

Given Heating of liquid CO_2 at constant pressure

Find

(a) Change in specific internal energy ($\frac{\text{kJ}}{\text{kg}}$) using:

- Property table
- Saturated liquid approximation
- Specific heat at average temperature

(b) Change in specific enthalpy ($\frac{\text{kJ}}{\text{kg}}$) using:

- Property table
- Saturated liquid approximation
- Specific heat at average temperature

System

N/A

Assumptions

- Liquid CO_2 is incompressible
- Specific heat constant with temperature change

Basic Equations

N/A

Solution

$$\text{CO}_2, \quad P_1 = 20 \text{ bar} \rightarrow P_2 = 20 \text{ bar}$$

$$T_1 = -40^\circ\text{C} \rightarrow T_2 = -20^\circ\text{C}$$

$$T_{\text{sat}}(20 \text{ bar}) = -19.503^\circ\text{C} \Rightarrow T < T_{\text{sat}} \Rightarrow \text{sub-cooled (compressed) liquid}$$

(a) Compressed liquid table:

$$u_1 = -1.641 \frac{\text{kJ}}{\text{kg}}, \quad u_2 = 39.601 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta u = u_2 - u_1 \Rightarrow \boxed{\Delta u = 41.242 \frac{\text{kJ}}{\text{kg}}}$$

Saturated liquid approximation:

$$u_1 \approx u_f(-40^\circ\text{C}) = -0.8997 \frac{\text{kJ}}{\text{kg}}, \quad u_2 \approx u_f(-20^\circ\text{C}) = 39.636 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta u = u_2 - u_1 \Rightarrow \boxed{\Delta u = 40.536 \frac{\text{kJ}}{\text{kg}}}$$

Specific heat:

$$C_{\text{CO}_2} = 2.082 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$\Rightarrow \boxed{\Delta u = 41.64 \frac{\text{kJ}}{\text{kg}}}$$

$$\Delta u = C_{\text{CO}_2}(T_2 - T_1) = 2.082 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} * (-20 - (-40)) \text{ K}$$

(b) Compressed liquid table:

$$h_1 = 0.146 \frac{\text{kJ}}{\text{kg}}, \quad h_2 = 41.540 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta h = h_2 - h_1 \Rightarrow \boxed{\Delta h = 41.394 \frac{\text{kJ}}{\text{kg}}}$$

Saturated liquid approximation:

$$h_1 \approx h_f(-40^\circ\text{C}) + v_f(-40^\circ\text{C}) [P_1 - P_{\text{sat}}(-40^\circ\text{C})]$$

$$= 0 \frac{\text{kJ}}{\text{kg}} + 0.0008957 \frac{\text{m}^3}{\text{kg}} * (20 - 10.045) \times 100 \text{ kPa}$$

$$= 0.89167 \frac{\text{kJ}}{\text{kg}}$$

$$h_2 \approx h_f(-20^\circ\text{C}) + v_f(-20^\circ\text{C}) [P_2 - P_{\text{sat}}(-20^\circ\text{C})]$$

$$= 41.546 \frac{\text{kJ}}{\text{kg}} + 0.0009693 \frac{\text{m}^3}{\text{kg}} * (20 - 19.696) \times 100 \text{ kPa}$$

$$= 41.575 \frac{\text{kJ}}{\text{kg}}$$

$$\Delta h = h_2 - h_1 \Rightarrow \boxed{\Delta h = 40.683 \frac{\text{kJ}}{\text{kg}}}$$

Specific heat:

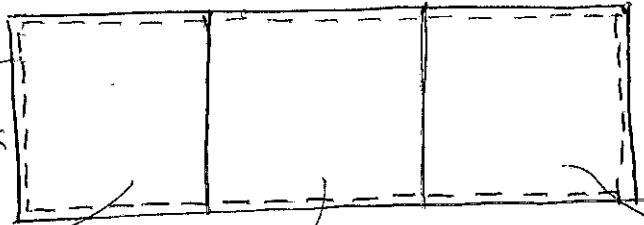
$$\Delta h = \Delta u + v \Delta P \Rightarrow \boxed{\Delta h = 41.64 \frac{\text{kJ}}{\text{kg}}}$$

Given

Heat transfer among three liquids inside a well-insulated, closed rigid tank

FindTemperature ($^{\circ}\text{C}$) at equilibriumSystem

Liquid inside the rigid tank is the system



$$m_A = 5 \text{ kg}$$

$$C_A = 2 \frac{\text{kJ}}{\text{kg-K}}$$

$$T_{A,i} = 100^{\circ}\text{C}$$

$$m_B = 2 \text{ kg}$$

$$C_B = 4 \frac{\text{kJ}}{\text{kg-K}}$$

$$T_{B,i} = 50^{\circ}\text{C}$$

$$m_C = 1 \text{ kg}$$

$$C_C = 7 \frac{\text{kJ}}{\text{kg-K}}$$

$$T_{C,i} = 40^{\circ}\text{C}$$

Assumptions- Closed system ($m = \text{constant}$)- Rigid tank: $W_{\text{boundary}} = 0$ - Insulated: $Q = 0$

- Ignore change in KE and PE

- Liquids are incompressible

- Specific heat constant with temperature change

Basic Equations

$$\cancel{Q} - \cancel{W} = \Delta E = \Delta U + \overset{\text{no}}{\Delta KE} + \overset{\text{no}}{\Delta PE}$$

$$\hookrightarrow (W_{\text{boundary}} + W_{\text{other}})$$

SolutionConsidering energy balance: $\Delta U = 0$

$$\Delta U_A + \Delta U_B + \Delta U_C = 0$$

$$m_A C_A (T_f - T_{A,i}) + m_B C_B (T_f - T_{B,i}) + m_C C_C (T_f - T_{C,i}) = 0$$

Temperature at equilibrium:

$$T_f = \frac{m_A C_A T_{A,i} + m_B C_B T_{B,i} + m_C C_C T_{C,i}}{m_A C_A + m_B C_B + m_C C_C}$$

$$T_f = 67.2^{\circ}\text{C}$$

Given

Air inside a closed, rigid tank

Find

(a) Temperature (K) of air at State 2

(b) Heat transfer (kJ) during the process

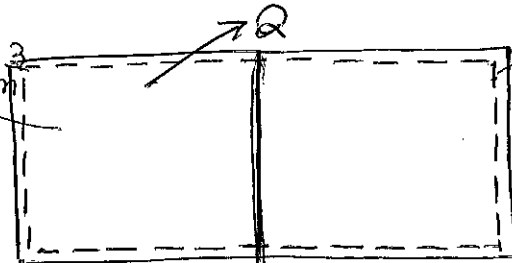
System

$$P_1 = 5 \text{ bar}, V_1 = 0.2 \text{ m}^3$$

$$T_1 = 500 \text{ K}$$



$$P_2 = 1.25 \text{ bar}, V_2 = 0.4 \text{ m}^3$$



Air inside the tank is the system

Assumptions- Closed system ($m = \text{constant}$)- Rigid tank: $W_{\text{boundary}} = 0$

- Ignore change in KE and PE

- Air behaves as an ideal gas

- Specific heat constant with temperature change

Basic Equations

$$Q - \overset{0}{W} = \Delta E = \Delta U + \overset{0}{\Delta KE} + \overset{0}{\Delta PE}$$

$$\hookrightarrow (W_{\text{boundary}} + W_{\text{other}})$$

Solution(a) State 1 $P_1 = 5 \text{ bar}, V_1 = 0.2 \text{ m}^3, T_1 = 500 \text{ K}$ Ideal gas equation: $P_1 V_1 = m_1 R_{\text{air}} T_1$

$$R_{\text{air}} = \frac{\bar{R}}{M_{\text{air}}} = \frac{8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}}{28.97 \frac{\text{kg}}{\text{kmol}}} = 0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$m_1 = \frac{P_1 V_1}{R_{\text{air}} T_1} = \frac{(5 \times 100) \text{ kPa} \cdot 0.2 \text{ m}^3}{0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \cdot 500 \text{ K}} = 0.697 \text{ kg}$$

State 2 $P_2 = 1.25 \text{ bar}, V_2 = 0.4 \text{ m}^3, m_2 = m_1$ Ideal gas equation: $P_2 V_2 = m_2 R_{\text{air}} T_2$

Temperature at State 2: $T_2 = \frac{P_2 V_2}{m_2 R_{air}}$

$$T_2 = \frac{(1.25 \times 100) \text{ kPa} * 0.4 \text{ m}^3}{0.697 \text{ kg} * 0.287 \frac{\text{kJ}}{\text{kg-K}}} \Rightarrow \boxed{T_2 = 250 \text{ K}}$$

(b) Considering energy balance, heat transfer during the process: $Q = \Delta U = U_2 - U_1 = m_2 u_2 - m_1 u_1$

$$Q = (m_1 = m_2) (u_2 - u_1) = (m_1 = m_2) C_{v, \text{air}} (T_2 - T_1)$$

For an ideal gas: $C_p - C_v = R$

$$C_{v, \text{air}} = C_{p, \text{air}} - R_{\text{air}} = 1.017 \frac{\text{kJ}}{\text{kg-K}} - 0.287 \frac{\text{kJ}}{\text{kg-K}} \\ = 0.730 \frac{\text{kJ}}{\text{kg-K}}$$

$$Q = 0.697 \text{ kg} * 0.730 \frac{\text{kJ}}{\text{kg-K}} * (250 - 500) \text{ K}$$

$$\boxed{Q = -127.2 \text{ kJ}} \quad Q < 0 \text{ heat transfer out of the system}$$

Given

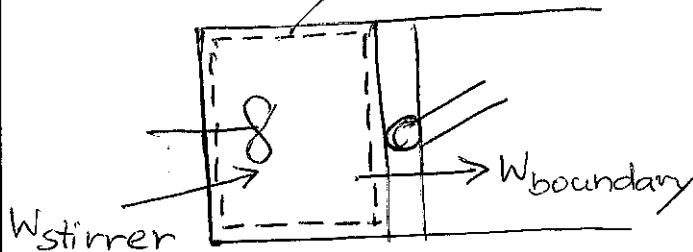
Air inside a closed ^{insulated} piston-cylinder device with a stirrer

Find

Work (kJ) for the stirrer

System

Air inside the cylinder is the system



$$P_1 = 1.5 \text{ bar}, V_1 = 0.02 \text{ m}^3$$

$$T_1 = 400 \text{ K}$$

↓

$$P_2 = P_1, T_2 = 950 \text{ K}$$

Assumptions

- Closed system ($m = \text{constant}$)
- Quasi-equilibrium process

- Ignore change in KE and PE
- Air behaves as an ideal gas
- Insulated: $Q = 0$

Basic Equations

$$W_{\text{boundary}} = \int P dV$$

$$Q - W = \Delta E = \Delta U + \Delta \cancel{KE} + \Delta \cancel{PE}$$

$$\hookrightarrow (W_{\text{boundary}} + W_{\text{other}})$$

Solution

Considering energy balance, work during the process: $-W = \Delta U = U_2 - U_1 = m_2 u_2 - m_1 u_1$

$$-(W_{\text{boundary}} + W_{\text{other}}) = (m_1 = m_2)(u_2 - u_1)$$

State 1 $P_1 = 1.5 \text{ bar}, V_1 = 0.02 \text{ m}^3, T_1 = 400 \text{ K}$

Ideal gas equation: $P_1 V_1 = m_1 R_{\text{air}} T_1$

$$R_{\text{air}} = \frac{\bar{R}}{M_{\text{air}}} = \frac{8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}}{28.97 \frac{\text{kg}}{\text{kmol}}} = 0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$m_1 = \frac{P_1 V_1}{R_{\text{air}} T_1} = \frac{(1.5 \times 100) \text{ kPa} * 0.02 \text{ m}^3}{0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} * 400 \text{ K}} = 0.0261 \text{ kg}$$

State 2 $P_2 = P_1 = 1.5 \text{ bar}$, $T_2 = 950 \text{ K}$, $m_2 = m_1$

Ideal gas equation: $P_2 V_2 = m_2 R_{\text{air}} T_2$

$$V_2 = \frac{m_2 R_{\text{air}} T_2}{P_2} = \frac{0.0261 \text{ kg} * 0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} * 950 \text{ K}}{(1.5 \times 100) \text{ kPa}}$$

$$= 0.0474 \text{ m}^3$$

Boundary work during the process:

$$W_{\text{boundary}} = \int_{V_1}^{V_2} P dV = (P_1 = P_2) (V_2 - V_1) = (1.5 \times 100) \text{ kPa} * (0.0474 - 0.02) \text{ m}^3$$

$$= +4.11 \text{ kJ} \quad W > 0 \text{ work done by air}$$

$$T_1 = 400 \text{ K}, \quad u_1 = 286.1 \frac{\text{kJ}}{\text{kg}}$$

$$T_2 = 950 \text{ K}, \quad u_2 = 716.5 \frac{\text{kJ}}{\text{kg}}$$

Work for the stirrer:

$$-W_{\text{stirrer}} = -W_{\text{other}} = (m_1 = m_2) (u_2 - u_1) + W_{\text{boundary}}$$

$$= 0.0261 \text{ kg} * (716.5 - 286.1) \frac{\text{kJ}}{\text{kg}} + 4.11 \text{ kJ}$$

$$= 15.3 \text{ kJ}$$

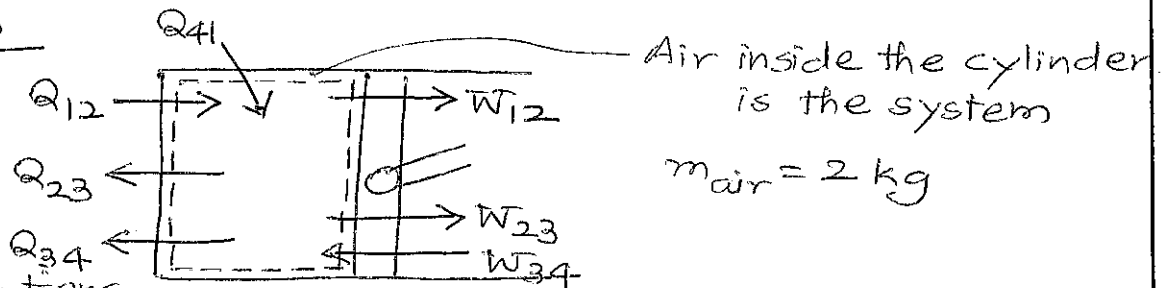
$$\Rightarrow \boxed{W_{\text{stirrer}} = -15.3 \text{ kJ}} \quad W < 0 \text{ work done on air}$$

Given

Air inside a closed piston-cylinder device undergoing four processes

Find

- Temperature (K) at State 3 and State 4
- Work (kJ) for each process
- Heat transfer (kJ) for each process
- P-V diagram

SystemAssumptions

- Closed system ($m = \text{constant}$)
- Quasi-equilibrium process
- Ignore KE and PE change
- Air behaves as an ideal gas

Basic Equations

$$W_{\text{boundary}} = \int P dV$$

$$Q - W = \Delta E = \Delta U + \Delta \cancel{KE} + \Delta \cancel{PE}$$

($W_{\text{boundary}} + W_{\text{other}}$)

Solution

(a) State 1 $P_1 = 5 \text{ bar}$, $T_1 = 600 \text{ K}$

Ideal gas equation: $P_1 V_1 = m_{\text{air}} R_{\text{air}} T_1$

$$R_{\text{air}} = \frac{\bar{R}}{M_{\text{air}}} = \frac{8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}}{28.97 \frac{\text{kg}}{\text{kmol}}} = 0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$V_1 = \frac{m_{\text{air}} R_{\text{air}} T_1}{P_1} = \frac{2 \text{ kg} * 0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} * 600 \text{ K}}{(5 \times 100) \text{ kPa}}$$

$$V_1 = 0.689 \text{ m}^3$$

State 2 $P_2 = 4 \text{ bar}$, $T_2 = T_1 = 600 \text{ K}$

$$V_2 = \frac{m_{\text{air}} R_{\text{air}} T_2}{P_2} = \frac{2 \text{ kg} * 0.287 \frac{\text{kJ}}{\text{kg-K}} * 600 \text{ K}}{(4 \times 100) \text{ kPa}}$$

$$V_2 = 0.861 \text{ m}^3$$

State 3 $P_3 = 3 \text{ bar}$, $P_2 V_2^{1.4} = P_3 V_3^{1.4} \Rightarrow V_3 = 1.06 \text{ m}^3$

$$T_3 = \frac{P_3 V_3}{m_{\text{air}} R_{\text{air}}} = \frac{(3 \times 100) \text{ kPa} * 1.06 \text{ m}^3}{2 \text{ kg} * 0.287 \frac{\text{kJ}}{\text{kg-K}}}$$

Temperature at State 3: $T_3 = 554 \text{ K}$

Alternatively

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2} \right)^{\frac{n-1}{n}} \Rightarrow T_3 = 600 \text{ K} * \left(\frac{3 \text{ bar}}{4 \text{ bar}} \right)^{\frac{1.4-1}{1.4}} \Rightarrow T_3 \approx 554 \text{ K}$$

State 4 $P_4 = P_3 = 3 \text{ bar}$, $V_4 = V_1 = 0.689 \text{ m}^3$

$$T_4 = \frac{P_4 V_4}{m_{\text{air}} R_{\text{air}}} = \frac{(3 \times 100) \text{ kPa} * 0.689 \text{ m}^3}{2 \text{ kg} * 0.287 \frac{\text{kJ}}{\text{kg-K}}}$$

Temperature at State 4: $T_4 = 360 \text{ K}$

Alternatively

$$\frac{T_4}{T_3} = \frac{V_4}{V_3} \Rightarrow T_4 = 554 \text{ K} * \frac{0.689 \text{ m}^3}{1.060 \text{ m}^3} \Rightarrow T_4 \approx 360 \text{ K}$$

(b) Boundary work for each process:

$$W_{12} = \int_{V_1}^{V_2} P dV \quad m_{\text{air}} = \frac{P_1 V_1}{R_{\text{air}} T_1} = \frac{P_2 V_2}{R_{\text{air}} T_2} \Rightarrow PV = \text{constant}$$

$$W_{12} = \int_{V_1}^{V_2} \left(\frac{\text{constant}}{V} \right) dV = \underbrace{(P_1 V_1 = P_2 V_2)}_{m_{\text{air}} R_{\text{air}} (T_1 = T_2)} \ln \frac{V_2}{V_1} \quad \frac{V_2}{V_1} = \frac{P_1}{P_2}$$

$$= 2 \text{ kg} * 0.287 \frac{\text{kJ}}{\text{kg-K}} * 600 \text{ K} \ln \frac{5 \text{ bar}}{4 \text{ bar}}$$

$$W_{12} = +76.8 \text{ kJ} \quad W > 0 \Rightarrow \text{work done by the system}$$

$$W_{23} = \int_{V_2}^{V_3} \left(\frac{\text{constant}}{V^{1.4}} \right) dV = \frac{P_2 V_2 - P_3 V_3}{n-1} = \frac{m_{\text{air}} R_{\text{air}} (T_2 - T_3)}{n-1}$$

$$= \frac{2 \text{ kg} * 0.287 \frac{\text{kJ}}{\text{kg-K}} * (600 - 554) \text{ K}}{1.4 - 1}$$

$$\boxed{W_{23} = +66 \text{ kJ}} \quad W > 0 \Rightarrow \text{work done by the system}$$

$$W_{34} = \int_{V_3}^{V_4} P dV = (P_3 = P_4)(V_4 - V_3) = (3 \times 100) \text{ kPa} * (0.889 - 1.060) \text{ m}^3$$

$$\boxed{W_{34} = -111.3 \text{ kJ}} \quad W < 0 \Rightarrow \text{work done on the system}$$

$$W_{41} = \int_{V_4}^{V_1} P dV \Rightarrow \boxed{W_{41} = 0}$$

(c) Considering energy balance, heat transfer for each process: $Q = W + \Delta U$

$$Q_{12} = W_{12} + (U_2 - U_1) = W_{12} + m_{\text{air}}(u_2 - u_1) \quad \begin{matrix} T_1 = T_2 \\ u = f(T) \text{ only} \\ \text{for ideal gas} \end{matrix}$$

$$\boxed{Q_{12} = +76.8 \text{ kJ}} \quad Q > 0 \Rightarrow \text{heat transfer into the system}$$

$$Q_{23} = W_{23} + (U_3 - U_2) = W_{23} + m_{\text{air}}(u_3 - u_2)$$

$$T_2 = 600 \text{ K} \Rightarrow u_2 = 434.8 \frac{\text{kJ}}{\text{kg}}$$

$$T_3 = 554 \text{ K} \quad \text{Interpolating: } u_3 = 399.9 \frac{\text{kJ}}{\text{kg}}$$

$$Q_{23} = +66 \text{ kJ} + 2 \text{ kg} * (399.9 - 434.8) \frac{\text{kJ}}{\text{kg}}$$

$$\boxed{Q_{23} = -3.8 \text{ kJ}} \quad Q < 0 \Rightarrow \text{heat transfer out of the system}$$

$$Q_{34} = W_{34} + (U_4 - U_3) = W_{34} + m_{\text{air}}(u_4 - u_3)$$

$$T_4 = 360 \text{ K} \Rightarrow u_4 = 257.2 \frac{\text{kJ}}{\text{kg}}$$

$$Q_{34} = -111.3 \text{ kJ} + 2 \text{ kg} * (257.2 - 399.9) \frac{\text{kJ}}{\text{kg}}$$

$$\boxed{Q_{34} = -396.7 \text{ kJ}} \quad Q < 0 \Rightarrow \text{heat transfer out of the system}$$

$$Q_{41} = \cancel{W_{41}}^0 + (U_1 - U_4) = m_{\text{air}}(u_1 - u_4)$$

$$Q_{41} = 2 \text{ kg} * (434.8 - 257.2) \frac{\text{kJ}}{\text{kg}}$$

$$\boxed{Q_{41} = +355.2 \text{ kJ}} \quad Q > 0 \Rightarrow \text{heat transfer into the system}$$

Note

$$W_{\text{net}} = W_{12} + W_{23} + W_{34} + \cancel{W_{41}}^0 = +31.5 \text{ kJ}$$

$$Q_{\text{net}} = Q_{12} + Q_{23} + Q_{34} + Q_{41} = +31.5 \text{ kJ}$$

$$\Delta U_{\text{net}} = \Delta U_{12} + \Delta U_{23} + \Delta U_{34} + \Delta U_{41} = 0$$

$$Q_{\text{net}} - W_{\text{net}} = \cancel{\Delta U_{\text{net}}}^0 \Rightarrow Q_{\text{net}} = W_{\text{net}}$$

This is a "cyclic" system

(d)

