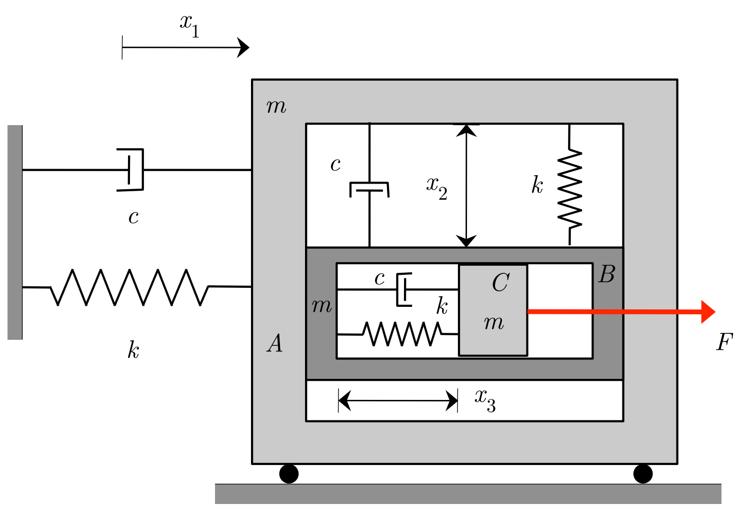
Example A1.6

Given: Consider the three-degree-of-freedom system shown below made up of three particles A, B and C, each of mass m with the system moving within a horizontal plane. Let x_1 describe the absolute motion of particle A, x_2 describe the motion of particle B relative to A and x_3 describe the motion of particle C relative to B. All springs are unstretched when $x_1 = x_2 = x_3 = 0$. Assume all surfaces to be smooth.

Find: For this problem:

a) Determine the generalized mass coefficients, m_{ij}, for the system corresponding to the generalized coordinates x_1, x_2 and x_3.

b) Determine the generalized force, Q_i, for the system corresponding to the generalized coordinates x_1, x_2 and x_3.



Body A: $\vec{r}_1 = x_1\hat{i} \quad \Rightarrow \quad d\vec{r}_1 = dx_1\hat{i} \quad \Rightarrow \quad \dot{\vec{r}}_1 = \dot{x}_1\hat{i}$

Body B: $\vec{r}_2 = x_1\hat{i} + x_2\hat{j} \quad \Rightarrow \quad d\vec{r}_2 = dx_1\hat{i} + dx_2\hat{j} \quad \Rightarrow \quad \dot{\vec{r}}_2 = \dot{x}_1\hat{i} + \dot{x}_2\hat{j}$

Body C: $\vec{r}_3 = \left(x_1 + x_3\right)\hat{i} + x_2\hat{j} \quad \Rightarrow \quad d\vec{r}_2 = \left(dx_1 + dx_3\right)\hat{i} + dx_2\hat{j} \quad \Rightarrow \quad \dot{\vec{r}}_2 = \left(\dot{x}_1 + \dot{x}_3\right)\hat{i} + \dot{x}_2\hat{j}$

From this, we can write:

$$T = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}m\left(\dot{x}_1^2 + \dot{x}_2^2\right) + \frac{1}{2}m\left[\left(\dot{x}_1 + \dot{x}_3\right)^2 + \dot{x}_2^2\right]$$

$$= \frac{1}{2}(3m)\dot{x}_1^2 + \frac{1}{2}(2m)\dot{x}_2^2 + \frac{1}{2}(m)\dot{x}_3^2 + \frac{1}{2}(2m)\dot{x}_1\dot{x}_3$$

From this, we have:

$m_{11} = 3m$

$m_{22} = 2m$

$m_{33} = m$

$m_{12} = m_{21} = m_{23} = m_{32} = 0$

$m_{13} = m_{31} = m$

Therefore,

$$[m] = \begin{bmatrix} 3m & 0 & m \\ 0 & 2m & 0 \\ m & 0 & m \end{bmatrix}$$

Also, we have:

$$dW = \vec{F} \bullet d\vec{r}_3 = \left(F\hat{i}\right) \bullet d\vec{r}_3 = F\left(dx_1 + dx_3\right)$$

Therefore:

$Q_1 = Q_3 = F$

$Q_2 = 0$