Example A2.16

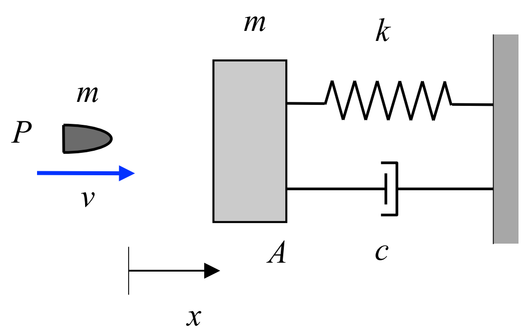
Given: When particle A is at rest and with the spring unstretched, a projectile P traveling with a speed of v impacts and immediately sticks to A.

Find: For this problem:

a) Determine the speed of A immediately after impact. (HINT: Use conservation of momentum for P and A together to determine this speed. Ignore the influence of the spring and dashpot on the motion of the block during impact.)

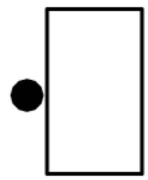
b) Using the coordinate x, determine the equation of motion for the system for times following the impact of P and A.

c) Determine the response found from the equation of motion in b) above. What is the maximum displacement of A during this response?



Use the following parameters: $v = 10$ m/sec, $m = 4$ kg, $k = 3200$ N/m and $c = 64$ kg/sec.

During impact:



$$\sum F_x = 0 \Rightarrow m v_{p1} + m \cancel{v_{A1}}^{\,0} = m v_{p2} + m v_{A2}$$

$$\hookrightarrow \quad mV = 2m V_{A2} \Rightarrow V_{A2} = \frac{1}{2} V = 5 \frac{m}{s}$$

After impact

$$\begin{cases} T = \frac{1}{2}(2m)\dot{x}^2 \\ U = \frac{1}{2} k x^2 \\ R = \frac{1}{2} c \dot{x}^2 \end{cases}$$

$$\hookrightarrow \quad 2m\ddot{x} + c\dot{x} + kx = 0$$

$$\div m : \quad \ddot{x} + \underbrace{\frac{c}{2m}}_{2\zeta\omega_n} \dot{x} + \underbrace{\frac{k}{2m}}_{\omega_n^2} x = 0$$

$$\therefore \; \omega_n = \sqrt{\frac{k}{2m}} = \sqrt{\frac{3200}{(2)(4)}} = 20 \, \frac{\text{rad}}{\text{sec}}$$

$$2\zeta\omega_n = \frac{c}{2m} \Rightarrow \zeta = \frac{c}{2\sqrt{km}} = \frac{64}{2\sqrt{(3200)(4)(2)}}$$

$$= \frac{1}{160} < 1 \Rightarrow \text{UNDERdamped}$$

Free response for $\zeta < 1$

$$x(t) = e^{-\zeta\omega_n t}\left[C\cos\omega_d t + S\sin\omega_d t\right]; \; \omega_d = \omega_n\sqrt{1-\zeta^2}$$

- $x(0) = 0 = C$
- $\dot{x}(t) = -\zeta\omega_n e^{-\zeta\omega_n t}\left[\cancelto{0}{C}\cos\omega_d t + S\sin\omega_d t\right]$

$$+ \; \omega_d e^{-\zeta\omega_n t}\left[-\cancelto{0}{C}\sin\omega_d t + S\cos\omega_d t\right]$$

$$\dot{x}(0) = \frac{V}{2} = \omega_d S \Rightarrow S = \frac{V}{2\omega_d}$$

$$\therefore \; x(t) = \frac{V}{2\omega_d} e^{-\zeta\omega_n t} \sin\omega_d t \quad \longleftarrow \; x(t)$$

$$\dot{x}(t) = \frac{V}{2\omega_d} e^{-\zeta\omega_n t}\left[-\zeta\omega_n \sin\omega_d t + \omega_d \cos\omega_d t\right]$$

To find X_{max}

$$\dot{x}(t) = 0 \Rightarrow \zeta\omega_n \sin\omega_d t = \omega_d \cos\omega_d t$$

$$\hookrightarrow \tan\omega_d t = \frac{\omega_d}{\zeta\omega_n} = \frac{\omega_n\sqrt{1-\zeta^2}}{\zeta\omega_n} = \frac{\sqrt{1-\zeta^2}}{\zeta}$$

$$\hookrightarrow t_{max} = \left(\frac{1}{\omega_n\sqrt{1-\zeta^2}}\right)\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

&

$$X_{max} = \frac{V}{2\omega_d} e^{-\zeta\omega_n t_{max}} \sin\omega_d t_{max} \quad \longleftarrow \; X_{max}$$