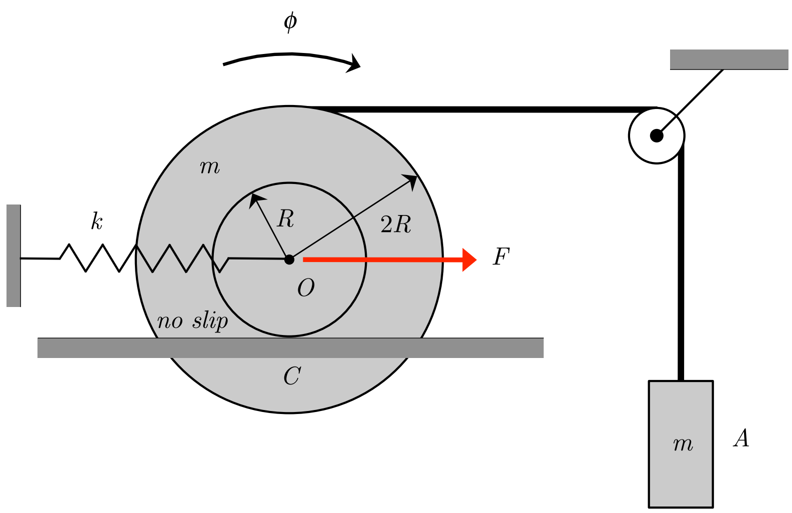
Example A1.5

Given: The stepped-spool below has a mass of m and amass moment of inertia of I_O about point O. Let ϕ represent the angle of rotation of the disk with the spring being unstretched when $\phi = 0$.

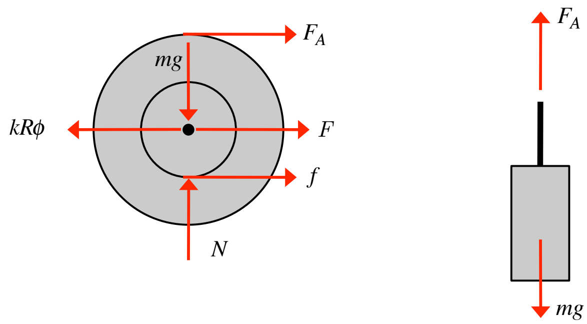
Find: For this problem:

a) Using the Newton-Euler formulation, determine the equation of motion for the system in terms of the coordinate ϕ. Draw the free body diagrams of the drum and block individually before writing down the Newton-Euler equations.

b) Write the equations on motion derived in b) in matrix form. Identify the mass, damping and stiffness matrices in these equations



Free body diagrams:



<u>Drum</u>: $\sum M_C = F(R) + F_A(3R) - kR\phi(R) = I_C\ddot{\phi}$ (1)

<u>Block</u>: $\sum F_y = F_A - mg = m\ddot{y}_A \;\Rightarrow\; F_A = m(g + \ddot{y}_A)$ (2)

where $I_C = I_O + mR^2$. From kinematics:

$$\ddot{y}_A = -3R\ddot{\phi} \quad (3)$$

Combining equations (1)-(3):

$$F(R) + m(g - 3R\ddot{\phi})(3R) - kR\phi(R) = I_C\ddot{\phi} \;\Rightarrow$$

$$\boxed{(I_O + 10mR^2)\ddot{\phi} + kR^2\phi = 3mgR + FR}$$

Using the power equation, we first write down the kinetic and potential energy:

$$T = \frac{1}{2}I_C\dot{\phi}^2 + \frac{1}{2}m\dot{y}_A^2 = \frac{1}{2}I_C\dot{\phi}^2 + \frac{1}{2}m(3R\dot{\phi})^2 = \frac{1}{2}(I_O + 10mR^2)\dot{\phi}^2$$

$$U = \frac{1}{2}k(R\phi)^2 - mg(3R\phi)$$

and the expression for differential work:

$$dW = \vec{F} \bullet d\vec{r}_O = (F\hat{i}) \bullet (Rd\phi\hat{i}) = FR\,d\phi \;\Rightarrow\; \frac{dW}{dt} = FR\dot{\phi}$$

Using the power equation:

$$\frac{dT}{dt} + \frac{dU}{dt} = \frac{dW}{dt} \;\Rightarrow\; (I_O + 10mR^2)\ddot{\phi}\dot{\phi} + kR^2\phi\dot{\phi} - 3mgR\dot{\phi} = FR\dot{\phi} \;\Rightarrow$$

$$\left[(I_O + 10mR^2)\ddot{\phi} + kR^2\phi - 3mgR - FR\right]\dot{\phi} = 0 \;\Rightarrow$$

$$\boxed{(I_O + 10mR^2)\ddot{\phi} + kR^2\phi = 3mgR + FR}$$