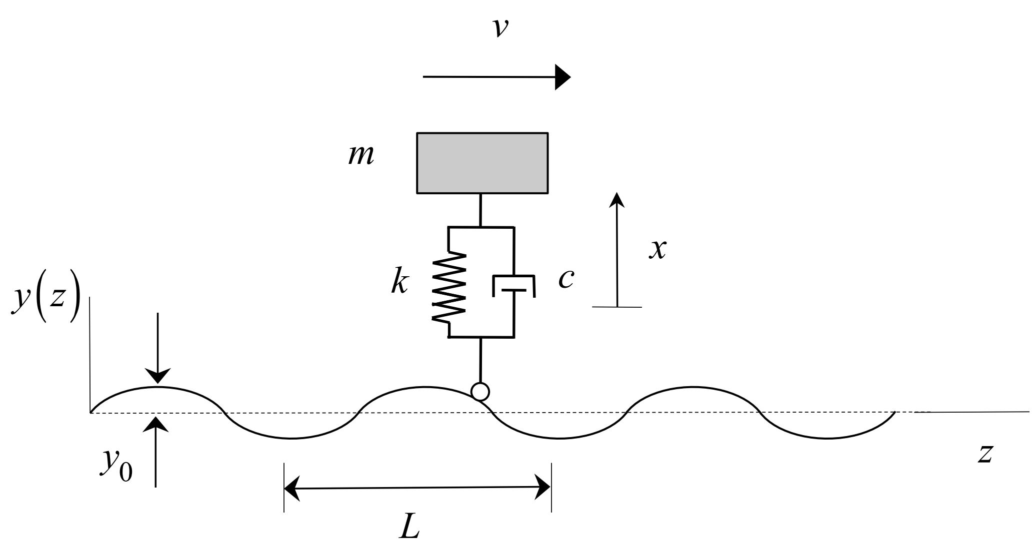
Example A5.1

Given: An automobile moves along a wavy roadway with a constant speed, where the surface of the roadway is idealized by $y(z) = y_0 sin 2\pi z/L$. The single-DOF model below is to be used to represent the vertical motion of the automobile body. Let the coordinate x represent the motion of the automobile body relative to the static equilibrium position.

Find: For given values of L, m, y_0 and v, you are asked to design the automobile suspension (that is, choose k and c) according to the criteria below:

- The amplitude of the vertical motion of the body is no larger than 4 percent of the road roughness amplitude y_0.
- The transmissibility of the vertical force to the suspension by the roadway is no larger than 1.2 when the automobile is traveling at a speed corresponding to the undamped natural frequency of the system.



SOLUTION

Using the following kinetic energy, potential energy and Rayleigh dissipation function:

$$T = \frac{1}{2} m(\dot{x} - \dot{y})^2$$

$$U = \frac{1}{2} k(x - y)^2$$

$$R = \frac{1}{2} c(\dot{x} - \dot{y})^2$$

with $y(t) = y_0 sin\Omega t$ and $\Omega = 2\pi v / L$ in Lagrange's equations gives:

$$\begin{aligned} m\ddot{x} + c\dot{x} + kx &= c\dot{y} + ky \\ &= c\Omega y_0 cos\Omega t + ky_0 sin\Omega t \\ &= f_0 sin(\Omega t + \psi) \end{aligned}$$

where:

$$
\begin{aligned}
f_0 &= y_0\sqrt{(c\Omega)^2 + k^2} \\
&= ky_0\sqrt{1+\left(\frac{c\Omega}{k}\right)^2} \\
&= ky_0\sqrt{1+\left(\frac{c}{\sqrt{km}}\frac{\Omega}{\sqrt{k/m}}\right)^2} \\
&= ky_0\sqrt{1+\left(2\zeta\frac{\Omega}{\omega_n}\right)^2}
\end{aligned}
$$

(The above can be used from the results from a previous homework set.)

As shown in lecture, the amplitude of steady-state response $X(\Omega)$ can be written in terms of the transmissibility function:

$$
\frac{X}{y_0} = T(\Omega) = \frac{\sqrt{1+\left(2\zeta\frac{\Omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\Omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\Omega}{\omega_n}\right]^2}}
$$

When the automobile speed corresponds to that of the undamped natural frequency ($\Omega = 2\pi v / L = \omega_n$), it is desired to have an amplitude ratio of $X / y_0 = 1.2$; that, is:

$$
1.2 = \frac{\sqrt{1+4\zeta^2}}{2\zeta} \quad \Rightarrow \quad 1.2^2 = \frac{1+4\zeta^2}{4\zeta^2} \quad \Rightarrow \quad \zeta = \sqrt{\frac{1}{1.2^2(4)-4}} = 0.754
$$

Also, we need to choose ω_n such that $T = 0.04$ for the design speed v (where $r = \Omega / \omega_n$):

$$
0.04 = \frac{\sqrt{1+4\zeta^2 r^2}}{\sqrt{\left[1-r^2\right]^2 + 4\zeta^2 r^2}} \quad \Rightarrow
$$

$$
0.04^2 = \frac{1+4\zeta^2 r^2}{r^4 + (4\zeta^2 - 2)r^2 + 1} \quad \Rightarrow
$$

$$
\begin{aligned}
& r^4 + (4\zeta^2 - 2)r^2 + 1 = \frac{1}{0.04^2}\left[1+4\zeta^2 r^2\right] \quad \Rightarrow \\
& r^4 - (2496\zeta^2 + 2)r^2 - 624 = 0 \quad \Rightarrow \\
& r^4 - 1421r^2 - 624 = 0 \quad \Rightarrow \\
& r^2 = \frac{1421 + \sqrt{1421^2 + (4)(624)}}{2} = 1421.4 \quad \Rightarrow \\
& r = \sqrt{1421.4} = 37.7 = \frac{2\pi v / L}{\omega_n} \quad \Rightarrow
\end{aligned}
$$

$$\omega_n = \frac{2\pi v / L}{37.7} = 0.053\frac{\pi v}{L} = \sqrt{\frac{k}{m}} \quad \Rightarrow$$

$$k = m\left(0.053\frac{\pi v}{L}\right)^2$$

Also,

$$2\zeta\omega_n = \frac{c}{m} \quad \Rightarrow \quad c = 2\zeta m\omega_n = 2(0.754)m\left(\frac{2\pi v / L}{37.7}\right) = 0.08\pi\frac{mv}{L}$$