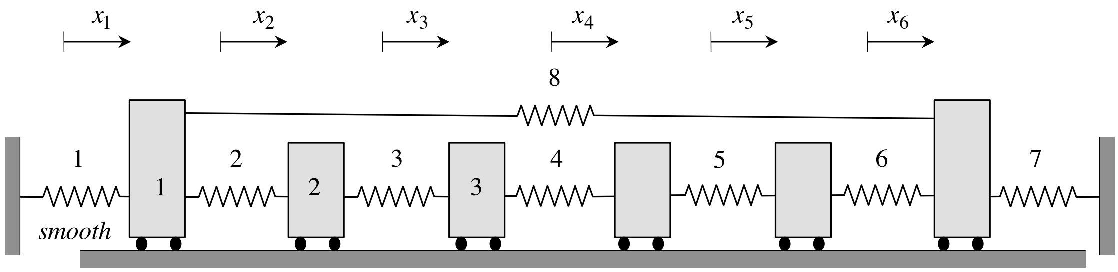
Example V.5.1

For the six-DOF system shown below, no information is given on the masses of the particles nor on the stiffnesses of the springs.



Only the configuration of the system is known, along with the two lowest natural frequencies of $\omega_1 = 8.154 rad/sec$ and $\omega_2 = 16.43 rad/sec$ and corresponding mass normalized modal vectors of:

$$\vec{X}^{(1)} = \begin{Bmatrix} 0.0737 \\ 0.1164 \\ 0.2210 \\ 0.2375 \\ 0.2300 \\ 0.0619 \end{Bmatrix} \frac{1}{\sqrt{kg}} \qquad \vec{X}^{(2)} = \begin{Bmatrix} 0.0574 \\ 0.1491 \\ 0.3032 \\ -0.0342 \\ -0.1937 \\ -0.0108 \end{Bmatrix} \frac{1}{\sqrt{kg}}$$

are known. For this system:

1. Approximate the lowest natural frequency if the stiffness of spring "3" is increased by $2 \times 10^3 N/m$.
2. Determine the sensitivity of the second natural frequency to an increase in the mass matrix element M_{33}.
3. Suppose a forcing is applied to the system with an excitation frequency Ω that is near the second natural frequency ($\Omega \approx \omega_2 = 16.43 rad/sec$), which creates an undesirable resonance situation. To alleviate this resonance problem, we decide to stiffen one of the springs to move (increase) ω_2 away from the excitation frequency; however, we need to be careful to not move $\omega_1 = 8.154 rad/sec$ too close to Ω. With which spring will we be most effective in increasing ω_2 and yet keeping the change in ω_1 small?

Tabular results for eigenvalue stiffness sensitivities are provided on the next page.

	$\frac{\partial \omega_1}{\partial \hat{k}_j}(rad/\sec)/(kN/m)$	$\frac{\partial \omega_2}{\partial \hat{k}_j}(rad/\sec)/(kN/m)$	$\left(\frac{\partial \omega_2}{\partial \hat{k}_j}\right)/\left(\frac{\partial \omega_1}{\partial \hat{k}_j}\right)$
$\hat{k}_1$	0.3331×10^{-3}	0.2020×10^{-3}	0.6066
$\hat{k}_2$	0.1118×10^{-3}	0.2559×10^{-3}	2.2888
$\hat{k}_3$	0.6709×10^{-3}	0.7227×10^{-3}	1.0771
$\hat{k}_4$	0.0167×10^{-3}	3.4644×10^{-3}	207.52
$\hat{k}_5$	0.0034×10^{-3}	0.7742×10^{-3}	***224.46***
$\hat{k}_6$	1.7327×10^{-3}	1.0180×10^{-3}	0.5875
$\hat{k}_7$	0.2349×10^{-3}	0.1166×10^{-3}	0.4963
$\hat{k}_8$	0.0085×10^{-3}	0.1415×10^{-3}	16.578

a) For an <u>additional</u> stiffness $\hat{k}_3$:

$$\frac{\partial \omega_1}{\partial \hat{k}_3} = \frac{1}{2\omega_1} \underset{\sim}{\tilde{\phi}}^{(1)T} [\Delta K_3] \underset{\sim}{\tilde{\phi}}^{(1)}$$

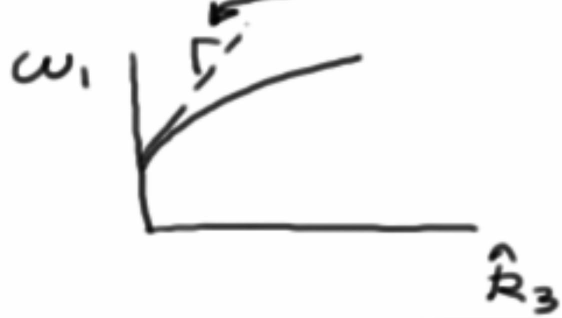
$$\Delta U = \frac{1}{2} \hat{k}_3 (x_3 - x_2)^2$$

$$= \frac{1}{2} \hat{k}_3 \underset{\sim}{x}^T \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{[\Delta K_3]} \underset{\sim}{x}$$

$$\frac{\partial \omega_1}{\partial \hat{k}_3} = \frac{1}{2\omega_1}\left(\tilde{\phi}_3^{(1)} - \tilde{\phi}_2^{(1)}\right)^2$$

$$= \frac{1}{(2)(8.154)}\left[0.2210 - 0.1164\right]^2$$

$$= 0.6709 \times 10^{-3}$$



$$\omega_1 = \omega_1\Big|_{\hat{k}_3=0} + \boxed{\frac{\partial \omega_1}{\partial \hat{k}_3}\Big|_{\hat{k}_3=0} \hat{k}_3}$$

$$\underbrace{+ \frac{1}{2}\frac{\partial^2 \omega_1}{\partial \hat{k}_3^2}\Big|_{\hat{k}_3=0} \hat{k}_3^2 + \cdots}_{\text{neglect}}$$

Taylor series exp.

$$\cong 8.154 + (0.6709 \times 10^{-3})(2 \times 10^{3})$$

$$\boxed{\omega_1 \cong 9.495 \text{ rad/sec}}$$

(b) $[M] = [M_0] + m_3 \underbrace{[\Delta M]}$

$$\begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & 1 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{bmatrix}$$

$$\frac{\partial \omega_2}{\partial m_3} = -\frac{\omega_2}{2} \tilde{\underset{\sim}{\phi}}^{(2)T} [\Delta M] \tilde{\underset{\sim}{\phi}}^{(2)}$$

$$= -\frac{\omega_2}{2} \tilde{\phi}_3^{(2)2}$$

$$= -\frac{16.43}{2}(0.3032)^2$$

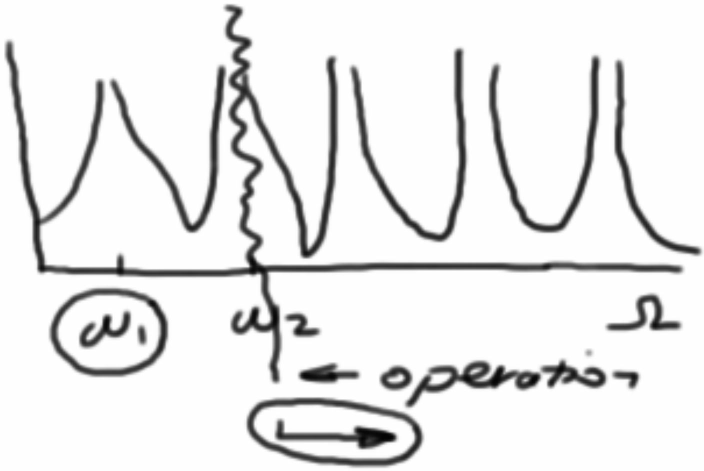
$$= \underbrace{-0.7552 \frac{\text{rad/sec}}{\text{kg}}}_{<0 \Rightarrow}$$

ω_2 will decrease with increase in m_3

$$\omega_2 = \omega_2\big|_{m_3=0} + \underbrace{\frac{\partial \omega_2}{\partial m_3}}_{<0} m_3$$

decrease !

(c)



Calculate $\frac{\partial \omega_1}{\partial \hat{k}_j}$ & $\frac{\partial \omega_2}{\partial \hat{k}_j}$

for all springs.

Choose spring with the largest ratio

$$\frac{\partial \omega_2}{\partial \hat{k}_j} \Big/ \frac{\partial \omega_1}{\partial \hat{k}_j}$$

→ we should choose $\hat{k}_5$ (or $\hat{k}_4$) to better design system to avoid resonance.