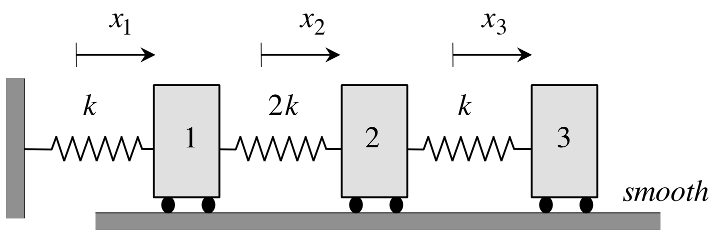
Example V.4.1

Determine lower and upper bounds on the fundamental natural frequency for the 3-DOF shown below. Also, use the matrix iteration method to provide accurate estimates for the lowest natural frequency and corresponding modal vector.



The solution of the exact characteristic equation gives the following three natural frequencies for the system:

j	$\omega_j \sqrt{m/k}$
1	0.42486838
2	1.19205922
3	1.97445730

$$[D] = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 3/2 \\ 1 & 3 & 5/2 \end{bmatrix} \frac{m}{k}$$; see lecture notes

- $\text{trace}[D] = D_{11} + D_{22} + D_{33}$

$$= \left(1 + 3 + \frac{5}{2}\right)\frac{m}{k}$$

$$= \frac{13}{2}\frac{m}{k}$$

$$\therefore \quad \omega_1 > \frac{1}{\sqrt{\text{trace}[D]}} = \sqrt{\frac{2}{13}}\sqrt{\frac{k}{m}}$$

$$= 0.3922\sqrt{k/m}$$

- Say we <u>arbitrarily</u> choose

$$\underline{v} = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$$

$$\underline{v}^T[K]\,\underline{v} = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}^T \begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix} k$$

$$= 4k$$

$$\underline{V}^T[M]\,\underline{V} = \begin{Bmatrix}1\\2\\3\end{Bmatrix}^T \begin{bmatrix}1 & & \\ & 2 & \\ & & 1\end{bmatrix} \begin{Bmatrix}1\\2\\3\end{Bmatrix} m$$

$$= 18m$$

$$\therefore \quad \omega_1 \le \left\{ \frac{\underline{V}^T[K]\,\underline{V}}{\underline{V}^T[M]\,\underline{V}} \right\}^{1/2} = \sqrt{\frac{4k}{18m}} = 0.4714\sqrt{\frac{k}{m}}$$

$\therefore$ we can say

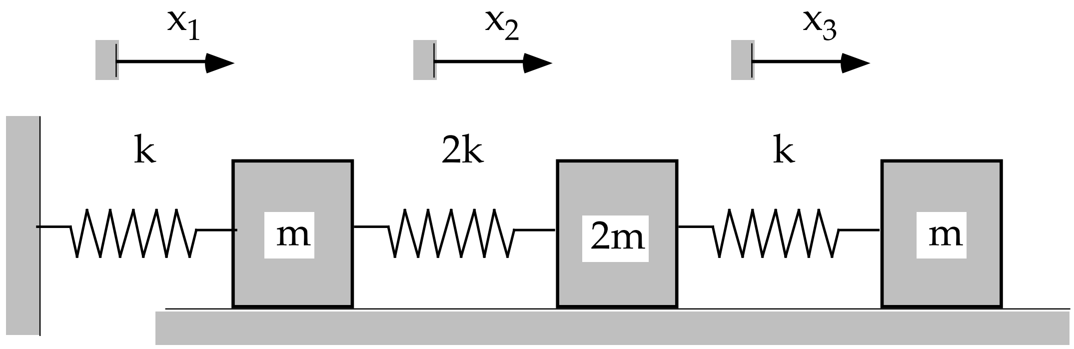
$$0.3922\sqrt{\frac{k}{m}} < \omega_1 \le 0.4714\sqrt{\frac{k}{m}}$$

From lecture notes we see that

$$\omega_1 = 0.4249\sqrt{\frac{k}{m}}$$

which is in agreement with the above bounds.

Use the power method to approximate the fundamental natural frequency and corresponding modal vector for the 3-DOF shown below:



Recall that the solution of the characteristic equation for the above system gives the following natural frequencies and modal vectors:

j	$\omega_j\sqrt{m/k}$	$\phi_1^{(j)}$	$\phi_2^{(j)}$	$\phi_3^{(j)}$
1	0.42486838	1	1.40974342	1.72027583
2	1.19205922	1	0.78949740	-1.87526761
3	1.97445730	1	-0.44924082	0.15499177

SOLUTION

From an earlier example, we have:

$$[D] = [K]^{-1}[M] = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 3/2 \\ 1 & 3 & 5/2 \end{bmatrix} \frac{m}{k}$$

Making an initial guess of $\underline{v}_0 = \{1, 1, 1\}^T$:

$$\underline{v}_1 = [D]\underline{v}_0 = \frac{m}{k} \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 3/2 \\ 1 & 3 & 5/2 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = \frac{m}{k} \begin{Bmatrix} 4 \\ 5.5 \\ 6.5 \end{Bmatrix}$$

$$\hat{\underline{v}}_1 = \frac{\underline{v}_1}{(v_1)_1} = \left(\frac{1}{4m/k} \right) \frac{m}{k} \begin{Bmatrix} 4 \\ 5.5 \\ 6.5 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1.3750 \\ 1.6250 \end{Bmatrix}$$

$$\underline{v}_2 = [D]\hat{\underline{v}}_1 = \frac{m}{k}\begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 3/2 \\ 1 & 3 & 5/2 \end{bmatrix}\begin{Bmatrix} 1 \\ 1.3750 \\ 1.6250 \end{Bmatrix} = \frac{m}{k}\begin{Bmatrix} 5.3750 \\ 7.5625 \\ 9{,}1875 \end{Bmatrix}$$

$$\hat{\underline{v}}_2 = \frac{\underline{v}_2}{(v_1)_2} = \left(\frac{1}{5.3750m\ /k}\right)\frac{m}{k}\begin{Bmatrix} 5.3750 \\ 7.5625 \\ 9{,}1875 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1.4070 \\ 1.7093 \end{Bmatrix}$$

etc. .....

Results from power method:

k	$(\mathbf{v_1})_\mathbf{k}$	$(\omega_1)_k = \left(1/\sqrt{(v_1)_k}\right)\sqrt{k/m}$
1	4.00000000	0.50000000
2	5.37500000	0.43133109
3	5.52325581	0.42550279
4	5.53789473	0.42494003
5	5.53953620	0.42487707
6	5.53973441	0.42486947
7	5.53975911	0.42486852
8	5.53976223	0.42486840
9	5.53976262	0.42486839
10	5.53976267	0.42486838
11	5.53976268	0.42486838

and

$$\hat{\underline{v}}_{11} = \begin{Bmatrix} 1 \\ 1.40974342 \\ 1.72027583 \end{Bmatrix}$$

This agrees well with the numerical results from Matlab.