Example A1.12

Given: The absolute coordinates y_1, y_2 and y_3 are used to describe the motion of A, B and the center of mass G of the homogeneous wheel.

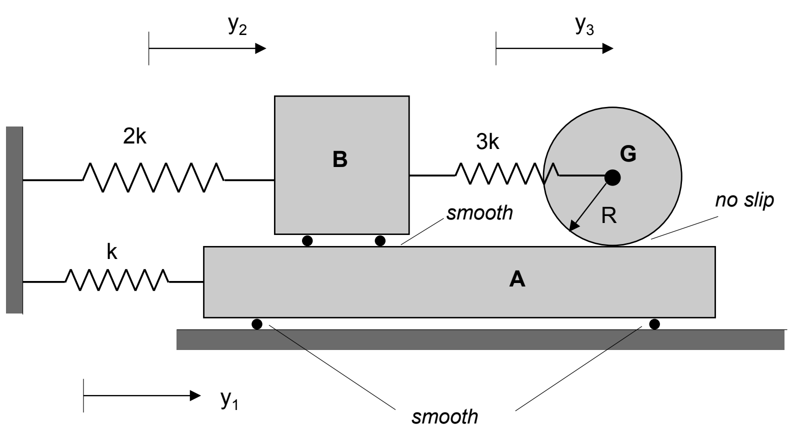
Find: For this problem:

a) Write down the potential energy function U for the system. Use the following equation to find the stiffness matrix for the system:

$$K_{ij} = \left[\frac{\partial^2 U}{\partial q_i \partial q_j} \right]_{q_0}$$

b) Find the flexibility matrix [A] using the influence coefficient method.

c) Using $[K]$ and $[A]$ from above, verify that $[A][K] = [I]$.

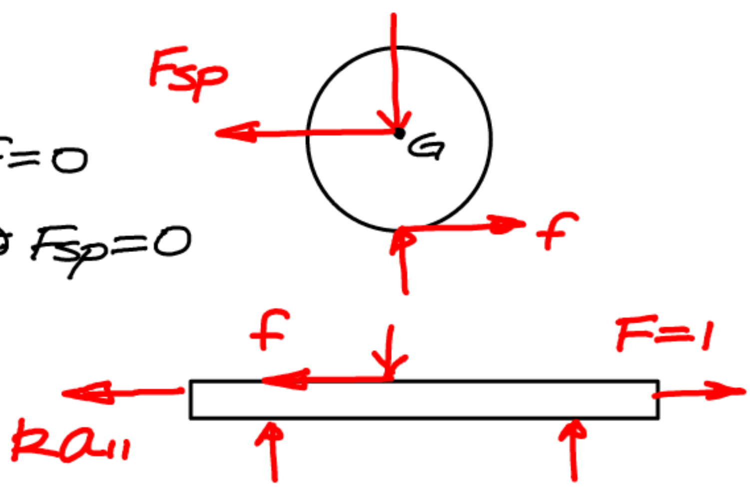


(a)

$$U = \frac{1}{2} k y_1^2 + \frac{1}{2}(2k) y_2^2 + \frac{1}{2}(3k)(y_3 - y_2)^2$$

$$[K] = \left[\frac{\partial^2 U}{\partial y_i \partial y_j}\right]_0 = \begin{bmatrix} k & 0 & 0 \\ 0 & 5k & -3k \\ 0 & -3k & 3k \end{bmatrix}$$

(b) Apply unit force to A:



Wheel: $\sum M_G = fR = 0 \Rightarrow f = 0$

$\sum F_x = f - F_{sp} = 0 \Rightarrow F_{sp} = 0$

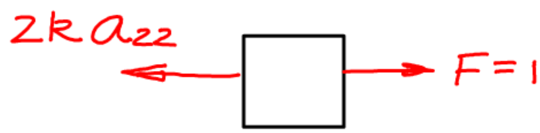
Since no net force on B or wheel:

$$a_{21} = a_{31} = 0$$

Block A: $\sum F_x = F - k a_{11} = 0 \Rightarrow a_{11} = \frac{1}{k}$

By reciprocity: $a_{12} = a_{13} = 0$

- Apply unit force on B:

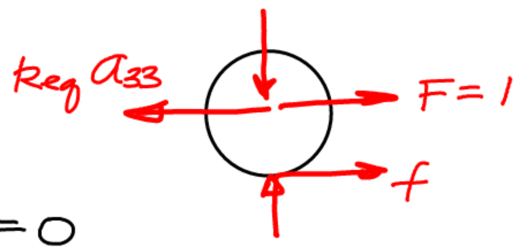


$$\sum F_x = -2k a_{22} + F = 0 \Rightarrow a_{22} = \frac{1}{2k}$$

Also $a_{32} = a_{22} = \frac{1}{2k}$

By reciprocity: $a_{23} = a_{32} = \frac{1}{2k}$

- Apply unit force to wheel



$$\sum M_G = fR = 0 \Rightarrow f = 0$$

$$\sum F_x = -k_{eq} a_{33} + F = 0$$

$\hookrightarrow \quad a_{33} = \frac{1}{k_{eq}}$

w/ $\frac{1}{k_{eq}} = \frac{1}{2k} + \frac{1}{3k} = \frac{3+2}{(2k)(3k)} = \frac{5}{6k}$

$\therefore \quad a_{33} = \frac{5}{6k}$

$$\therefore \quad [a] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{5}{6} \end{bmatrix} \frac{1}{k}$$

Check:

$$[a][K] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{5}{6} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & -3 \\ 0 & -3 & 3 \end{bmatrix} \frac{k}{k}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \overset{\checkmark}{=} [I]$$