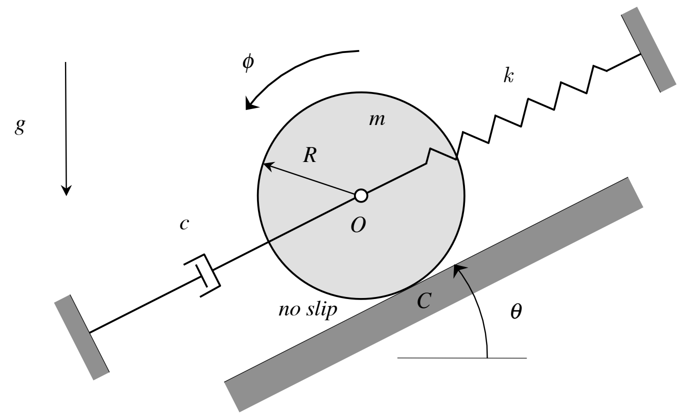
**Example A1.7**

**Given:** A homogeneous disk of mass $m$ and outer radius $R$ is able to roll without slipping on an inclined ramp. A spring (of stiffness $k$) and a dashpot (of damping constant $c$) connect the center of the disk O to ground. The spring is unstretched when $\phi = 0$.

**Find:** Use Lagrange‘s equations to derive the EOM for this single-DOF system in terms of the generalized coordinate $\phi$.



***SOLUTION***

$$T = \frac{1}{2}mv_O^2 + \frac{1}{2}I_O\dot{\phi}^2 \overset{OR}{=} \frac{1}{2}I_C\dot{\phi}^2 \quad \text{with } I_O = \frac{1}{2}mR^2 \text{ and } I_C = I_O + mR^2 = \frac{3}{2}mR^2$$

$$U = \frac{1}{2}k(R\phi)^2 - mg(R\phi)sin\theta$$

$$R = \frac{1}{2}c(R\dot{\phi})^2$$

Applying Lagrange's equations:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}}\right) = \frac{d}{dt}\left[\frac{3}{2}mR^2\dot{\phi}\right] = \frac{3}{2}mR^2\ddot{\phi}$$

$$\frac{\partial T}{\partial \dot{\phi}} = 0$$

$$\frac{\partial R}{\partial \dot{\phi}} = cR^2\dot{\phi}$$

$$\frac{\partial U}{\partial \phi} = kR^2\phi - mgRsin\theta$$

Therefore:

$$\frac{3}{2}mR^2\ddot{\phi} + cR^2\dot{\phi} + kR^2\phi = mgRsin\theta$$