1 $\quad P_1 = 1$ bar, sat. vapor

$$T_1 = T_{sat}(P_1) = 99.61^\circ C$$

$$h_1 = h_g(P_1) = 2674.9 \ \frac{kJ}{kg}$$

$$s_1 = s_g(P_1) = 7.3588 \ \frac{kJ}{kg-K}$$

State 2s $\quad P_{2s} = 25 \text{ bar}, \; s_{2s} = s_1 = 7.3588 \frac{kJ}{kg-K}$

$s_{2s} > s_g(P_{2s}) \Rightarrow$ superheated vapor

Interpolating: $T_{2s} = 509.1°C, \; h_{2s} = 3483.9 \frac{kJ}{kg}$

State 2

$$\eta_{comp.} = \frac{|w_{isentropic}|}{|w_{actual}|} = \frac{h_{2s} - h_1}{h_2 - h_1}$$

$$\Rightarrow 0.85 = \frac{3483.9 - 2674.9}{h_2 - 2674.9} \Rightarrow h_2 = 3626.7 \frac{kJ}{kg}$$

$P_2 = 25 \text{ bar}, \; h_2 = 3626.7 \frac{kJ}{kg}$

$h_2 > h_g(P_2) \Rightarrow$ superheated vapor

Interpolating: $T_2 = 573.3°C, \; s_2 = 7.5361 \frac{kJ}{kg-K}$

(Double Interpolation

(a) Mass balance: $0 = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}$

Energy balance: $0 = 0 - \dot{W}_{comp.} + \dot{m}_1 h_1 - \dot{m}_2 h_2$

$\dot{W}_{comp.} = \dot{m}(h_1 - h_2)$

Specific work for the compressor: $w_{comp.} = \frac{\dot{W}_{comp.}}{\dot{m}} = h_1 - h_2$

$$\boxed{w_{comp.} = -951.8 \frac{kJ}{kg}}$$ $w < 0$ work input

(b) Entropy balance: $0 = \dot{m}_1 s_1 - \dot{m}_2 s_2 + 0 + \dot{\sigma}_{comp.}$

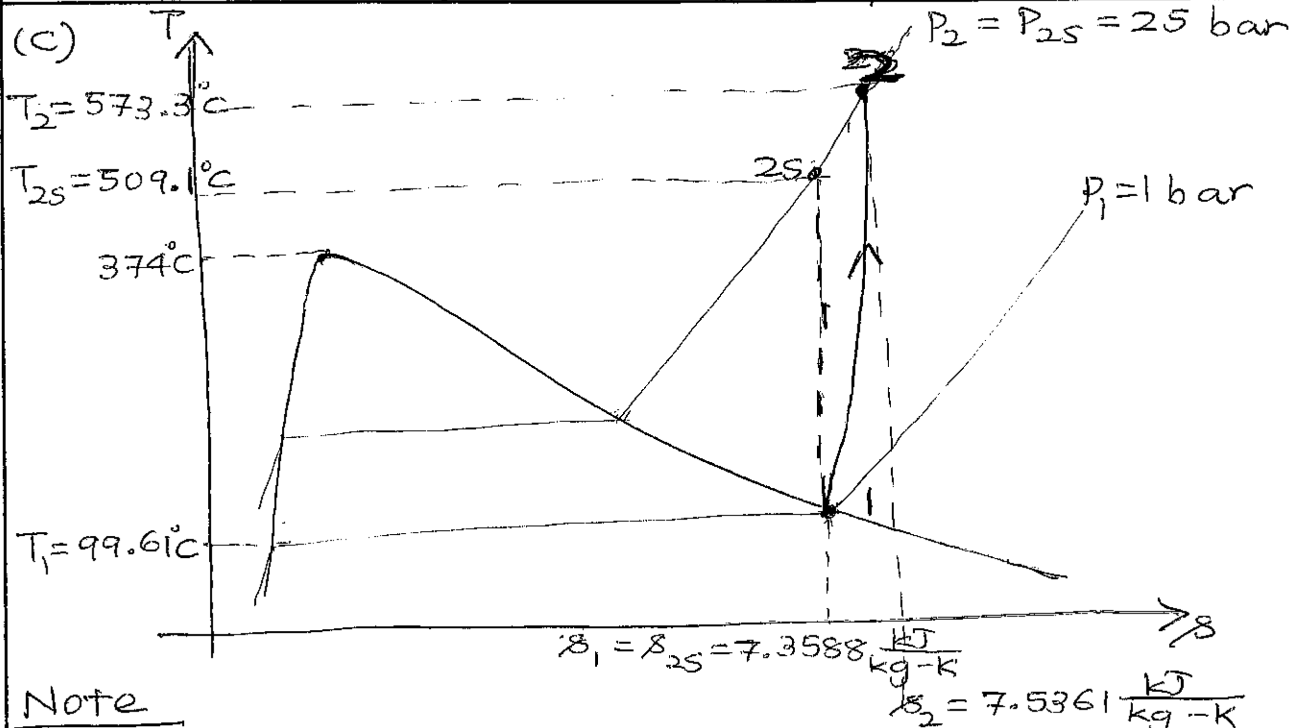
$\dot{\sigma}_{comp.} = \dot{m}(s_2 - s_1)$

Specific entropy generation for the compressor:

$$\sigma_{comp.} = \frac{\dot{\sigma}_{comp.}}{\dot{m}} = s_2 - s_1$$

$$\boxed{\sigma_{comp.} = +0.1773 \frac{kJ}{kg-K}}$$ $\sigma_{generation} > 0$ ✓

(c)



Note

* For the same pressure rise and isentropic efficiency

$$w_{pump} <<< w_{comp.}$$

$$\sigma_{pump} <<< \sigma_{comp.}$$

(liquid) (vapor)

* For nearly incompressible liquid in the pump

$$T_{2s} \cong T_2$$

For compressible vapor in the compressor

$$T_{2s} \neq T_2$$

Given

Expansion of air via an internally reversible polytropic process

Find

(a) Power output (kW) of the turbine

(b) Rate of heat transfer (kW) for the turbine

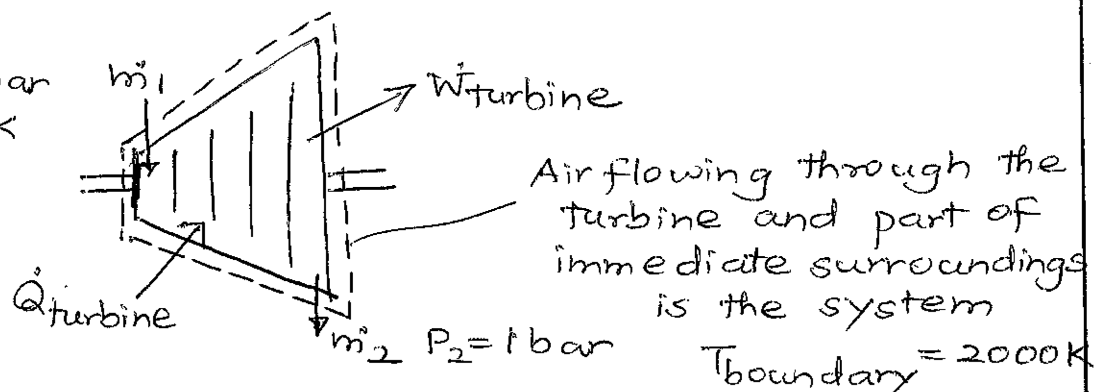
(c) Rate of total entropy generation $\left(\frac{kW}{K}\right)$

(d) T-s diagram

System

$P_1 = 11.75$ bar

$T_1 = 1500$ K



Assumptions

- Open system
- Steady state, steady flow
- 1-D, uniform flow
- Ignore change in KE and PE
- Air behaves as an ideal gas
- Neglect additional mass in the enlarged system

Basic Equations

$$\cancelto{0}{\left.\frac{dm}{dt}\right|_{system}} = \cancel{\sum_{in}} \dot{m}_{in} - \cancel{\sum_{out}} \dot{m}_{out}$$

$$\cancelto{0}{\left.\frac{dE}{dt}\right|_{system}} = \cancel{\sum_{in}} \dot{m}_{in}(h + \cancel{ke} + \cancel{pe})_{in} - \cancel{\sum_{out}} \dot{m}_{out}(h + \cancel{ke} + \cancel{pe})_{out} + \dot{Q} - \dot{W}$$

$$\cancelto{0}{\left.\frac{dS}{dt}\right|_{system}} = \cancel{\sum_{in}} \dot{m}_{in} s_{in} - \cancel{\sum_{out}} \dot{m}_{out} s_{out} + \cancel{\sum_{j}} \frac{\dot{Q}_j}{T_{j,boundary}} + \dot{\sigma}_{generation}$$

Solution

(a) For an internally reversible process, the specific work:

$$w_{\substack{int\text{-}rev\\flow}} = -\int v\,dP + \cancel{\Delta ke}^{\,no} + \cancel{\Delta pe}^{\,no}$$

$$w_{12} = -\int_{P_1}^{P_2} \left(\frac{\text{constant}}{P}\right)^{1/n} dP = \frac{n}{n-1}(P_1 v_1 - P_2 v_2)$$

$$= \frac{n}{n-1} R_{air}(T_1 - T_2) \qquad R_{air} = \frac{\bar{R}}{M_{air}} = 0.287\ \frac{kJ}{kg-K}$$

For polytropic process of an ideal gas:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \Rightarrow \frac{T_2}{1500\ K} = \left(\frac{1\ bar}{11.75\ bar}\right)^{\frac{0.3}{1.3}} \Rightarrow T_2 = 849.5\ K$$

$$w_{12} = \frac{1.3}{1.3-1} * 0.287\ \frac{kJ}{kg-K} * (1500 - 849.5)\ K$$

$$w_{12} = +809\ \frac{kJ}{kg}$$

Power output of the turbine: $\dot{W}_{turbine} = \dot{m}_{air}\, w_{12}$

$$\dot{W}_{turbine} = 1.5\ \frac{kg}{s} * 809\ \frac{kJ}{kg} \Rightarrow \boxed{\dot{W}_{turbine} = +1213.5\ kW}$$

(b) Energy balance: $\overset{\vee}{0} = \dot{Q}_{turbine} - \dot{W}_{turbine} + \dot{m}_1 h_1 - \dot{m}_2 h_2$

Rate of heat transfer for the turbine:

$$\dot{Q}_{turbine} = \dot{W}_{turbine} + \dot{m}_{air}(h_2 - h_1)$$

$T_1 = 1500\ K$, $h_1 = 1636\ \frac{kJ}{kg}$

$T_2 = 849.5\ K$, $h_2 = 876.6\ \frac{kJ}{kg}$

$$\dot{Q}_{turbine} = +1213.5\ kW + 1.5\ \frac{kg}{s} * (876.6 - 1636)\ \frac{kJ}{kg}$$

$$\Rightarrow \boxed{\dot{Q}_{turbine} = +74.4\ kW}$$

$\dot{Q} > 0$ heat transfer into the system

(c) Entropy balance: $0 = \dot{m}_1 s_1 - \dot{m}_2 s_2 + \frac{\dot{Q}_{turbine}}{T_b} + \dot{\sigma}_{total}$

Rate of total entropy generation:

$$\dot{\sigma}_{total} = \dot{m}_{air}(s_2 - s_1) - \frac{\dot{Q}_{turbine}}{T_b}$$

$$= \dot{m}_{air}\left(s_2^\circ - s_1^\circ - R_{air} \ln \frac{P_2}{P_1}\right) - \frac{\dot{Q}_{turbine}}{T_b}$$

$T_1 = 1500\ K$, $s_1^\circ = 3.445 \frac{kJ}{kg-K}$

$T_2 = 849.5\ K$, $s_2^\circ = 2.785 \frac{kJ}{kg-K}$

$$\dot{\sigma}_{total} = 1.5 \frac{kg}{s} * \left(2.785 - 3.445 - 0.287 \ln \frac{1\ bar}{11.75\ bar}\right) \frac{kJ}{kg-K} - \frac{74.4\ kW}{2000\ K}$$

$\Rightarrow \boxed{\dot{\sigma}_{total} = +0.0335 \frac{kW}{K}}$ $\dot{\sigma}_{generation} > 0$ ✓

Note

$\sigma_{internal} = 0$ and $\sigma_{total} > 0$

(d)