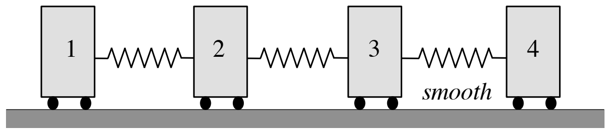
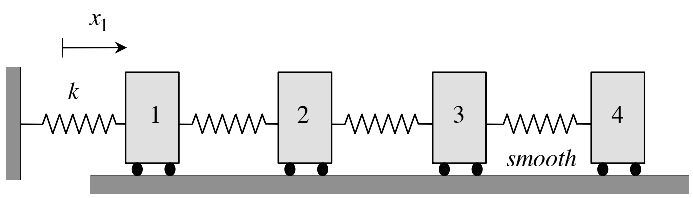
Example V.5.2
The natural frequencies and mass normalized modes of an N-DOF free-free component are known.

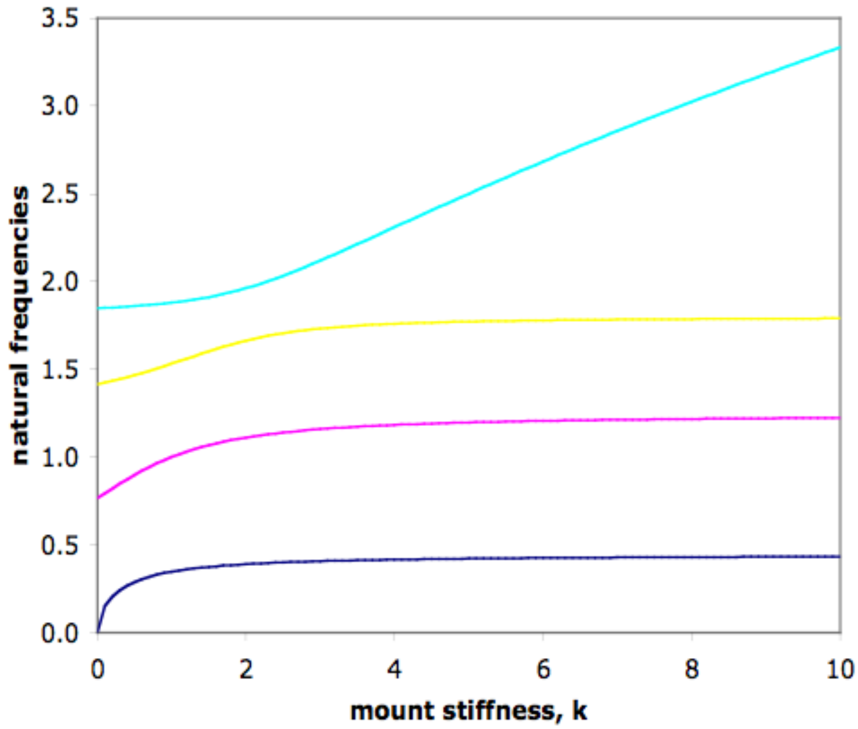


Since the free-free system has a rigid body mode, then $\omega_1 = 0$.

Suppose that we now attach this component to ground with a mount having a stiffness of k.



Determine the sensitivities of each natural frequency to the stiffness k of the mount. In the following figure are shown the exact four natural frequencies of the system as a function of the mount stiffness k.


3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
natural frequencies
0
2
4
6
8
10
mount stiffness, k

unt.

<u>Given</u>: ω_1, $\underset{\sim}{\tilde{\phi}}^{(1)} = \begin{Bmatrix} \tilde{\phi}_1^{(1)} \\ \tilde{\phi}_2^{(1)} \\ \tilde{\phi}_3^{(1)} \\ \tilde{\phi}_4^{(1)} \end{Bmatrix}$

$$\Delta U = \text{additional P.E.}$$

$$= \frac{1}{2} k x_1^2$$

$$= \frac{1}{2} \underset{\sim}{x}^T \underbrace{\begin{bmatrix} k & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{k[\Delta K]} \underset{\sim}{x}$$

From lecture notes:

$$\frac{\partial \omega_j}{\partial R} = \frac{1}{\omega_j} \underbrace{\tilde{\phi}^{(j)T} [\Delta K] \tilde{\phi}^{(j)}}$$

$$= \omega_j \underbrace{\phi_1^{(j)2}}_{\geqslant 0}$$

(i.e., $[\Delta K]$ pos. semi-def)

Size of $\frac{\partial \omega_j}{\partial R}$ dictated by:

$$\frac{1}{\omega_j}, \quad \boxed{\phi_1^{(j)2}}$$

<u>1st mode</u>: $\omega_1 = 0$

$$\frac{\partial \omega_1}{\partial R} = \infty$$

vertical tangency

<u>Otherwise</u>

$\boxed{\frac{1}{\omega_1}} \approx \Rightarrow$ sensitivities will tend to go down for higher modes

Higher freqs, less influenced by support stiffness