Example A4.13

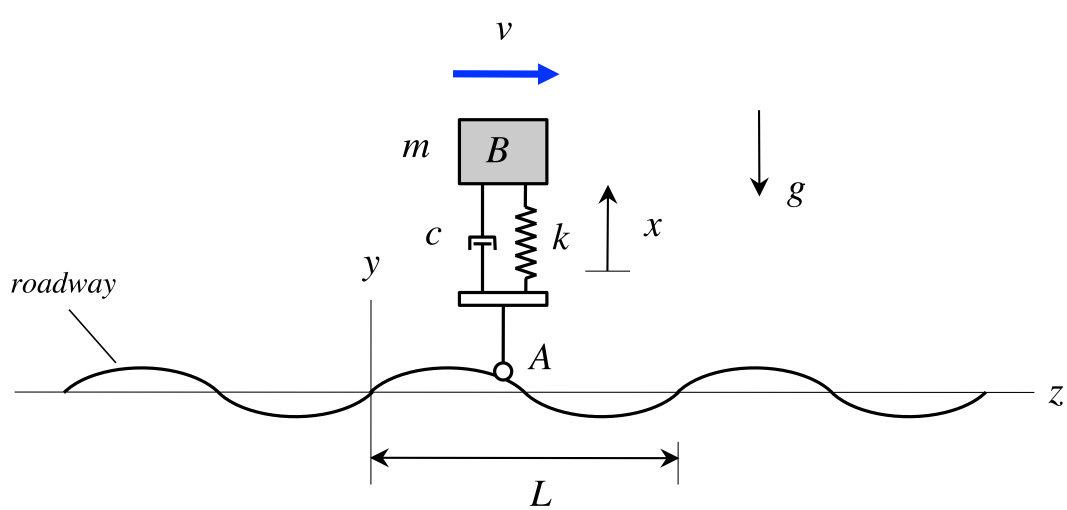
Given: Consider the single-DOF model of an automotive system shown below as the automobile moves along a wavy-surfaced roadway with a constant speed of v. The profile of the roadway is given by $y(z) = y_0 sin(\pi z/L)$. Let x represent the vertical motion of the automobile body B as measured from the stretched position of the body. Assume that the wheel A does not lose contact with the roadway surface as the system moves along the road.

Find: For this problem:

a) Derive the equation of motion (EOM) of the system in terms of the coordinate x. Transform this EOM to be in terms of the coordinate $r = x - mg/k$.

b) Write down the particular solution $r_P(t)$for your EOM. Make a hand sketch of the UNDAMPED response amplitude of $r_P(t)$ vs. the temporal excitation frequency. Scale the response amplitude by y_0 and the frequency by $\sqrt{k/m}$.

c) Let $F(t)$ represent the time-varying portion (i.e., excluding the influence of weight) of the force acting on A by the roadway. Derive an expression the particular solution $F_P(t)$. Make a hand sketch of the UNDAMPED response amplitude of $F_P(t)$ vs. the temporal excitation frequency. Scale the response amplitude by ky_0 and the frequency by $\sqrt{k/m}$.



SOLUTION

$$T = \frac{1}{2} m \dot{x}^2$$

$$U = \frac{1}{2} k (x - y)^2 - mgx$$

$$R = \frac{1}{2} c (\dot{x} - \dot{y})^2$$

with $z = vt$. Therefore, $y(t) = y_0 sin\left(\frac{\pi v}{L}t\right) = y_0 sin\Omega t$, where $\Omega = \pi v / L$.

Applying Lagrange's equations:

$$\begin{aligned} m\ddot{x} + c\dot{x} + kx &= mg + c\dot{y} + ky \\ &= mg + c\Omega y_0 cos\Omega t + ky_0 sin\Omega t \\ &= mg + ky_0\sqrt{1 + \left(2\zeta\frac{\Omega}{\omega_n}\right)^2} sin(\Omega t + \psi) \\ &= mg + f_0 sin(\Omega t + \psi) \end{aligned}$$

where $\zeta = c / 2\sqrt{km}$, $\omega_n = \sqrt{k/m}$ and:

$$f_0 = ky_0\sqrt{1 + \left(2\zeta\frac{\Omega}{\omega_n}\right)^2}$$

Let $r(t) = x(t) - mg/k$. With this, the EOM becomes:

$$m\ddot{r} + c\dot{r} + kr = f_0 sin(\Omega t + \psi)$$

Using results from lecture:

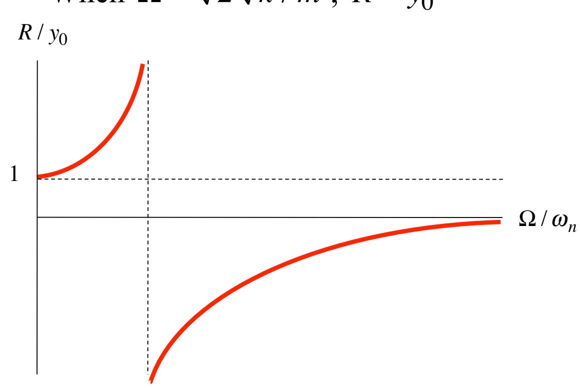
$$r_P(t) = R(\Omega) sin(\Omega t + \psi - \phi)$$

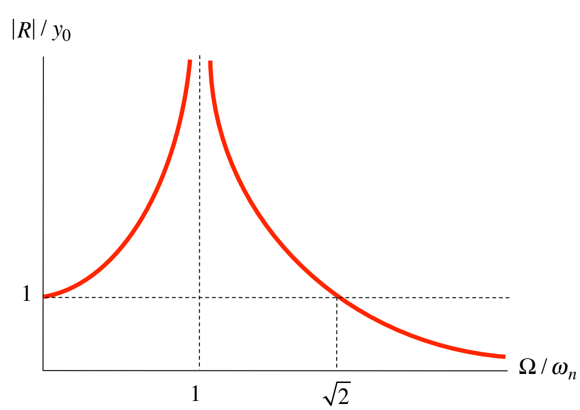
where:

$$R(\Omega) = \frac{f_0 / k}{\sqrt{\left[1 - \left(\frac{\Omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\Omega}{\omega_n}\right]^2}} = \frac{\sqrt{1 + \left(2\zeta\frac{\Omega}{\omega_n}\right)^2}}{\sqrt{\left[1 - \left(\frac{\Omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\Omega}{\omega_n}\right]^2}} y_0$$

For the hand sketch, you need to clearly identify the following:

- As $\Omega \to 0$, $R \to y_0$
- As $\Omega \to \infty$, $R \to 0$
- For $\Omega = \omega_n$, $R \to \infty$ for $\zeta = 0$
- When $\Omega = \sqrt{2}\sqrt{k/m}$, $R = y_0$





The oscillatory portion of the force on A is given by:

$$
\begin{aligned}
F_P(t) &= c\dot{r}_P(t) + r_P(t) \\
&= c\Omega R(\Omega)cos(\Omega t + \psi - \phi) + kR(\Omega)sin(\Omega t + \psi - \phi) \\
&= \sqrt{(c\Omega)^2 + k^2}\,R(\Omega)sin(\Omega t + \psi - \phi - \beta) \\
&= k\sqrt{1 + \left(2\zeta\frac{\Omega}{\omega_n}\right)^2}\,R(\Omega)sin(\Omega t + \psi - \phi - \beta) \\
&= ky_0\frac{1 + \left(2\zeta\frac{\Omega}{\omega_n}\right)^2}{\sqrt{\left[1 - \left(\frac{\Omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\Omega}{\omega_n}\right]^2}}\,sin(\Omega t + \psi - \phi - \beta) \\
&= |F_P|\,sin(\Omega t + \psi - \phi - \beta)
\end{aligned}
$$

Identify the following in your plots:

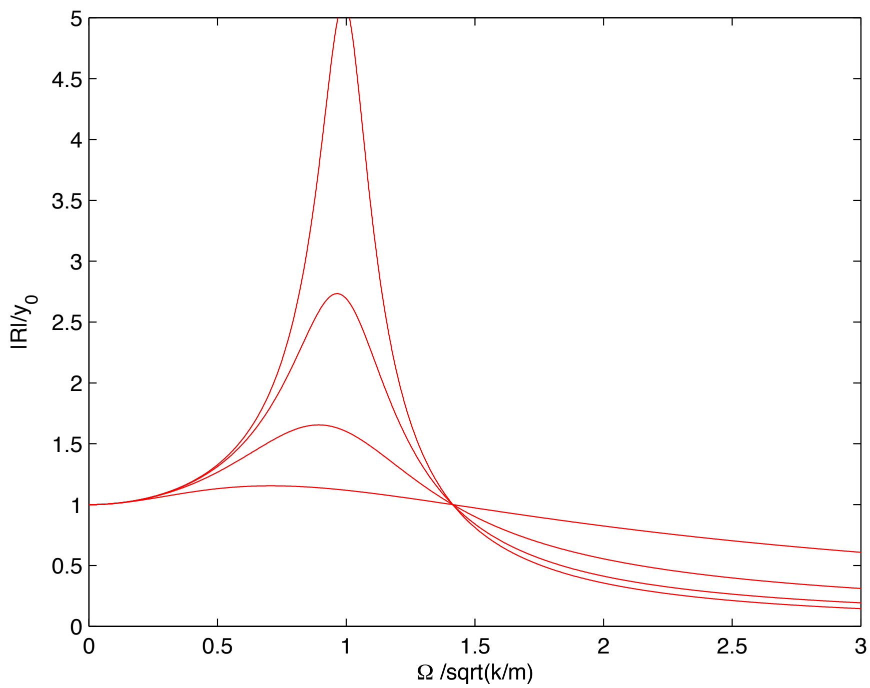
- As $\Omega \to 0$, $|F_P| \to ky_0$
- As $\Omega \to \infty$, $|F_P| \to 4\zeta^2 ky_0$
- For $\Omega = \omega_n$, $|F_P| \to \infty$ for $\zeta = 0$
- When $\Omega = \sqrt{2}\sqrt{k/m}$, $|F_P| = ky_0\sqrt{1 + 8\zeta^2}$

Matlab code:

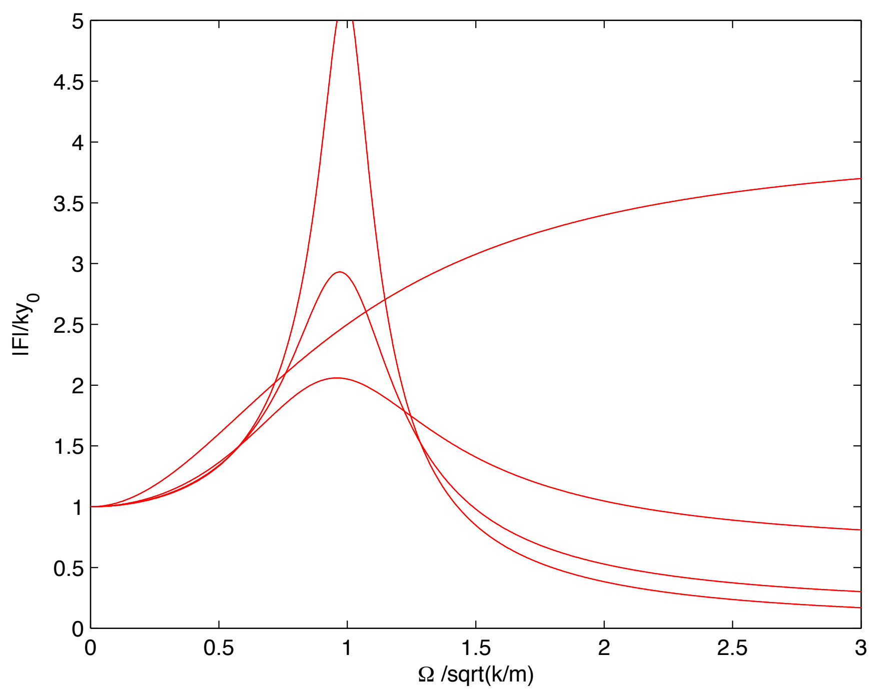
```
clear
W=linspace(0,5,2000);nW=length(W);
zz=[0.1,0.2,0.4,1];Nz=length(zz);
for jj=1:Nz
   R=sqrt((1+(2*zz(jj)*W).^2)./((1-W.^2).^2+(2*zz(jj)*W).^2));
   F=sqrt(1+(2*zz(jj)*W).^2).*R;

   figure(1)
   plot(W,R,'r'),hold on
   axis([0,3,0,5])
   xlabel('\Omega /sqrt(k/m)')
   ylabel('|R|/y_0')

   figure(2)
   plot(W,F,'r'),hold on
   axis([0,3,0,5])
   xlabel('\Omega /sqrt(k/m)')
   ylabel('|F|/ky_0')
end
figure(1), hold off
figure(2), hold off
```



5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
|R|/y_0
0
0.5
1
1.5
2
2.5
3
Ω /sqrt(k/m)



5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
|F|/ky_0
0
0.5
1
1.5
2
2.5
3
Ω /sqrt(k/m)