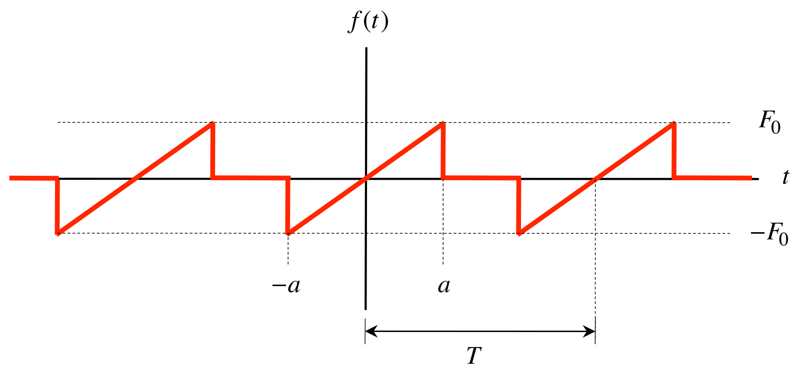
Example A4.16

Given: Consider the T-periodic function f(t) shown below.

Find: For this problem:

a) Determine the Fourier series of $f(t)$. Take advantage of any symmetry/anti-symmetry/zero mean value characteristics of $f(t)$ when finding the Fourier series coefficients.

b) Use Matlab, or equivalent application, to plot the truncated Fourier series found in a) above. Use a sufficiently large number of terms in the series when plotting to observe the convergence of this series.



SOLUTION

$$f(t) = f_0 + \sum_{k=1}^{\infty}\left[f_{ck}\cos k\Omega t + f_{sk}\sin k\Omega t\right]$$

with $\Omega = 2\pi / T$ where:

$$f_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt = f_{ave} = 0$$ (since the average value of f(t) is zero)

$$f_{ck} = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos k\Omega t\,dt = 0$$ (since f(t) is an ODD function in time)

$$f_{sk} = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin k\Omega t\,dt$$

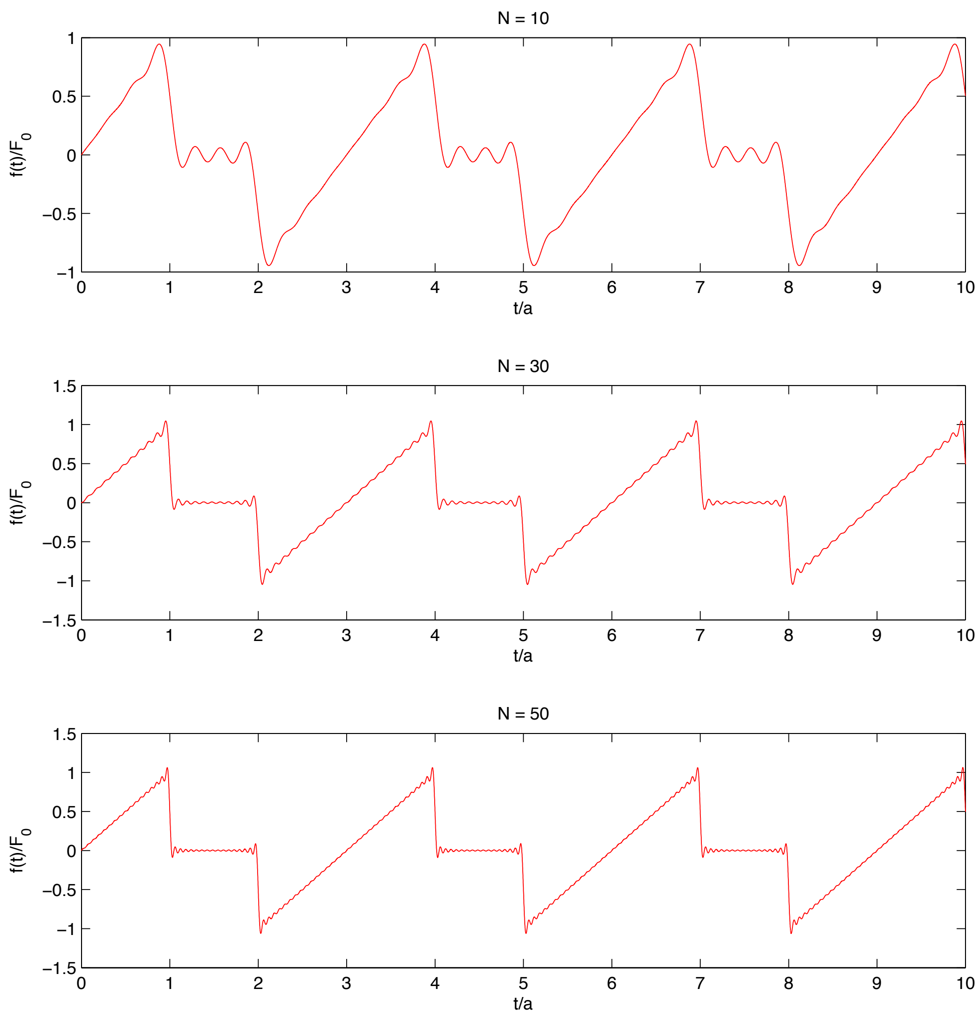
$$= \frac{4}{T}\int_{0}^{T/2} f(t)\sin k\Omega t\,dt \quad ; \quad f(t)\ \textit{is an ODD function}$$

$$= \frac{4F_0}{aT}\int_{0}^{a} t\sin k\Omega t\,dt$$

Using integration by parts gives:

$$
\begin{aligned}
f_{sk} &= \frac{4F_0}{aT}\left\{-\frac{t\cos k\Omega t}{k\Omega}\bigg|_0^a + \frac{1}{k\Omega}\int_0^a \cos k\Omega t\, dt\right\} \\
&= \frac{4F_0}{aT}\left\{-\frac{t\cos k\Omega t}{k\Omega}\bigg|_0^a + \frac{\sin k\Omega t}{k^2\Omega^2}\bigg|_0^a\right\} \\
&= \frac{4F_0}{aT}\left\{-\frac{a\cos k\Omega a}{k\Omega} + \frac{\sin k\Omega a}{k^2\Omega^2}\right\} \\
&= \frac{4F_0}{a(3a)}\left\{-\frac{a\cos k(2\pi/3a)a}{k(2\pi/3a)} + \frac{\sin k(2\pi/3a)a}{k^2(2\pi/3a)^2}\right\} \quad ; \quad T = 3a \ \ \& \ \ \Omega = 2\pi/T = 2\pi/3a \\
&= \frac{4F_0}{3a^2}\left\{-\frac{3a^2\cos(2k\pi/3)}{2k\pi} + \frac{9a^2\sin k(2k\pi/3)}{(2k\pi)^2}\right\} \\
&= 4F_0\left\{-\frac{\cos(2k\pi/3)}{2k\pi} + \frac{3\sin k(2k\pi/3)}{(2k\pi)^2}\right\}
\end{aligned}
$$

Plots of the truncated Fourier series are shown below for three values of N.


N = 10
f(t)/F_0
t/a
0
1
2
3
4
5
6
7
8
9
10
−1
−0.5
0.5
N = 30
N = 50
1.5
−1.5