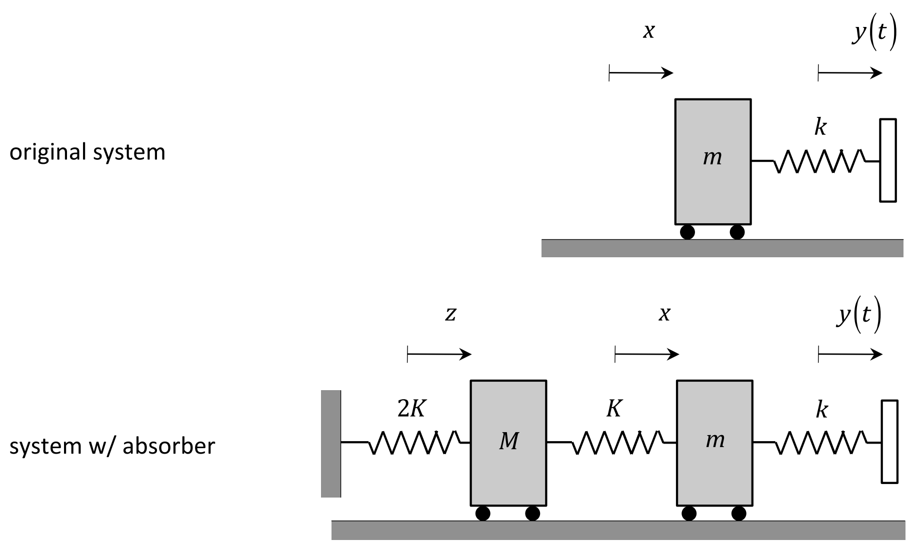
Example A5.2

Given: The single-DOF system shown below experiences a base excitation of $y(t) = y_0 sin\omega t$. For the particular application of this device, it turns out that the excitation frequency is unfortunately tuned exactly to the natural frequency of this system. It is desired to attach a vibration absorber to the original system, creating a two-DOF system with the constraint that the mass of the absorber is no larger than 20 percent of the original system mass; that is, $M \leq 0.2m$.

Find: For this problem:

a) Determine an appropriate set of values for K and M such that perfect vibration absorption is achieved while at the same time the separation between the two natural frequencies of the system is maximized. Write these values in terms of the stiffness k and mass m of the original system.

b) What is the separation between the pair of frequencies of the system with the absorber that you designed?



SOLUTION

For original system:

$$T = \frac{1}{2} m\dot{x}^2$$

$$U = \frac{1}{2} k(x - y)^2$$

From this, the EOM is:

$$m\ddot{x} + kx = ky(t) = ky_0 sin\omega t \quad \Rightarrow \quad \omega_n = \sqrt{k / m}$$

For system with absorber:

$$T = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} M\dot{z}^2$$

$$U = \frac{1}{2} k(x - y)^2 + \frac{1}{2} K(x - z)^2 + \frac{1}{2} (2K)z^2$$

Using the above in Lagrange's equations gives the following EOMs:

$$\begin{bmatrix} m & 0 \\ 0 & M \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} k+K & -K \\ -K & 3K \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} ky_0 \\ 0 \end{Bmatrix} sin\omega t = \vec{f}_0 sin\omega t$$

The particular solution of the above EOMs is written as: $\vec{x}_P(t) = \vec{X} sin\omega t$. Substituting into the above EOMs and solving for $\vec{X}$ gives:

$$[-\omega^2[M]+[K]]\vec{X} = \vec{f}_0 \quad \Rightarrow$$

$$\vec{X} = [-\omega^2[M]+[K]]^{-1}\vec{f}_0$$

$$= \begin{bmatrix} -m\omega^2+k+K & -K \\ -K & -M\omega^2+3K \end{bmatrix}^{-1} \begin{Bmatrix} ky_0 \\ 0 \end{Bmatrix}$$

$$= \frac{1}{\Delta}\begin{bmatrix} -M\omega^2+3K & K \\ K & -m\omega^2+k+K \end{bmatrix} \begin{Bmatrix} ky_0 \\ 0 \end{Bmatrix} = \frac{ky_0}{\Delta}\begin{Bmatrix} -M\omega^2+3K \\ K \end{Bmatrix}$$

where:

$$\Delta = det\begin{bmatrix} -m\omega^2+k+K & -K \\ -K & -M\omega^2+3K \end{bmatrix}$$

$$= (-m\omega^2+k+K)(-M\omega^2+3K) - K^2$$

$$= mM\omega^4 - \left[M(k+K)+3mK\right]\omega^2 + 3kK + 2K^2$$

For perfect vibration absorption at $\omega = \omega_n = \sqrt{k/m}$, we need to have:

$$X_1(\omega = \omega_n) = 0 = (-M\omega_n^2 + 3K)\frac{ky_0}{\Delta} \quad \Rightarrow \quad K = \frac{M\omega_n^2}{3} = \frac{M(k/m)}{3} = \frac{1}{3}\frac{M}{m}k$$

As we have seen from lecture, in order to maximize the natural frequency separation, we need to make M as large as possible within the other constraints of the problem. Therefore, we should choose:

$$M = 0.2m$$

giving:

$$K = \frac{1}{3}\left(\frac{0.2m}{m}\right)k = \frac{1}{15}k$$

Solving the characteristic equation for the system with absorber:

$$0 = m(0.2m)\omega^4 - \left[(0.2m)\left(k+\frac{k}{15}\right)+3m\left(\frac{k}{15}\right)\right]\omega^2 + 3k\left(\frac{k}{15}\right)+2\left(\frac{k}{15}\right)^2$$

$$= 0.2m^2\omega^4 - \frac{31}{75}mk\omega^2 + \frac{47}{225}k^2$$

$$= 0.2\mu^2 - \frac{31}{75}\mu + \frac{47}{225}$$

where $\mu = \omega\sqrt{m/k}$. Solving for the two roots gives:

$$\mu_{1,2} = \frac{31/75 \pm \sqrt{(31/75)^2 + (4)(0.2)(47/225)}}{(2)(0.2)} = 0.8806, 1.1861$$

Therefore,

$$\omega_{1,2} = \sqrt{\mu_{1,2}} = 0.9384\sqrt{\frac{k}{m}}, 1.0891\sqrt{\frac{k}{m}}$$

giving a frequency separation of:

$$\Delta\omega = \omega_2 - \omega_1 = 1.0891\sqrt{\frac{k}{m}} - 0.9384\sqrt{\frac{k}{m}} = 0.1507\sqrt{\frac{k}{m}}$$